Introduction to RooFit

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7 Constructing joint models

- *Using discrete variable to classify data*
- *Simultaneous fits on multiple datasets*

Datasets and discrete observables

- Discrete observables play an important role in management of datasets
  - Useful to classify 'sub datasets' inside datasets
  - Can collapse multiple, logically separate datasets into a single dataset by adding them and labeling the source with a discrete observable
  - Allows to express operations such a simultaneous fits as operation on a single dataset

Dataset A
x
5.0
3.7
1.2
4.3

Dataset B
x
5.0
3.7
1.2

Dataset A+B	
x	**source**
5.0	A
3.7	A
1.2	A
4.3	A
5.0	B
3.7	B
1.2	B

Discrete variables in RooFit – RooCategory

- Properties of RooCategory variables
  - Finite set of named states → self documenting
  - Optional integer code associated with each state

```
// Define a cat. with explicitly numbered states
w.factory("b0flav[B0=-1,B0bar=1]") ;

// Define a category with labels only
w.factory("tagCat[Lepton,Kaon,NT1,NT2]") ;
w.factory("sample[CPV,BMixing]") ;
```

- Used for classification of data, or to describe occasional discrete fundamental observable (e.g. B^0 flavor)

Datasets and discrete observables – part 2

- Example of constructing a joint dataset from 2 inputs

Dataset A
x
5.0
3.7
1.2
4.3

Dataset B
x
5.0
3.7
1.2

Dataset A+B	
x	**source**
5.0	A
3.7	A
1.2	A
4.3	A
5.0	B
3.7	B
1.2	B

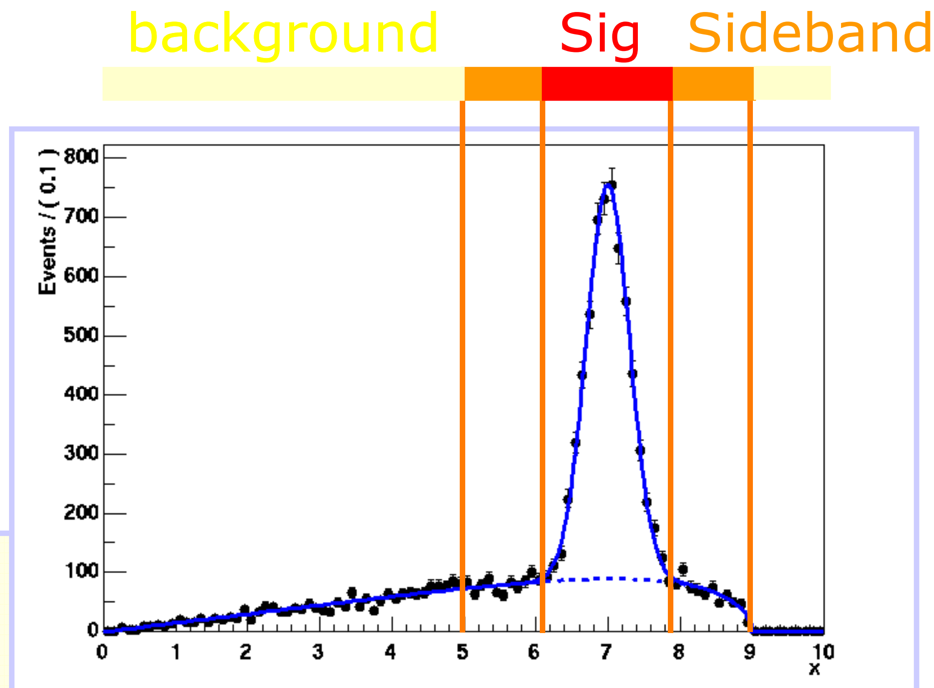
Wouter Verkerk

```
RooDataSet simdata("simdata","simdata",x,source,
                   Import("A",*dataA),Import("B",*dataB)) ;
```

- But can also derive classification from info within dataset
  - E.g. (10<x<20 = "signal", 0<x<10 | 20<x<30 = "sideband")
  - Encode classification using real→discrete mapping functions

A universal real→discrete mapping function

- Class **RooThresholdCategory** maps ranges of input **RooRealVar** to states of a **RooCategory**




```
// Mass variable
RooRealVar m(“m”,”mass,0,10.);

// Define threshold category
RooThresholdCategory region(“region”,”Region of M”,m,”Background”);
region.addThreshold(9.0, “SideBand”) ;
region.addThreshold(7.9, “Signal”) ;
region.addThreshold(6.1,”SideBand”) ;
region.addThreshold(5.0,”Background”) ;
```

Default state

Define region boundaries

Discrete multiplication function

- **`RooSuperCategory/RooMultiCategory`** provides category multiplication

```
// Define 'product' of tagCat and runBlock
RooSuperCategory prod("prod","prod",RooArgSet(tag,flav))
```

flav
B0
B0bar

tag
Lepton
Kaon
NT1
NT2

X

prod	
{B0;Lepton}	{B0bar;Lepton}
{B0;Kaon}	{B0bar;Kaon}
{B0;NT1}	{B0bar;NT1}
{B0;NT2}	{B0bar;NT2}

Discrete→Discrete mapping function

- **`RooMappedCategory`** provides cat → cat mapping

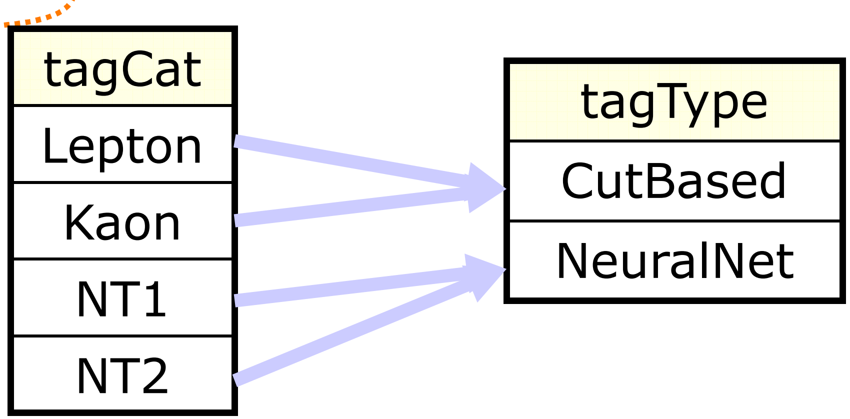
Define input category

Create mapped category

Add mapping rules

```
RooCategory tagCat("tagCat","Tagging category") ;
tagCat.defineType("Lepton") ;
tagCat.defineType("Kaon") ;
tagCat.defineType("NetTagger-1") ;
tagCat.defineType("NetTagger-2") ;

RooMappedCategory tagType(“tagType”,”type”,tagCat) ;

tagType.map(“Lepton”,”CutBased”) ;
tagType.map(“Kaon”,”CutBased”) ;
tagType.map(“NT*”,”NeuralNet”) ;
```

Wildcard expressions allowed



Exploring discrete data

- Like real variables of a dataset can be plotted, discrete variables can be tabulated

Tabulate contents of dataset by category state

```
RooTable* table=data->table(b0flav) ;
table->Print() ;

Table b0flav : aData
  +-------+------+
  |    B0 | 4949 |
  | B0bar | 5051 |
  +-------+------+
```

Extract contents by label

```
Double_t nB0 = table->get("B0") ;
```

Extract contents fraction by label

```
Double_t b0Frac = table->getFrac("B0");
```

Tabulate contents of selected part of dataset

```
data->table(tagCat,"x>8.23")->Print() ;

  Table tagCat : aData(x>8.23)
  +-------------+-----+
  |      Lepton | 668 |
  |        Kaon | 717 |
  | NetTagger-1 | 632 |
  | NetTagger-2 | 616 |
  +-------------+-----+
```

Exploring discrete data

- *Discrete functions*, built from categories in a dataset can be tabulated likewise

Tabulate **`RooSuperCategory`** states

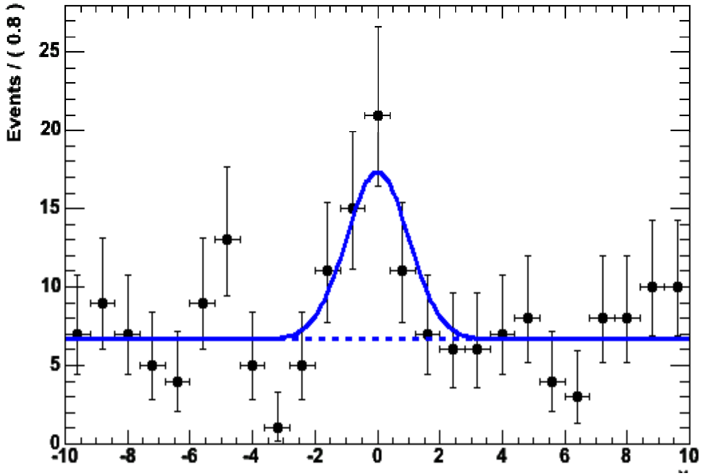
```
data->table(b0Xtcat)->Print() ;

  Table b0Xtcat : aData
  +----------------------+-------+
  |          {B0;Lepton} | 1226 |
  |       {B0bar;Lepton} | 1306 |
  |            {B0;Kaon} | 1287 |
  |         {B0bar;Kaon} | 1270 |
  |     {B0;NetTagger-1} | 1213 |
  |  {B0bar;NetTagger-1} | 1261 |
  |     {B0;NetTagger-2} | 1223 |
  |  {B0bar;NetTagger-2} | 1214 |
  +----------------------+-------+
```

Tabulate **`RooMappedCategory`** states

```
data->table(tcatType)->Print() ;

  Table tcatType : aData
  +-----------------+-------+
  |         Unknown |    0 |
  |       Cut based | 5089 |
  |  Neural Network | 4911 |
  +-----------------+-------+
```

Fitting multiple datasets simultaneously

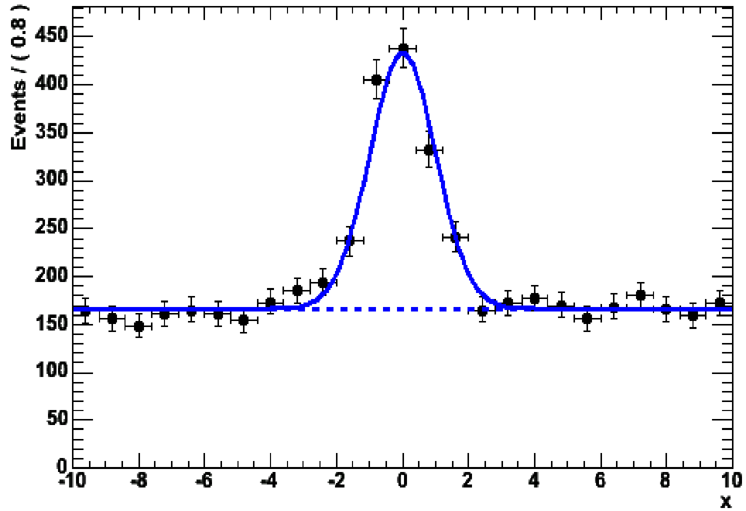
- Simultaneous fitting efficient solution to incorporate information from control sample into signal sample
- Example problem: search rare decay
  - Signal dataset has small number entries.



  - Statistical uncertainty on shape in fit contributes significantly to uncertainty on fitted number of signal events
  - However can constrain shape of signal from control sample (e.g. another decay with similar properties that is not rare), so no need to relay on simulations

Fitting multiple datasets simultaneously

- Fit to control sample yields accurate information on shape of signal



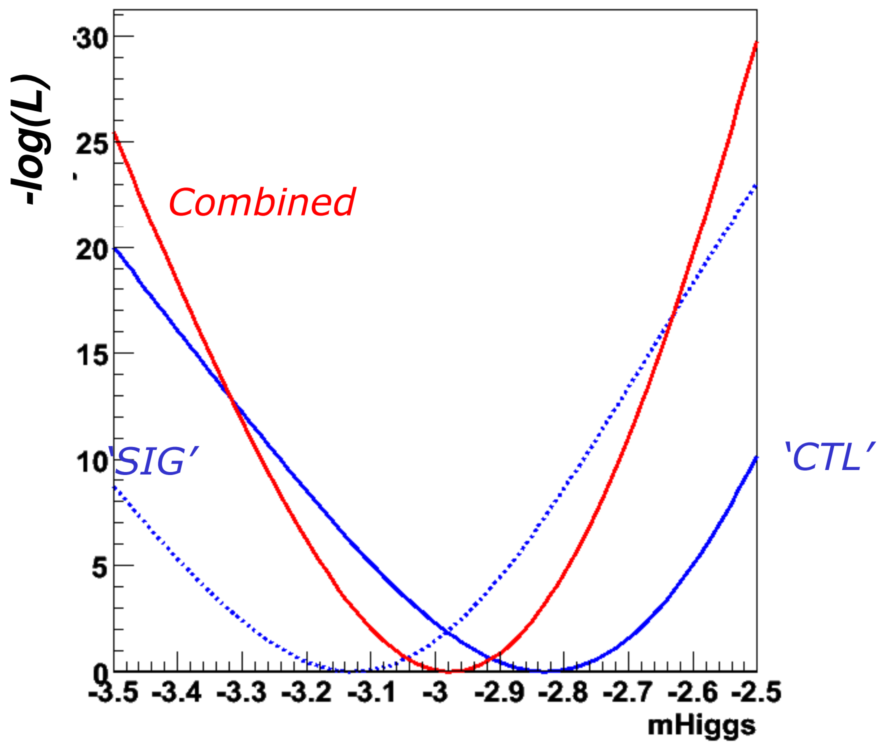
- Q: What is the most practical way to combine shape measurement on control sample to measurement of signal on physics sample of interest

- A: Perform a simultaneous fit
  - Automatic propagation of errors & correlations
  - Combined measurement
    (i.e. error will reflect contributions from both physics sample and control sample

Discrete observable as data subset classifier

- Likelihood level definition of a simultaneous fit

$$-\log(L) = \sum_{i=1,n} -\log(PDF_A(D_A^i)) + \sum_{i=1,m} -\log(PDF_B(D_B^i))$$



- Minimize -logL(a,b,c)= -logL(a,b)+ -logL(b,c)
  - Errors, correlations on common par. b automatically propagated

Discrete observable as data subset classifier

- Likelihood level definition of a simultaneous fit

$$-\log(L) = \sum_{i=1,n} -\log(PDF_A(D_A^i)) + \sum_{i=1,m} -\log(PDF_B(D_B^i))$$

- PDF level definition of a simultaneous fit

$$-\log(L) = \sum_{i=1,n} -\log(simPDF(D_{A+B}^i))$$

RooSimultaneous implements 'switch' PDF:

```
case (indexCat) {
  A: return pdfA ;
  B: return pdfB ;
}
```

Likelihood of switchPdf with composite dataset *automatically* constructs sum of likelihoods above

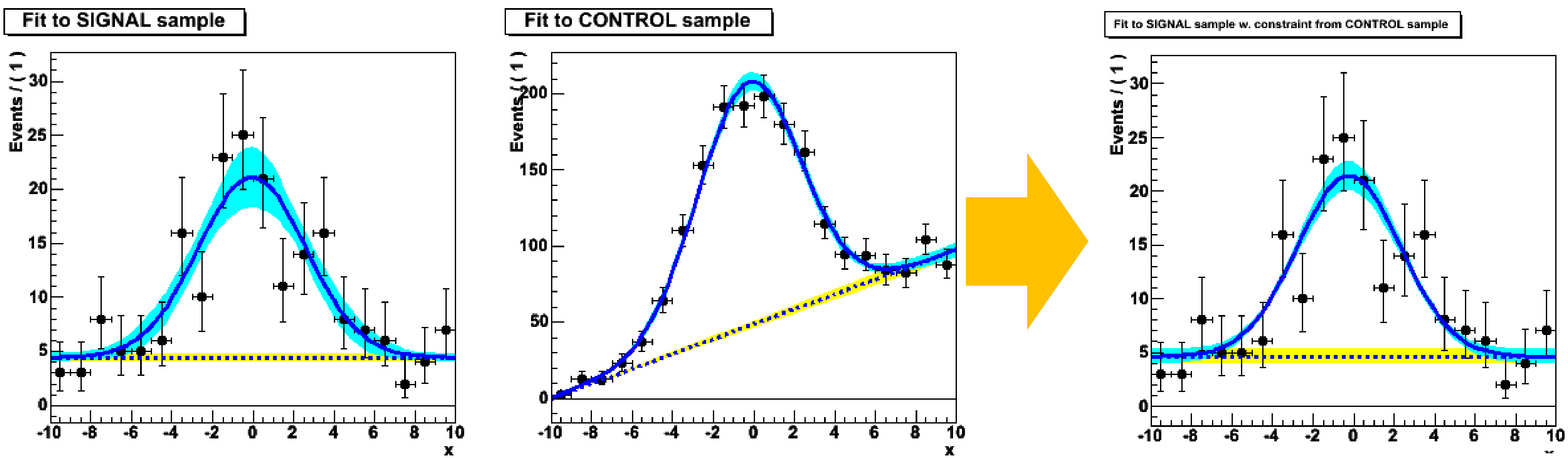
Dataset A+B	
x	**source**
5.0	A
3.7	A
1.2	A
4.3	A
5.0	B
3.7	B
1.2	B

Practical fitting – Simultaneous fit technique

- given data $D_{sig}(x)$ and model $F_{sig}(x;a,\boldsymbol{b})$ and
  data $D_{ctl}(x)$ and model $F_{ctl}(x;\boldsymbol{b},c)$

  – Construct $-\log[L_{sig}(a,\boldsymbol{b})]$ and $-\log[L_{ctl}(\boldsymbol{b},c)]$ and

•$D_{sig}(x)$, $F_{sig}(x;a,b)$ •$D_{ctl}(x)$, $F_{ctl}(x;b,c)$



Constructing joint pdfs

- Operator class SIMUL to construct joint models at the pdf level

```
// Pdfs for channels 'A' and 'B'
w.factory("Gaussian::pdfA(x[-10,10],mean[-10,10],sigma[3])") ;
w.factory("Uniform::pdfB(x)") ;

// Create discrete observable to label channels
w.factory("index[A,B]") ;

// Create joint pdf
w.factory("SIMUL::joint(index,A=pdfA,B=pdfB)") ;
```

- Can also construct joint datasets

```
RooDataSet *dataA, *dataB ;
RooDataSet dataAB("dataAB","dataAB",Index(w::index),
                  Import("A",*dataA),Import("B",*dataB)) ;
```

Building simultaneous fits in RooFit

- Code that construct example shown 2 slides back

```
// Signal pdf
w.factory("Gaussian::sig(x[-10,10],mean[0,-10,10],sigma[3,2,4])") ;
w.factory("Uniform::bkg(x)") ;
w.factory("SUM::model(Nsig[800,0,1000]*sig,Nbkg[0,1000]*bkg)") ;

// Background pdf
w.factory("Gaussian::sig_control(x[-10,10],mean[0,-10,10],sigma[3,2,4])") ;
w.factory("Chebychev::bkg_control(x,a0[1])") ;
w.factory("SUM::model_control(Nsig_control[500,0,10000]*sig_control,
                              Nbkg_control[500,0,10000]*bkg_control)") ;

// Joint pdf construction
w.factory("SIMUL::model_sim(index[sig,control],
                            sig=model, control=model_control)") ;

// Joint data construction
RooDataSet simdata("simdata","simdata",w::x,Index(w::index),
                   Import("sig",*data),Import("control",*data_control)) ;

// Joint fit
RooFitResult* rs = w::model_sim.fitTo(simdata,Save()) ;
```

Other scenarios in which simultaneous fits are useful

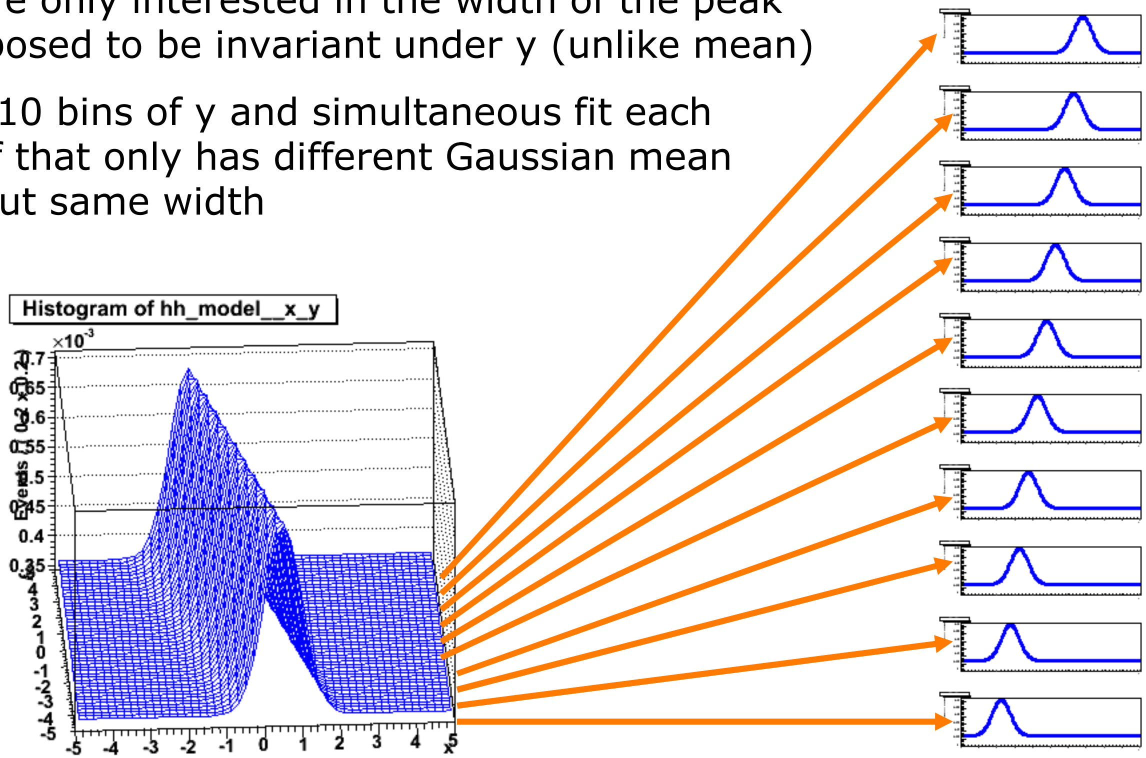
- Preceding example was 'asymmetric'
  - Very large control sample, small signal sample
  - Physics in each channel possibly different (but with some similar properties

- There are also 'symmetric' use cases
  - Fit multiple data sets that are functionally equivalent, but have slightly different properties (e.g. purity)
  - Example: Split B physics data in block separated by flavor tagging technique (each technique results in a different sensitivity to CP physics parameters of interest).
  - Split data in block by data taking run, mass resolutions in each run may be slightly different
  - For symmetric use cases pdf-level definition of simultaneous fit very convenient as you usually start with a single dataset with subclassing formation derived from its observables

- By splitting data into subsamples with p.d.f.s that can be tuned to describe the (slightly) varying properties you can increase the statistical sensitivity of your measurement

A more empirical approach to simultaneous fits

- Instead of investing a lot of time in developing multi-dimensional models → Split data in many subsamples, fit all subsamples simultaneously to slight variations of 'master' p.d.f
- Example: Given dataset D(x,y) where observable of interest is x.
  - Distribution of x varies slightly with y
  - Suppose we're only interested in the width of the peak which is supposed to be invariant under y (unlike mean)
  - Slice data in 10 bins of y and simultaneous fit each bin with p.d.f that only has different Gaussian mean parameter, but same width



A more empirical approach to simultaneous fits

- Fit to sample of preceding page would look like this
  - Each mean is fitted to expected value (-4.5 + ibin)

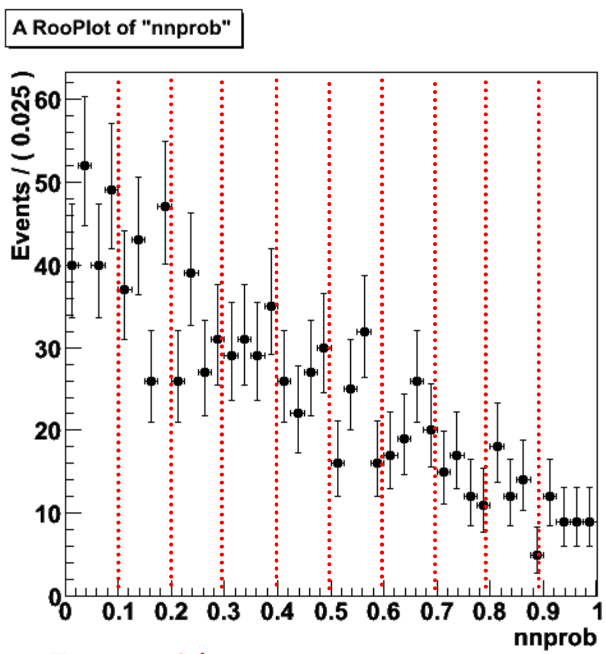
Floating Parameter	FinalValue +/-	Error
mean_bin1	-4.5302e+00 +/-	1.62e-02
mean_bin2	-3.4928e+00 +/-	1.38e-02
mean_bin3	-2.4790e+00 +/-	1.35e-02
mean_bin4	-1.4174e+00 +/-	9.64e-03
mean_bin5	-4.8945e-01 +/-	7.95e-03
mean_bin6	4.0716e-01 +/-	9.67e-03
mean_bin7	1.4733e+00 +/-	1.37e-02
mean_bin8	2.4912e+00 +/-	1.44e-02
mean_bin9	3.5028e+00 +/-	1.41e-02
mean_bin10	4.5474e+00 +/-	1.68e-02
sigma	2.7319e-01 +/-	2.46e-03

  - But joint measurement of sigma
  - NB: Correlation matrix is mostly diagonal as all mean_binXX parameters are completely uncorrelated!

A more empirical approach to simultaneous fits

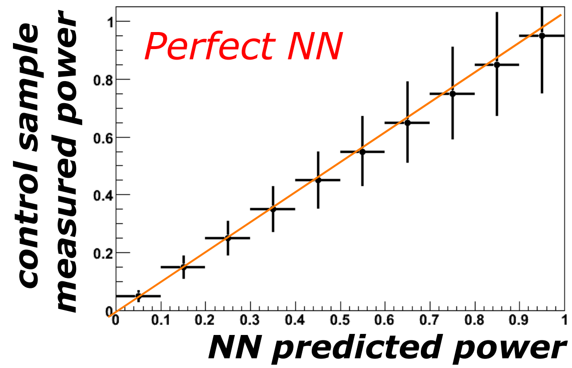
- Preceding example was simplistic for illustrational clarity, but more sensible use cases exist
  - Example: **Measurement CP violation in B decay**. Analyzing power of each event is diluted by factor (1-2w) where w is the mistake rate of the flavor tagging algorithm
  - Neural net flavor tagging algorithm provides a tagging probability for each event in data. Could use prob(NN) as w, but then we rely on good calibration of NN, don't want that
  - In a simultaneous fit to CPV+Mixing samples, can measure *average* w from the latter. Now not relying on NN calibration, but not exploiting event-by-event variation in analysis power.
  - Improved scenario: divide (CPV+mixing) data in 10 or 20 subsets corresponding to bins in prob(NN). Use identical p.d.f but only have separate parameter to express fitted mistag rate w_binXX.
  - Simultaneous fit will now exploit difference in analyzing power of events and be insensitive to calibration of flavor tagging NN.
  - If calibration of NN was OK fitting mistag rate in each bin of probNN will be average probNN value for that bin

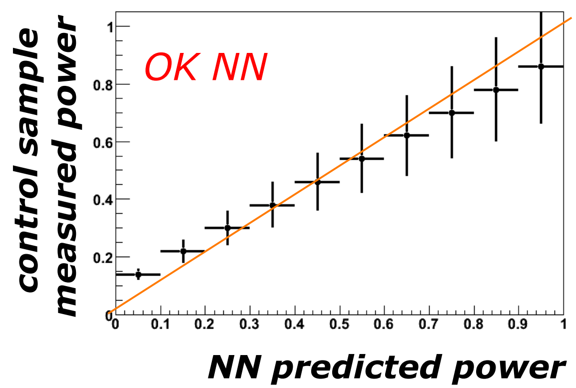
A more empirical approach to simultaneous fits

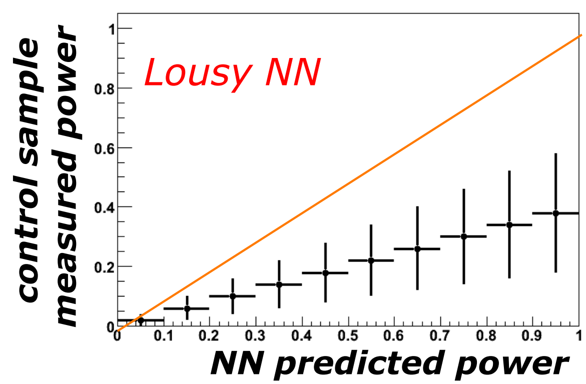


Event with little analyzing power

Event with great analyzing power







Better precision on CPV meas. because more sensitive events in sample

In all 3 cases fit not biased by NN calibration

Worse precision on CPV meas. because less sensitive events in sample

Building simultaneous fits from a template

- In the 'symmetric' use case the models assigned to each state are very similar in structure – Usually just one parameter name is different
- Easiest way to construct these from a template pdf and a prescription on how to tailor the template for each index state
- Use operator SIMCLONE instead of SIMUL

```
// Template pdf - B0 decay with mixing
w.factory("TruthModel::tm(t[-20,20])") ;
w.factory("BMixDecay::sig(t,mixState[mixed=-1,unmixed=1],
                          tagFlav[B0=1,B0bar=-1], tau[1.54,1,2],
                           dm[0.472,0.1,0.8],w[0.1,0,0.5],dw[0],tm)") ;

// Construct index category
w.factory("tag[Lep,Kao,NT1,NT2]") ;

// Construct simultaneous pdf with separate mistag rate for each category
w.factory("SIMCLONE::model(sig,$SplitParam({w,dw},tagCat)") ;
```

Building simultaneous fits from a template

- Result

```
RooWorkspace(w) w contents

variables
---------
(dm,dw,dw_Kao,dw_Lep,dw_NT1,dw_NT2,mixState,t,tagCat,tagFlav,tau,w,w_Kao,w_Lep,w_NT1,w_NT2)

p.d.f.s
-------
RooBMixDecay::sig[ mistag=w delMistag=dw mixState=mixState tagFlav=tagFlav tau=tau dm=dm t=t ] = 0.2
RooSimultaneous::model[ indexCat=tagCat Lep=sig_Lep Kao=sig_Kao NT1=sig_NT1 NT2=sig_NT2 ] = 0.2
  RooBMixDecay::sig_Kao[ mistag=w_Kao delMistag=dw_Kao ... t=t ] = 0.2
  RooBMixDecay::sig_Lep[ mistag=w_Lep delMistag=dw_Lep ... t=t ] = 0.2
  RooBMixDecay::sig_NT1[ mistag=w_NT1 delMistag=dw_NT1 ... t=t ] = 0.2
  RooBMixDecay::sig_NT2[ mistag=w_NT2 delMistag=dw_NT2 ... t=t ] = 0.2

analytical resolution models
----------------------------
RooTruthModel::tm[ x=t ] = 1
```

8 Working with Likelihood

- *Using discrete variable to classify data*
- *Simultaneous fits on multiple datasets*

Fitting and likelihood minimization

- What happens when you do **`pdf->fitTo(*data)`**
  - 1) Construct object representing –log of (extended) likelihood
  - 2) Minimize likelihood w.r.t floating parameters using MINUIT

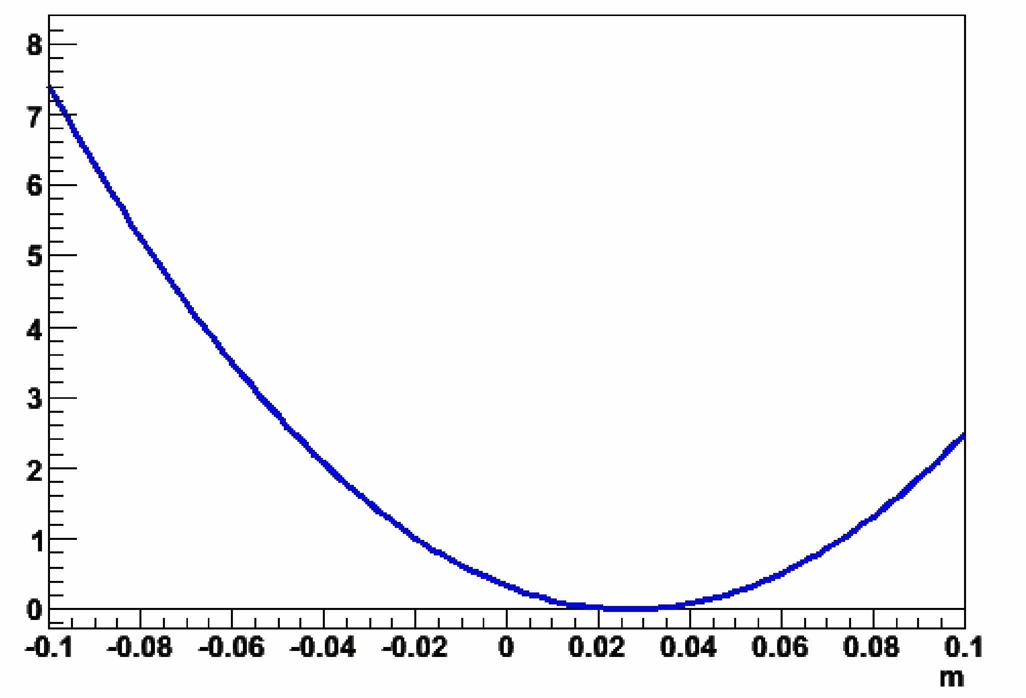
- Can also do these two steps explicitly by hand

```
// Construct function object representing –log(L)
RooAbsReal* nll = pdf.createNLL(data) ;

// Minimize nll w.r.t its parameters
RooMinuit m(*nll) ;
m.migrad() ;
m.hesse() ;
```

Plotting the likelihood

- A likelihood function is a regular RooFit function
- Can e.g. plot is as usual

```
RooAbsReal* nll = w::model.createNLL(data) ;

RooPlot* frame = w::param.frame() ;
nll->plotOn(frame,ShiftToZero()) ;
```



Constructing a χ^2 function

- Along similar lines it is also possible to construct a χ^2 function
  - Only takes binned datasets (class **RooDataHist**)
  - Normalized p.d.f is multiplied by Ndata to obtain χ^2

```
// Construct function object representing –log(L)
RooAbsReal* chi2 = pdf.createChi2(data) ;

// Minimize nll w.r.t its parameters
RooMinuit m(chi2) ;
m.migrad() ;
m.hesse() ;
```

  - MINUIT error definition for χ^2 automatically adjusted to 1 (it is 0.5 for likelihoods) as default error level is supplied through virtual method of function base class **RooAbsReal**

Automatic optimizations in the calculation of the likelihood

- Several automatic computational optimizations are applied the calculation of likelihoods inside RooNLLVar
  - Components that have all constant parameters are pre-calculated
  - Dataset variables not used by the PDF are dropped
  - PDF normalization integrals are only recalculated when the ranges of their observables or the value of their parameters are changed
  - Simultaneous fits: When a parameters changes only parts of the total likelihood that depend on that parameter are recalculated
    - Lazy evaluation: calculation only done when intergal value is requested
- Applicability of optimization techniques is re-evaluated for each use
  - Maximum benefit for each use case
- ‘Typical’ large-scale fits see significant speed increase
  - Factor of 3x – 10x not uncommon.

Features of class RooMinuit

- Class RooMinuit is an *interface* to the ROOT implementation of the MINUIT minimization and error analysis package.

- RooMinuit takes care of
  - Passing value of miminized RooFit function to MINUIT
  - Propagated changes in parameters both from **`RooRealVar`** to MINUIT and back from MINUIT to **`RooRealVar`**, i.e. it keeps the state of RooFit objects synchronous with the MINUIT internal state
  - Propagate error analysis information back to **`RooRealVar`** parameters objects
  - Exposing high-level MINUIT operations to RooFit uses (MIGRAD,HESSE,MINOS) etc…
  - Making optional snapshots of complete MINUIT information (e.g. convergence state, full error matrix etc)

Demonstration of RooMinuit use

```
// Start Minuit session on above nll
RooMinuit m(nll) ;

// MIGRAD likelihood minimization
m.migrad() ;

// Run HESSE error analysis
m.hesse() ;

// Set sx to 3, keep fixed in fit
sx.setVal(3) ;
sx.setConstant(kTRUE) ;

// MIGRAD likelihood minimization
m.migrad() ;

// Run MINOS error analysis
m.minos()

// Draw 1,2,3 'sigma' contours in sx,sy
m.contour(sx,sy) ;
```

What happens if there are problems in the NLL calculation

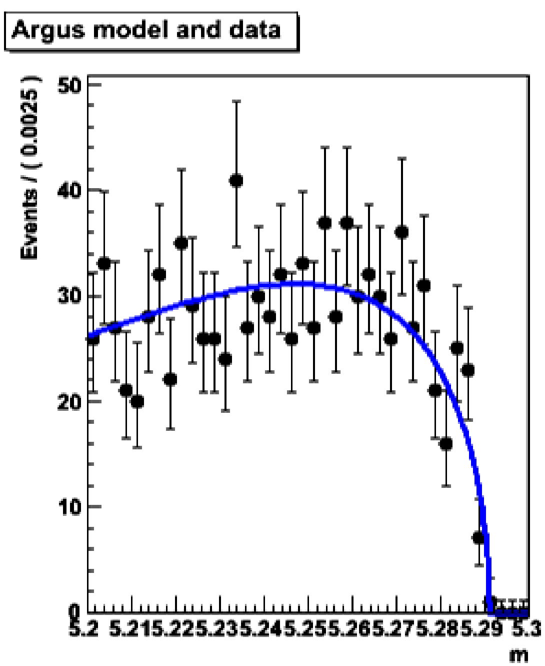
- Sometimes the likelihood cannot be evaluated do due an error condition.
  - PDF Probability is zero, or less than zero at coordinate where there is a data point 'infinitely improbable'
  - Normalization integral of PDF evaluates to zero
- Most problematic during MINUIT operations. How to handle error condition
  - All error conditions are gather and reported in consolidated way by RooMinuit
  - Since MINUIT has no interface deal with such situations, RooMinuit passes instead a large value to MINUIT to force it to retreat from the region of parameter space in which the problem occurred

```
[#0] WARNING:Minization -- RooFitGlue: Minimized function has error status.
Returning maximum FCN so far (99876) to force MIGRAD to back out of this region.
Error log follows. Parameter values: m=-7.397
RooGaussian::gx[ x=x mean=m sigma=sx ] has 3 errors
```

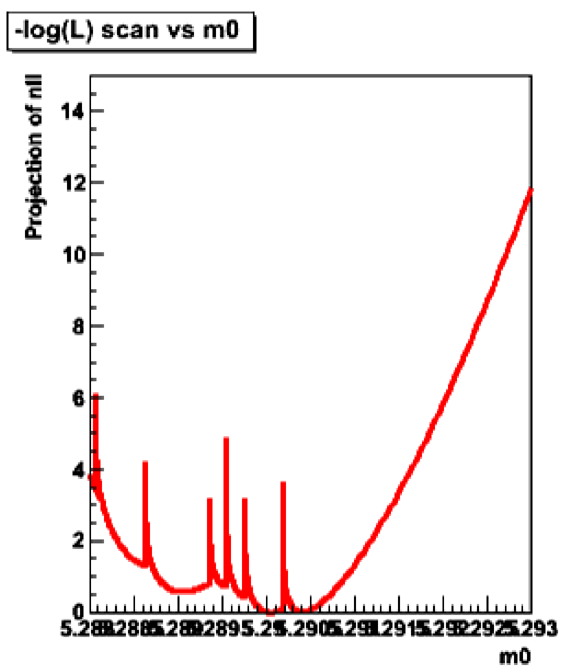
What happens if there are problems in the NLL calculation

- Classic example in B physics: floating the end point of the ARGUS function
  - Probability density of ARGUS above end point is zero → If end point is moved to low value in fit you end up with events above end point → Probility is zero → Likelihood is –log(0) = infinity

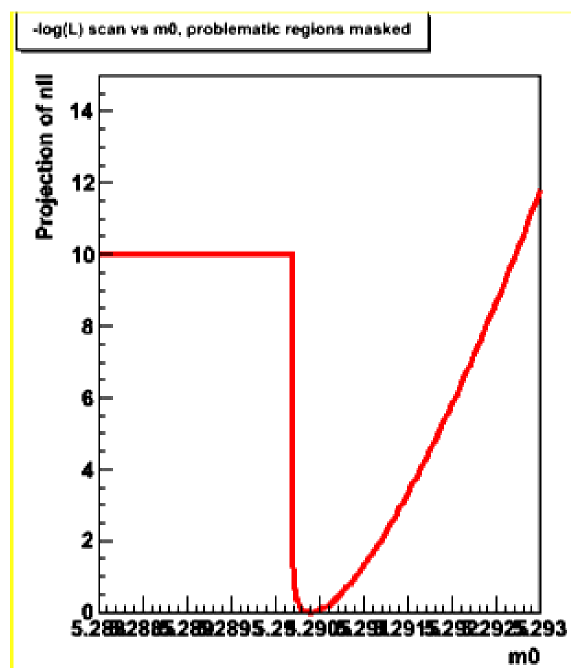
pdf and data



-log(L) vs m0
dropping problematic events



-log(L) vs m0
with 'wall' (RooFit default)



What happens if there are problems in the NLL calculation

- Can request more verbose error logging to debug problem
  - Add PrintEvalError(N) with N>1

```
[#0] WARNING:Minization -- RooFitGlue: Minimized function has error status.
Returning maximum FCN so far (-1e+30) to force MIGRAD to back out of this region.
Error log follows
Parameter values: m=-7.397
RooGaussian::gx[ x=x mean=m sigma=sx ]
     getLogVal() top-level p.d.f evaluates to zero or negative number
                 @ x=x=9.09989, mean=m=-7.39713, sigma=sx=0.1
     getLogVal() top-level p.d.f evaluates to zero or negative number
                 @ x=x=6.04652, mean=m=-7.39713, sigma=sx=0.1
     getLogVal() top-level p.d.f evaluates to zero or negative number
                 @ x=x=2.48563, mean=m=-7.39713, sigma=sx=0.1
```

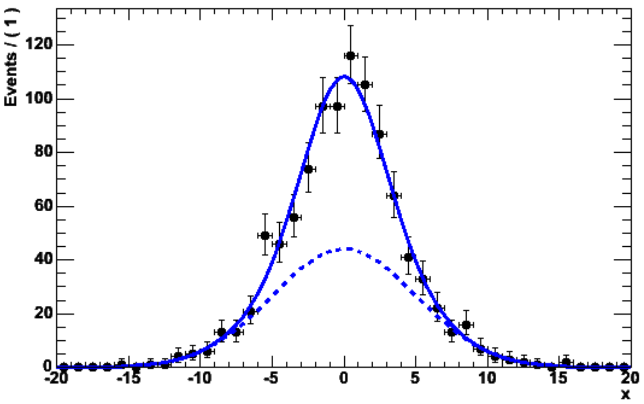
Working with profile likelihood

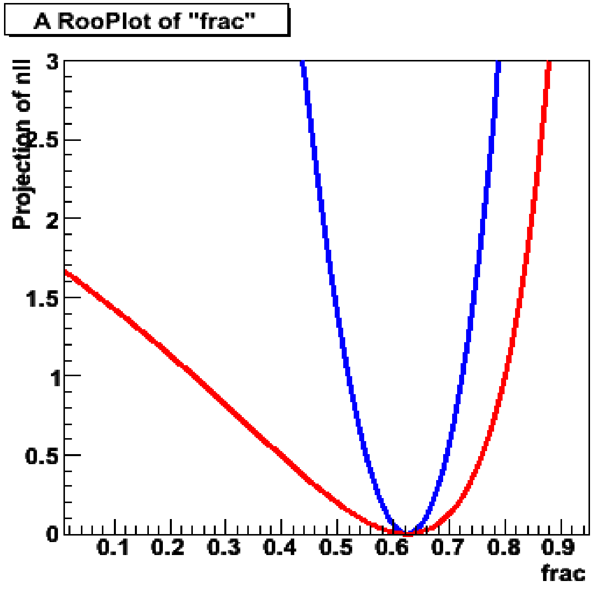
- A profile likelihood ratio $$\lambda(p) = \frac{L(p,\hat{\hat{q}})}{L(\hat{p},\hat{q})}$$ ← **Best L for given p** ← **Best L**

  can be represent by a regular RooFit function (albeit an expensive one to evaluate)

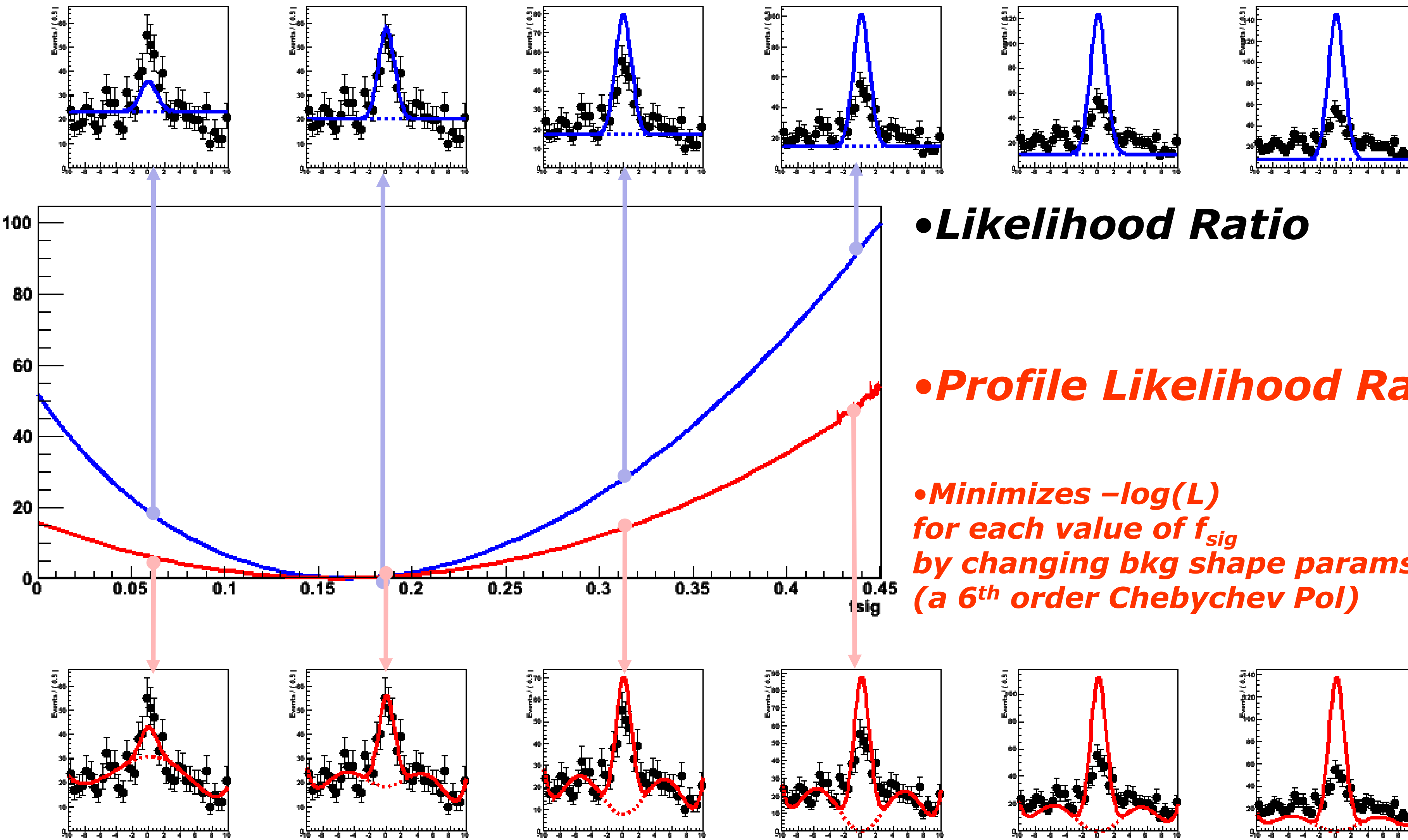
```
RooAbsReal* ll = model.createNLL(data,NumCPU(8)) ;
RooAbsReal* pll = ll->createProfile(params) ;
```

```
RooPlot* frame = w::frac.frame() ;
nll->plotOn(frame,ShiftToZero()) ;
pll->plotOn(frame,LineColor(kRed)) ;
```





Dealing with nuisance parameters in Likelihood ratio intervals

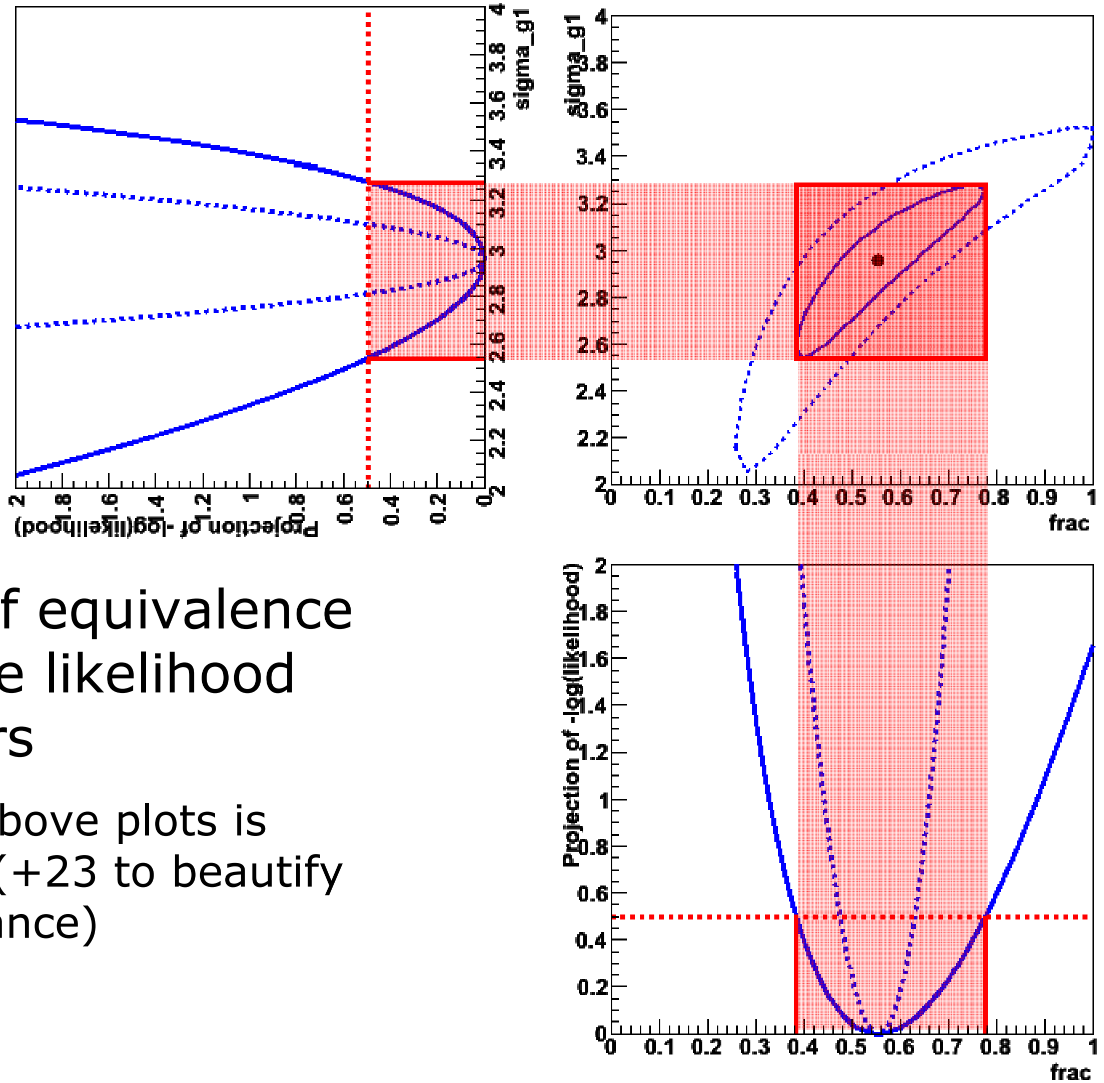


•Likelihood Ratio

•Profile Likelihood Ratio

•Minimizes −log(L) for each value of f_{sig} by changing bkg shape params (a 6th order Chebychev Pol)

On the equivalence of profile likelihood and MINOS



- Demonstration of equivalence of (RooFit) profile likelihood and MINOS errors
  - Macro to make above plots is 34 lines of code (+23 to beautify graphics appearance)

Constructing joint likelihood

- When you have a simultaneous pdf you can create a joint likelihood from the joint pdf

```
RooAbsReal* nllJoint = w::joint.createNLL(dataAB) ;
```

- Also possible to make likelihood functions of the components first and then add them

```
RooAbsReal* nllA = w::A.createNLL(*dataA) ; w.import(nllA) ;
RooAbsReal* nllB = w::B.createNLL(*dataB) ; w.import(nllB) ;
w.factory(sum::nllJoint(nllA,nllB)) ;
```

- Likelihood constructed either way is the same.
- Minimization of joint likelihood == Joint fit

Adding parameter pdfs to the likelihood

- Systematic/external uncertainties can be modeled with regular RooFit pdf objects.
- To incorporate in likelihood, simply multiply with orig pdf

```
w.factory("Gaussian::g(x[-10,10],mean[-10,10],sigma[3])") ;

w.factory("PROD::gprime(f,Gaussian(mean,1.15,0.30))") ;
```

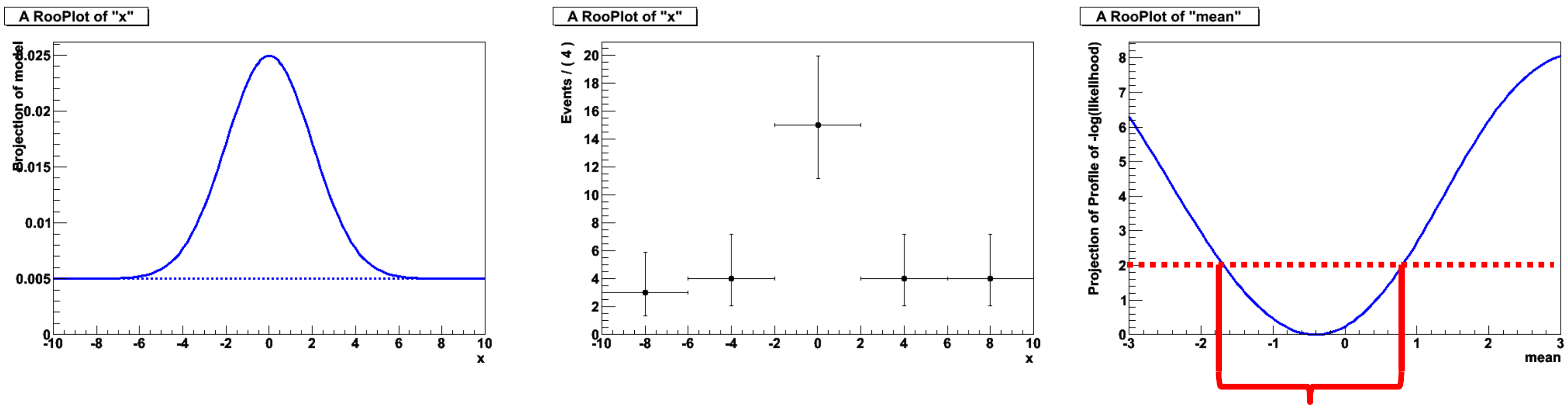
$$-\log L(\mu,\sigma) = -\sum_{data} -\log(f(x_i;\mu,\sigma) - \log(Gauss(\mu,1.15,0.30))$$

  - Any pdf can be supplied, e.g. Gaussian most common, but an also use class RooMultiVarGaussian to introduce a Gaussian uncertainty on multiple parameteres including a correlation

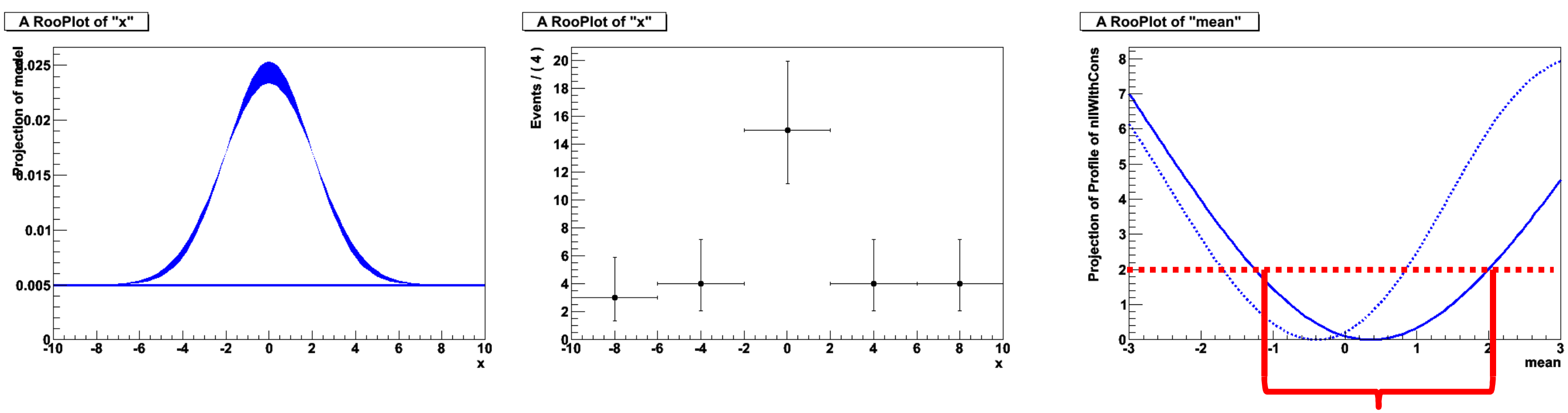
- Advantage of including systematic uncertainties in likelihood: error automatically propagated to error reported by MINUIT

Adding uncertainties to a likelihood

- Example 1 – Width known exactly



- Example 2 – Gaussian uncertainty on width

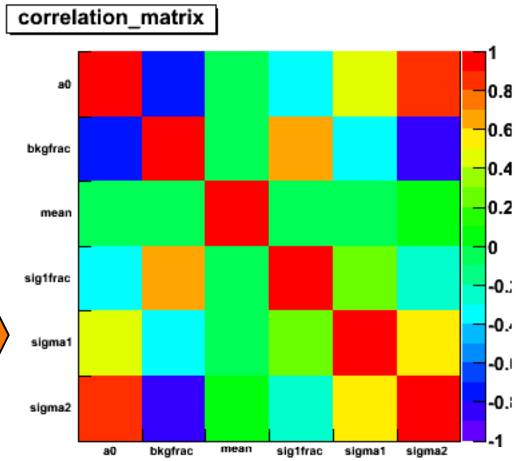


Using the fit result output

- The fit result class contains the full MINUIT output

- Easy visualization of correlation matrix

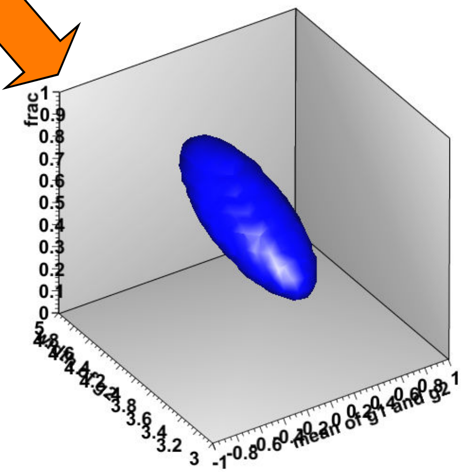
```
fitresult->correlationHist->Draw("colz") ;
```



- Construct multi-variate Gaussian pdf representing pdf on parameters

```
RooAbsPdf* paramPdf = fr->createHessePdf(RooArgSet(frac,mean,sigma));
```

  – Returned pdf represents HESSE parabolic approximation of fit



- Extract correlation, covariance matrix

```
TMatrixDSym cov = fr->covarianceMatrix() ;
TMatrixDSym cov = fr->covarianceMatrix(a,b) ;
```

  – Can also retrieve partial matrix (Schur compl.)

Another approach to joint fitting

- ‘Asymmetric’ simultaneous fit may spend majority of it CPU time calculating the likelihood of the control sample part
  - Because control sample have many more events
  - Example: joint fit between CPV golden modes and BMixing samples
- Alternate solution: Make joint fit using likelihood of signal sample and parameterized likelihood of control sample
  - Assumption: Likelihood can be described by a multi-variate Gaussian with correlations (i.e. log-likelihood is parabolic)
  - Very easy to do in RooFit using RooFitResult->createHessePdf()
  - Example on next page

Example of joint fit with parameterized likelihood

Regular joint fit

```
// Joint pdf construction
 w.factory("SIMUL::model_sim(index[sig,ctl],
                            sig=model, ctl=model_ctl)") ;

 // Joint data construction
 RooDataSet simdata("simdata","simdata",w::x,Index(w::index),
                   Import("sig",*data),Import("ctl",*data_ctl)) ;

 // Joint fit
 RooFitResult* rs = w::model_sim.fitTo(simdata,Save()) ;
```

Joint fit with parameterized L for ctl sample

```
// Fit to control sample only
RooFitResult* r = w::model_ctl.fitTo(*data_ctl,Save()) ;
RooAbsPdf* ctrlParamPdf = r->createHessePdf(w::model_ctl.getParameters());

// Make pdf of parameters and import in workspace
ctrlParamPdf->SetName("ctrlParamPdf") ;
w.import(*ctrlParamPdf) ;
w.factory("PROD::model_sim2(model,ctrlParamPdf)") ;

// Joint fit with parameterized likelihood for control sample
RooFitResult* rs = w::model_sim2.fitTo(*data,Save()) ;
```

9 Intervals & Limits

- *A brief introduction to RooStats*

RooStats Project – Overview

- Goals:
  - Standardize interface for major statistical procedures so that they can work on an arbitrary RooFit model & dataset and handle many parameters of interest and nuisance parameters.
  - Implement most accepted techniques from Frequentist, Bayesian, and Likelihood-based approaches
  - Provide utilities to perform combined measurements

- Design:
  - Essentially all methods start with the basic probability density function or likelihood function. *Building a good model is the hard part*. Want to re-use it for multiple methods → Use RooFit to construct models
  - Build series of tools that perform statistical procedures on RooFit models

RooStats Project – Structure

- **RooFit** (data modeling)
  - Data modeling language (pdfs and likelihoods). Scales to arbitrary complexity
  - Support for efficient integration, toy MC generation
  - Workspace
    - Persistent container for data models
    - Completely self-contained (including custom code)
    - Complete introspection and access to components
  - Workspace factory provides easy scripting language to populate the workspace

- **RooStats** (limits, interval calculators & utilities)
  - Profile Likelihood calculator
  - Neyman construction (FC)
  - Bayesian calculator (BAT & native MCMC)
  - Utilities (combinations, construct pdfs corresponding to standard number counting problems)

RooStats Project – Organization

- Joint ATLAS/CMS project
- Core developers
  - K. Cranmer (ATLAS)
  - Gregory Schott (CMS)
  - Wouter Verkerke (RooFit)
  - Lorenzo Moneta (ROOT)
- Open project, you are welcome to join
  - Max Baak, Mario Pelliccioni, Alfio Lazzaro contributing now
- Included since ROOT v5.22
  - Example macros in $ROOTSYS/tutorials/roostats
- Documentation
  - Code doc. via ROOT
  - Esers manual is in development

RooStats Project – Example

- Create a model - Example

$$Poisson(x \mid s \cdot r_s + b \cdot r_b) \cdot Gauss(r_s, 1, 0.05) \cdot Gauss(r_b, 1, 0.1)$$

•Create workspace with above model (using factory)

```
RooWorkspace* w = new RooWorkspace(“w”);
w->factory(“Poisson::P(obs[150,0,300],
                      sum::n(s[50,0,120]*ratioSigEff[1.,0,2.],
                             b[100,0,300]*ratioBkgEff[1.,0.,2.]))");
w->factory("PROD::PC(P, Gaussian::sigCon(ratioSigEff,1,0.05),
                        Gaussian::bkgCon(ratioBkgEff,1,0.1))");
```

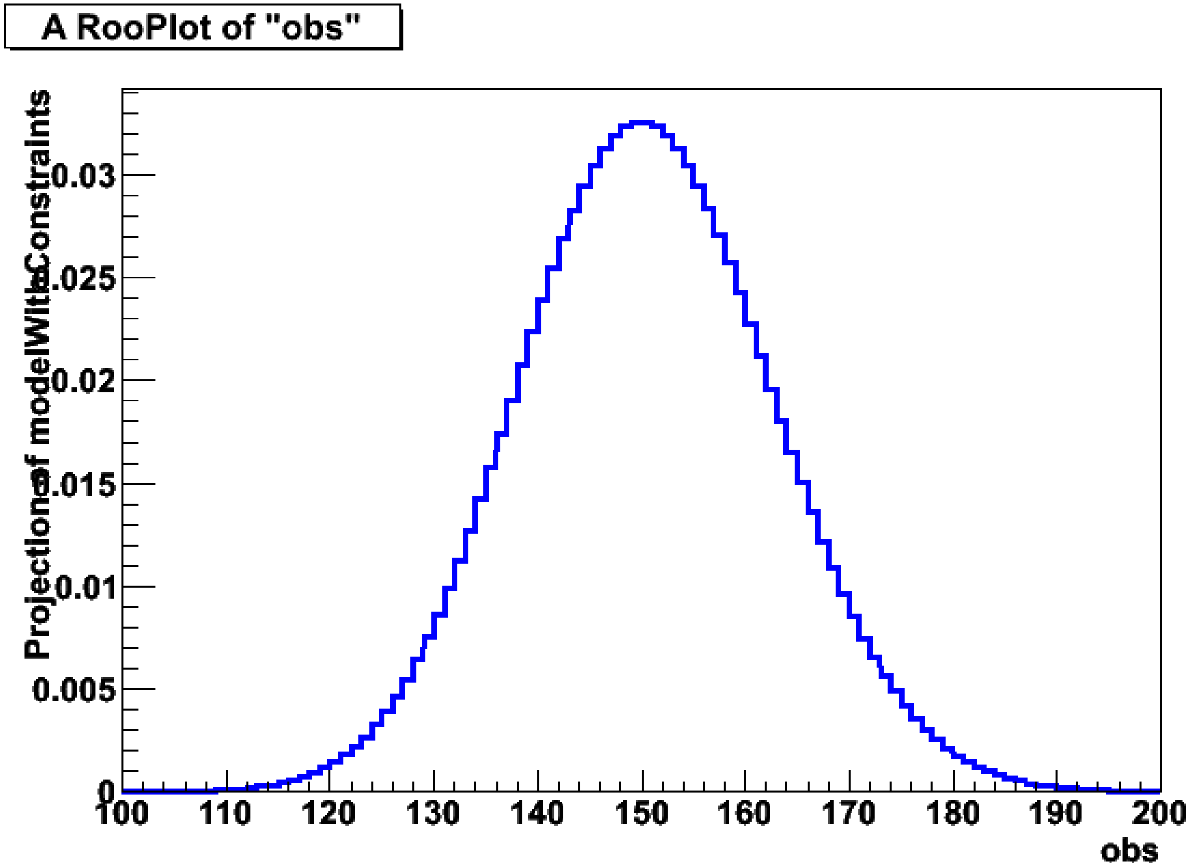
•Contents of workspace from above operation

```
RooWorkspace(w) w contents

variables
---------
(b,obs,ratioBkgEff,ratioSigEff,s)

p.d.f.s
-------
RooProdPdf::PC[ P * sigCon * bkgCon ] = 0.0325554
  RooPoisson::P[ x=obs mean=n ] = 0.0325554
    RooAddition::n[ s * ratioSigEff + b * ratioBkgEff ] = 150
  RooGaussian::sigCon[ x=ratioSigEff mean=1 sigma=0.05 ] = 1
  RooGaussian::bkgCon[ x=ratioBkgEff mean=1 sigma=0.1 ] = 1
```

RooStats Project – Example

- Simple use of model

```
RooPlot* frame = w::obs.frame(100,200) ;
w::PC.plotOn(frame) ;
frame->Draw()
```



RooStats Project – Example

- Confidence intervals calculated with model
  - Profile likelihood

```
ProfileLikelihoodCalculator plc;
plc.SetPdf(w::PC);
plc.SetData(data); // contains [obs=160]
plc.SetParameters(w::s);
plc.SetTestSize(.1);
ConfInterval* lrint = plc.GetInterval(); // that was easy.
```

  - Feldman Cousins

```
FeldmanCousins fc;
fc.SetPdf(w::PC);
fc.SetData(data); fc.SetParameters(w::s);
fc.UseAdaptiveSampling(true);
fc.FluctuateNumDataEntries(false);
fc.SetNBins(100); // number of points to test per parameter
fc.SetTestSize(.1);
ConfInterval* fcint = fc.GetInterval(); // that was easy.
```

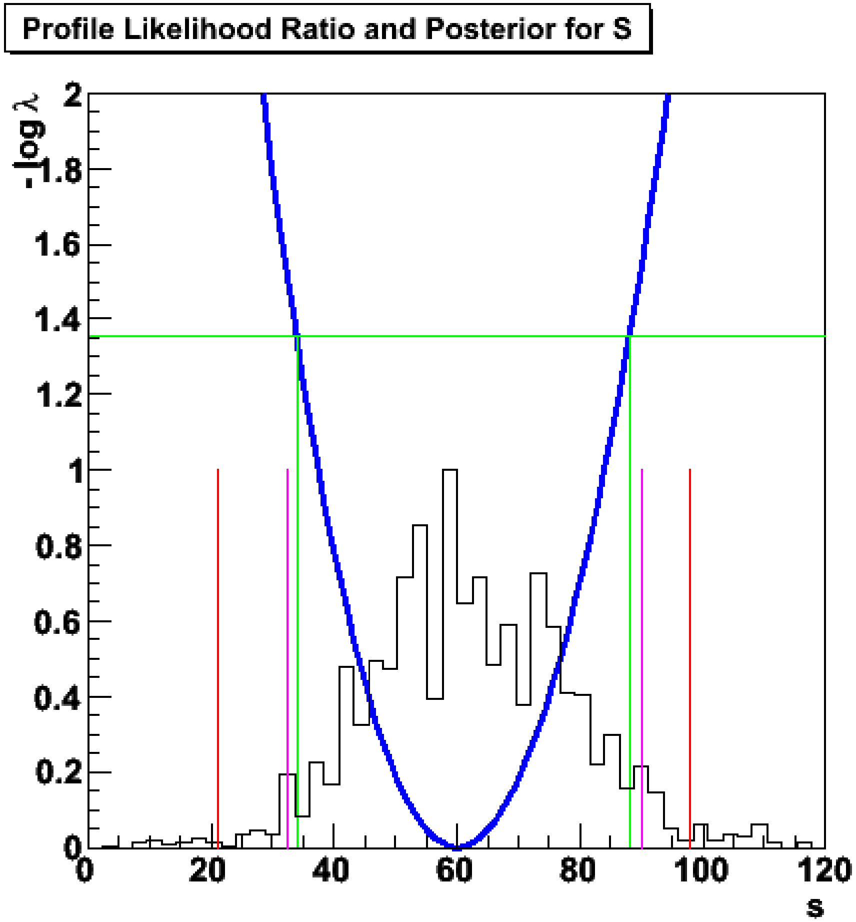
  - Bayesian (MCMC)

```
UniformProposal up;
MCMCCalculator mc;
mc.SetPdf(w::PC);
mc.SetData(data);  mc.SetParameters(s);
mc.SetProposalFunction(up);
mc.SetNumIters(100000); // steps in the chain
mc.SetTestSize(.1); // 90% CL
mc.SetNumBins(50); // used in posterior histogram
mc.SetNumBurnInSteps(40);
ConfInterval* mcmcint = mc.GetInterval();
```

RooStats Project – Example

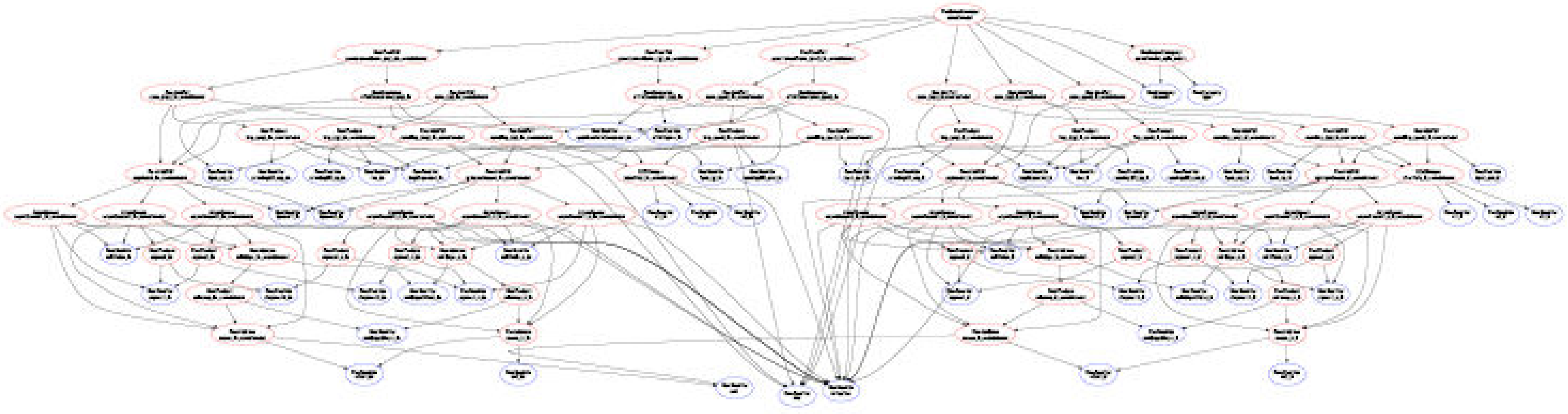
- Retrieving and visualizing output

```
double fcul = fcint->UpperLimit(w::s);
double fcll = fcint->LowerLimit(w::s);
```



RooStats Project – Example

- Some notes on example
  - Complete working example (with output visualization) shipped with ROOT distribution (*$ROOTSYS/tutorials/roofit/rs101_limitexample.C*)
  - **Interval calculators make no assumptions on internal structure of model**. Can feed model of arbitrary complexity to same calculator (computational limitations still apply!)



The end

- RooFit Documentation

- Starting point http://root.cern.ch/drupal/content/roofit
  - Quick start guide (20 pages) – Includes Workspace & Factory
  - Users Manual (140 pages)

- Tutorial macros
  - root.cern.ch → documentation → tutorials → roofit
  - There are over 80 macros illustrating many aspects of RooFit functionality

- Help
  - Post your question on the Stat & Maths tools forum of root.cern.ch