Follow up material for
PRACTICAL CLASS 7
for the Course
Laboratorio Analisi Dati
2017/2018
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[Some slides borrowed from RooFit tutorials]

We have seen in PRACTICAL CLASS 7 that :

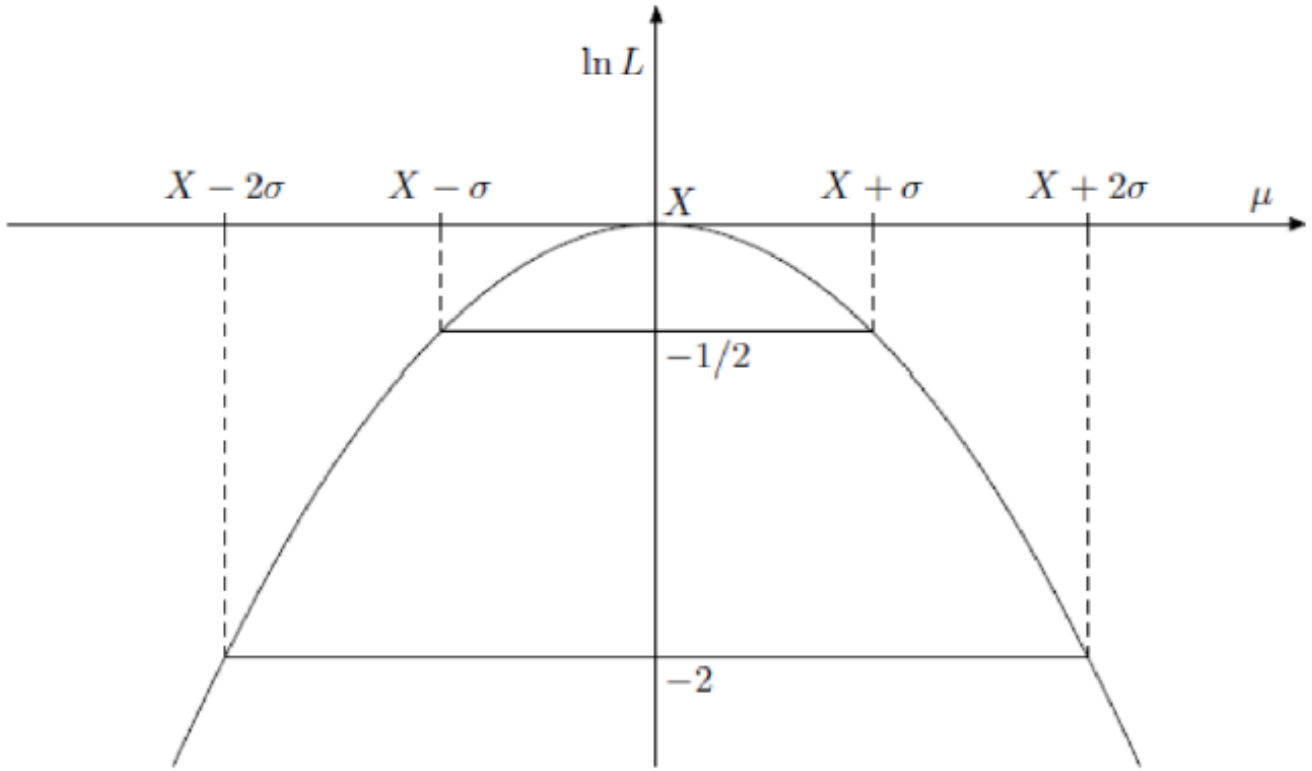
Likelihood minimization – class RooMinuit

- Class RooMinuit is an *interface* to the ROOT implementation of the MINUIT minimization and error analysis package.

- RooMinuit takes care of
  - Passing value of miminized RooFit function to MINUIT
  - Propagated changes in parameters both from `RooRealVar` to MINUIT and back from MINUIT to `RooRealVar`, i.e. it keeps the state of RooFit objects synchronous with the MINUIT internal state
  - Propagate error analysis information back to `RooRealVar` parameters objects
  - Exposing high-level MINUIT operations to RooFit uses (MIGRAD,HESSE,MINOS) etc…
  - Making optional snapshots of complete MINUIT information (e.g. convergence state, full error matrix etc)

```
// Start Minuit session on above nll
RooMinuit m(nll) ;

// MIGRAD likelihood minimization
m.migrad() ;

// Run HESSE error analysis
m.hesse() ;

// Set sx to 3, keep fixed in fit
sx.setVal(3) ;
sx.setConstant(kTRUE) ;

// MIGRAD likelihood minimization
m.migrad() ;

// Run MINOS error analysis
m.minos()

// Draw 1,2,3 'sigma' contours in sx,sy
m.contour(sx,sy) ;
```

We know that PDF can be converted into a Likelihood function *L* by exchanging the vector of observation with the vector of parameters.
For only 1 parameter μ the likelihood *L* is a function of μ and *lnL* is a parabola:

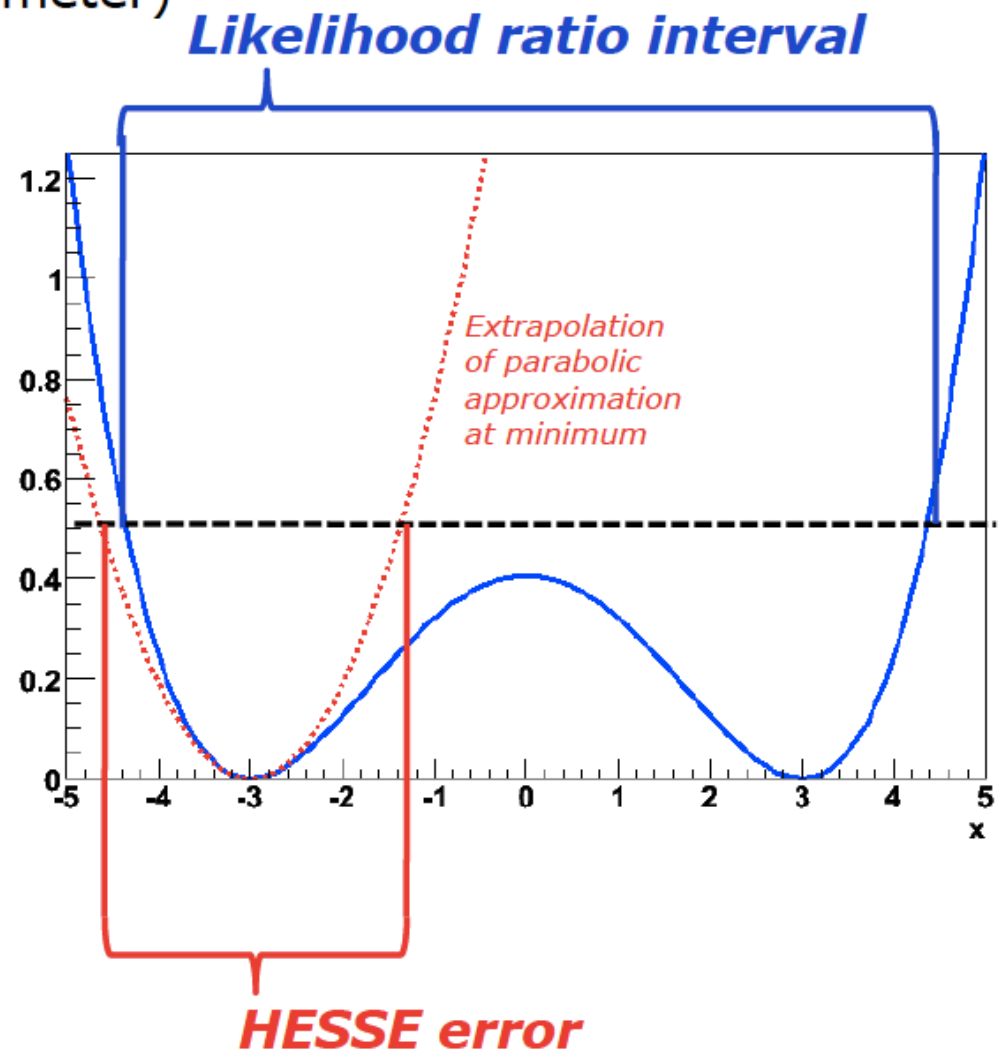


If we have many parameters, some of them are "of interest" (POI) and the other are called "nuisance parameters"; the latter are parameters in the problem which affect the result but which are not of primary interest.

Likelihood ratio intervals

- Definition of Likelihood Ratio interval (identical to MINOS for 1 parameter)

$$LR(\vec{x},\mu) = \frac{L(\vec{x},\mu)}{L(\vec{x},\hat{\mu})}$$

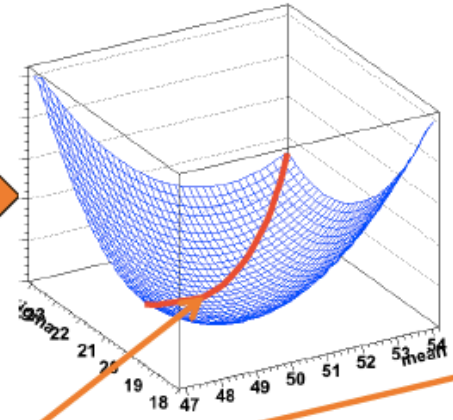


Dealing with nuisance pars in Likelihood ratio interval

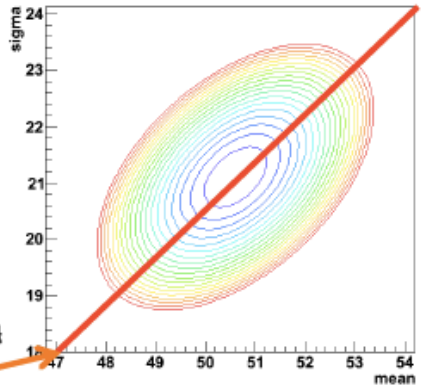
- Nuisance parameters in LR interval
  - For each value of the parameter of interest, search the full subspace of nuisance parameters for the point at which the likelihood is maximized.
  - Associate that value of the likelihood with that value of the parameter of interest → 'Profile likelihood'

MLE fit fit data

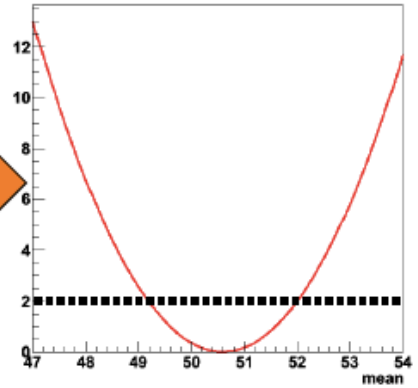
-logLR(mean,sigma)



-logLR(mean,sigma)



-logPLR(mean)



$$\lambda(\mu) = \frac{L(\mu, \hat{\hat{\sigma}}(\mu))}{L(\hat{\mu}, \hat{\sigma})}$$

- ***best L(μ) for any value of s***
- ***best L(μ,σ)***

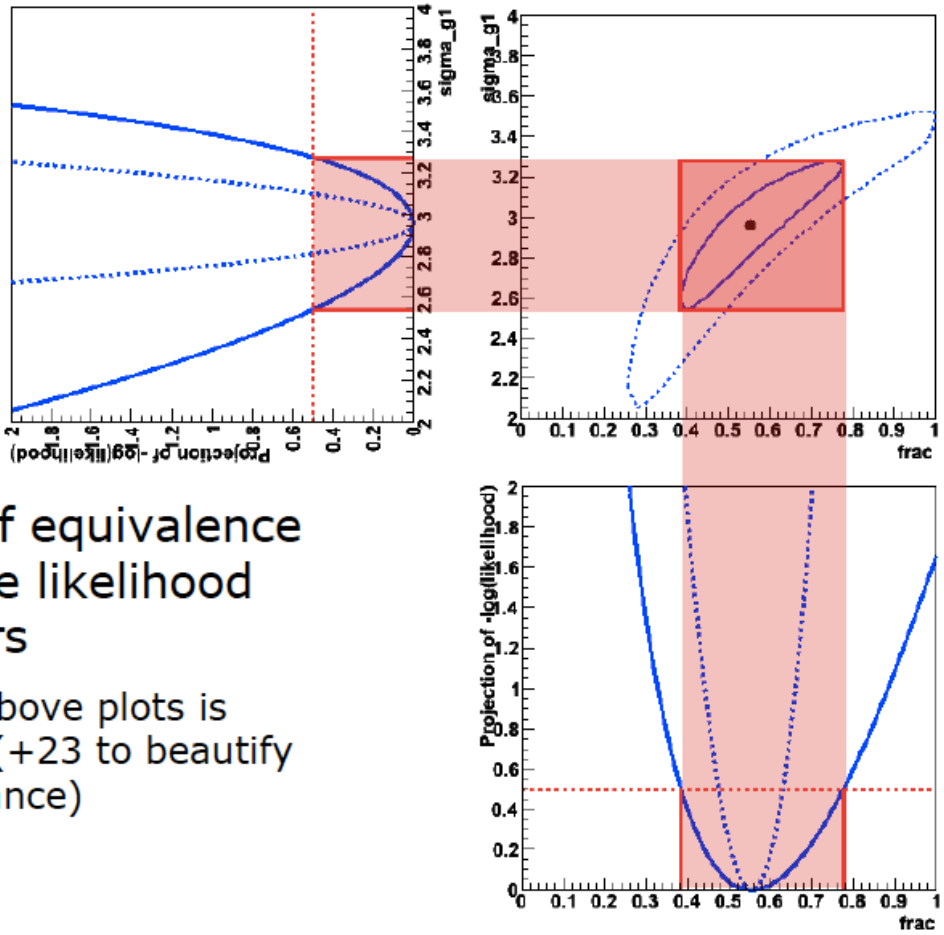
Working with profile likelihood

- A profile likelihood ratio $\lambda(p) = \dfrac{L(p,\hat{\hat{q}})}{L(\hat{p},\hat{q})}$ ← ***Best L for given p*** / ← ***Best L***

  can be represent by a regular RooFit function
  (albeit an expensive one to evaluate)

```
RooAbsReal* ll = model.createNLL(data,NumCPU(8)) ;
RooAbsReal* pll = ll->createProfile(params) ;
```

On the equivalence of profile likelihood and MINOS



- Demonstration of equivalence of (RooFit) profile likelihood and MINOS errors
  - Macro to make above plots is 34 lines of code (+23 to beautify graphics appearance)

Indeed we have checked by ourselves that the Profile Likelihood ratio defines an interval (1sigma uncertainty) of values for a single POI - when *lnL* is increased by 0.5 – that is the same as the one provided by MINOS.