

NOW 2010  
"Neutrino Oscillation Workshop"  
Conca Specchiulla, 4-11 September 2010

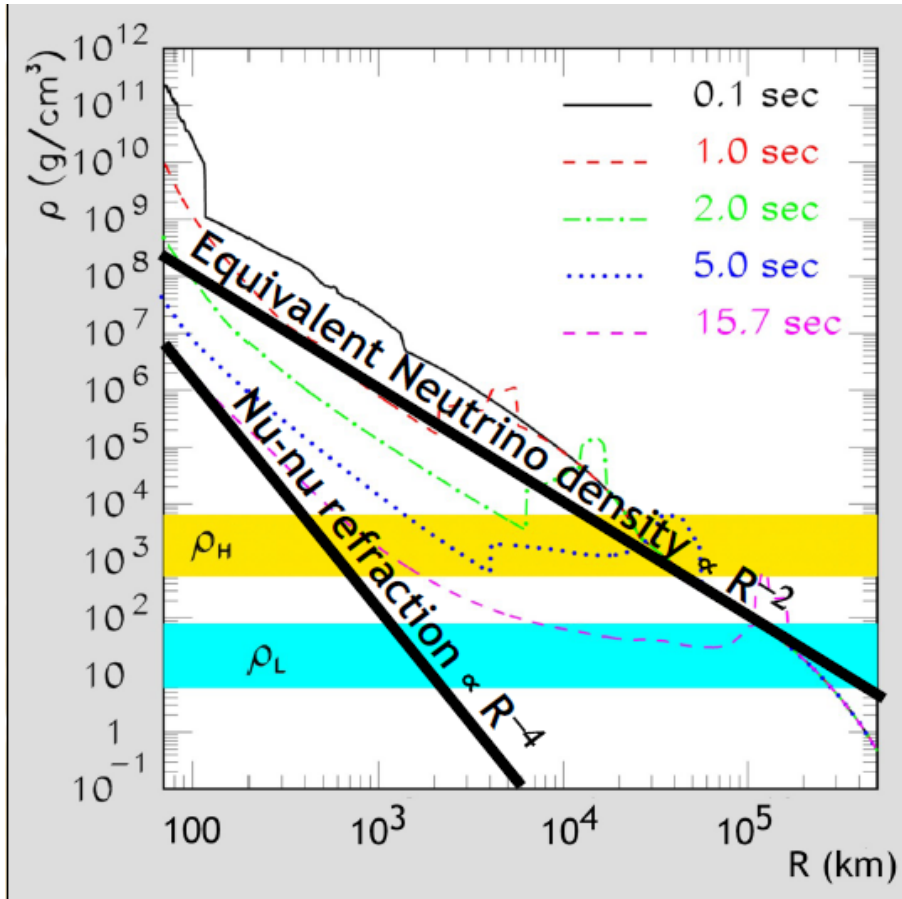
SELF-INDUCED SPECTRAL  
SPLITS IN SUPERNOVA  
NEUTRINOS:  
THREE-FLAVOR EFFECTS

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[Based on Dasgupta, A.M., Tamborra & Tomas, PRD81, 093008  
(2010), arXiv:1002:2943]

# NEUTRINO-NEUTRINO INTERACTIONS

In the region just above the neutrino-sphere the neutrino density exceeds the ordinary electron background. Neutrinos themselves form a background medium



**$\nu$ - $\nu$  NC interactions important!**

- Matter bkg potential

$$V = \sqrt{2}G_F N_e \sim R^{-3}$$

- $\nu$ - $\nu$  potential ↙ Multi-angle effects

$$\mu = \sqrt{2}G_F n_\nu \langle 1 - \cos \theta_{pq} \rangle \approx R^{-2} \times R^{-2} = R^{-4}$$

Lesson: self-interactions ( $\mu$ ) can induce large, non-MSW flavor change at small radii, despite large matter density  $\nu$

TALK BY G.RAFFELT

# FORMALISM AND EVOLUTION EQUATIONS

The evolution equation of the density matrix for each mode  $p$

$$i\partial_t \rho_p = [H_p, \rho_p]$$

Diagonal elements related to flavor content

$$\rho_{\alpha\alpha} = \frac{F_{\nu_\alpha}(E, r)}{F(E, r)}$$

$$\rho = \begin{pmatrix} \rho_{ee} & \rho_{e\mu} & \rho_{e\tau} \\ \rho_{e\mu}^* & \rho_{\mu\mu} & \rho_{\mu\tau} \\ \rho_{e\tau} & \rho_{\mu\tau}^* & \rho_{\tau\tau} \end{pmatrix}$$

Off-diagonal elements responsible for flavor conversions

- "Single-angle" Hamiltonian

$$H_p = \Omega_p + V + \mu \int \frac{d^3 q}{(2\pi)^3} (\rho_p - \bar{\rho}_p)$$

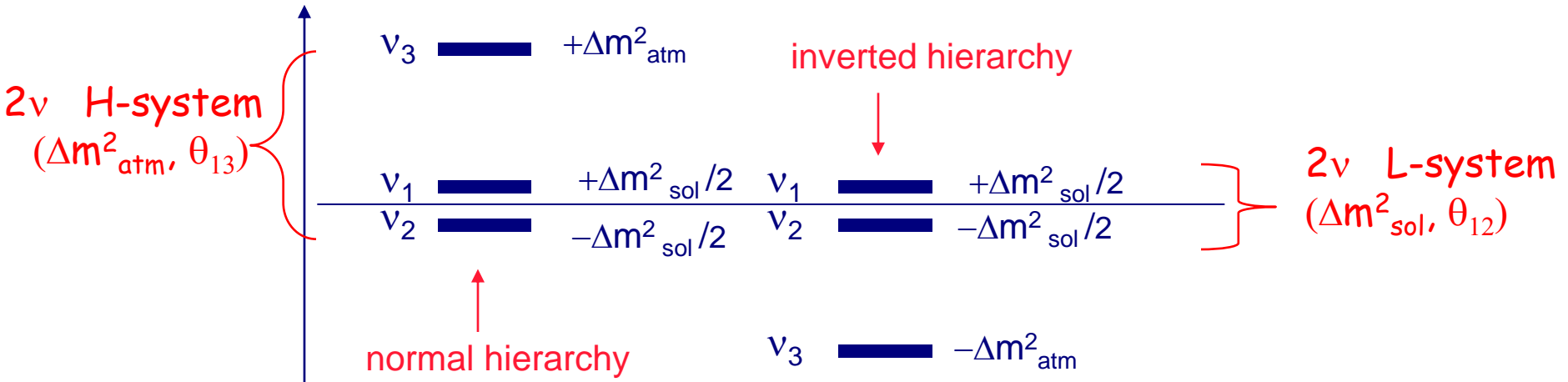
Matter term

$\nu$ - $\nu$  interaction term

$$\Omega_p = \left( -\frac{\Delta m_{sol}^2}{2}, \frac{\Delta m_{sol}^2}{2}, \Delta m_{atm}^2 \right) / 2|p| \quad \text{Vacuum oscillations}$$

# VACUUM OSCILLATIONS: 3 $\nu$ FRAMEWORK

- **Mixing parameters:**  $U = U(\theta_{12}, \theta_{13}, \theta_{23})$  as for CKM matrix
- **Mass spectrum**



$$\left\{ \begin{array}{l} \Delta m^2_{\text{atm}} = 2.0 \times 10^{-3} \text{ eV}^2 \\ \Delta m^2_{\text{sol}} = 8. \times 10^{-5} \text{ eV}^2 \\ \theta_{12} = 0.6 \\ \theta_{23} = \pi / 4 \\ \theta_{13} \leq 0.1 \end{array} \right.$$

# PENDULUM IN FLAVOR SPACE

[Hannestad, Raffelt, Sigl, Wong, astro-ph/0608695, Duan, Carlson, Fuller, Qian, astro-ph/0703776]

Neutrino mass hierarchy (and  $\theta_{13}$ ) set initial condition and fate

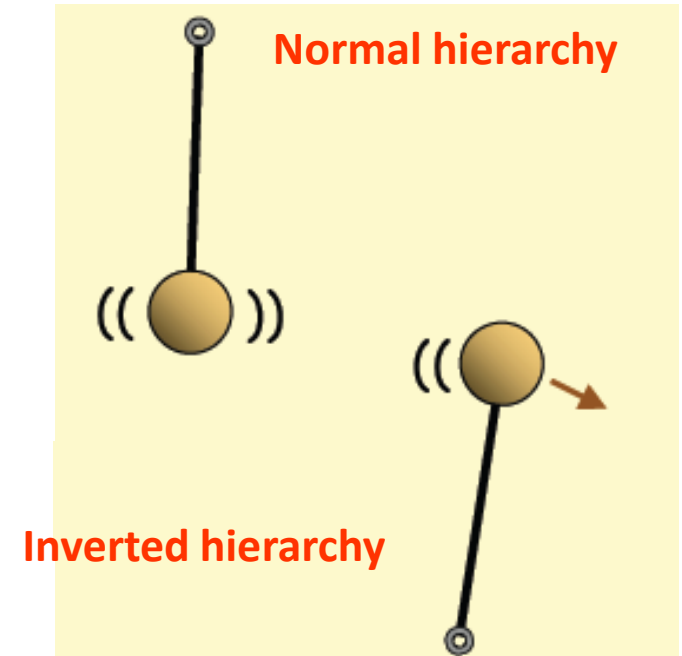
With only initial  $\nu_e$  and  $\bar{\nu}_e$ :

- **Normal hierarchy**

Pendulum starts in  $\sim$  downward (stable) positions and stays nearby. No significant flavor change.

- **Inverted hierarchy**

Pendulum starts in  $\sim$  upward (unstable) positions and eventually falls down. Significant flavor changes.



$\theta_{13}$  sets initial misalignment with vertical. Specific value not much relevant.

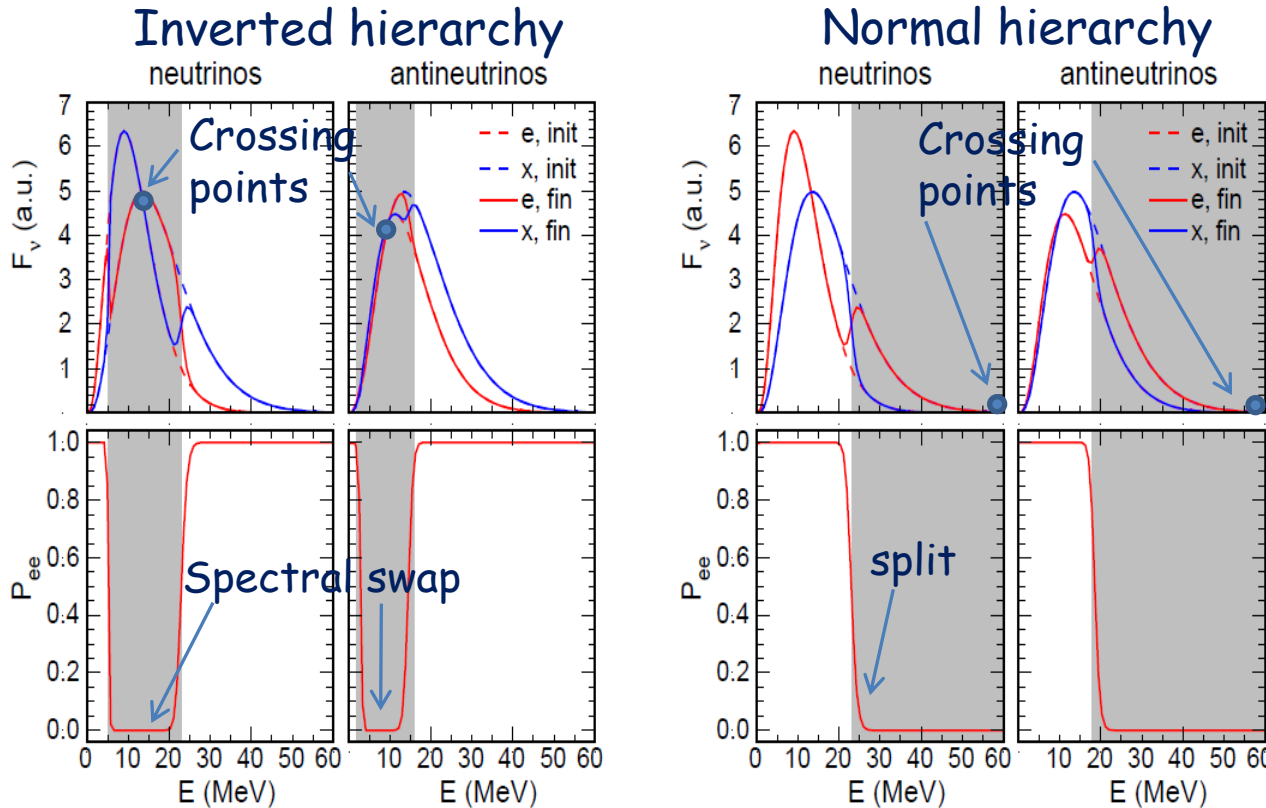
## Which mass hierarchy?

With only initial  $\nu_\mu$  and  $\bar{\nu}_\mu$  large flavor conversions in NH. The **unstable case** is when the initial ensemble consists of that flavor which is dominated by the heavier mass eigenstate.

# SPECTRAL SPLITS IN 2 FLAVORS (H-SYSTEM)

[Dasgupta, Dighe, Raffelt, Smirnov, 0904.3542]

$$F_{\nu_e} : F_{\bar{\nu}_e} : F_{\nu_x} = 0.85 : 0.75 : 1.00$$



(typical in cooling phase)

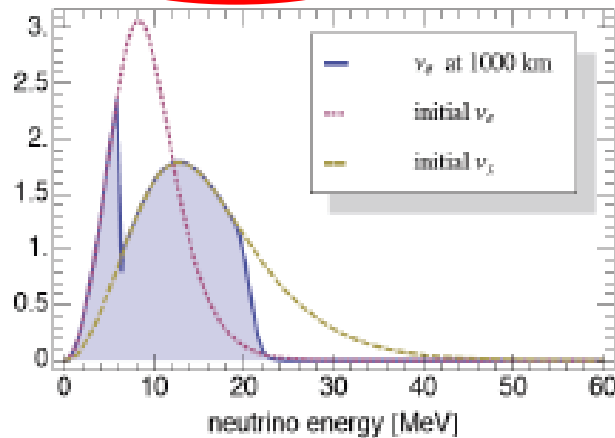
Spectral splits can develop around the **crossing points** of the original neutrino spectra.

A given crossing point is unstable if

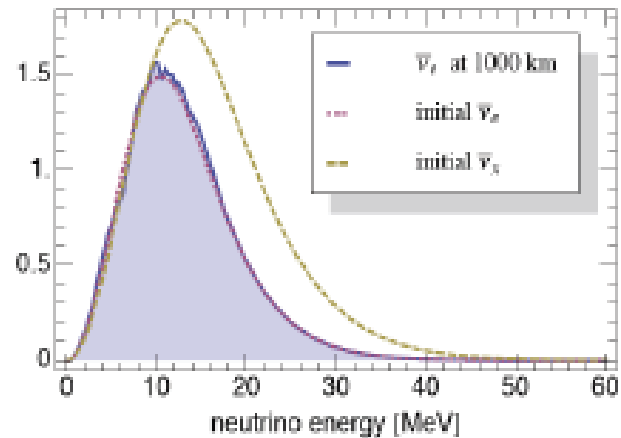
- $d(F_e - F_x)/dE < 0$  for inverted mass hierarchy
- $d(F_e - F_x)/dE > 0$  for normal mass hierarchy

# THREE-FLAVOR EFFECTS

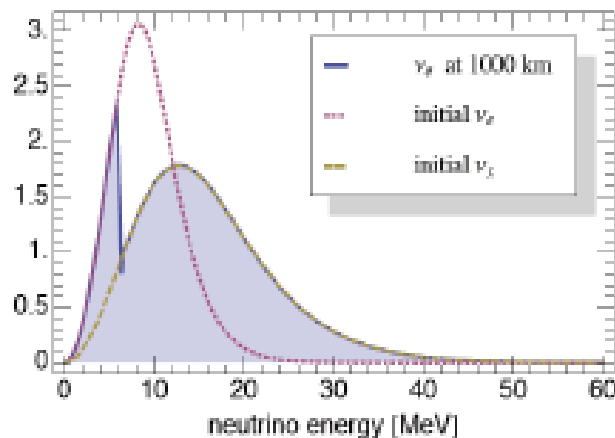
Inverted Hierarchy, 2 flavors,  $\nu_e$



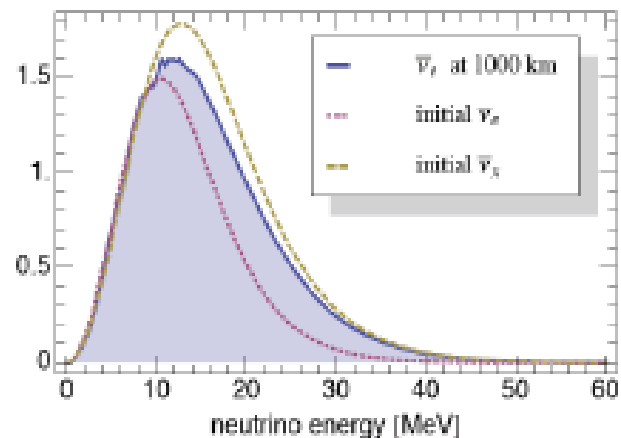
Inverted Hierarchy, 2 flavors,  $\bar{\nu}_e$



Inverted Hierarchy, full 3 flavors,  $\nu_e$



Inverted Hierarchy, full 3 flavors,  $\bar{\nu}_e$



# SPECTRAL SPLITS IN 2 FLAVORS (L-SYSTEM)

We would expect a behavior similar to that of the H-system in NH. However, **no flavor conversion occurs.**

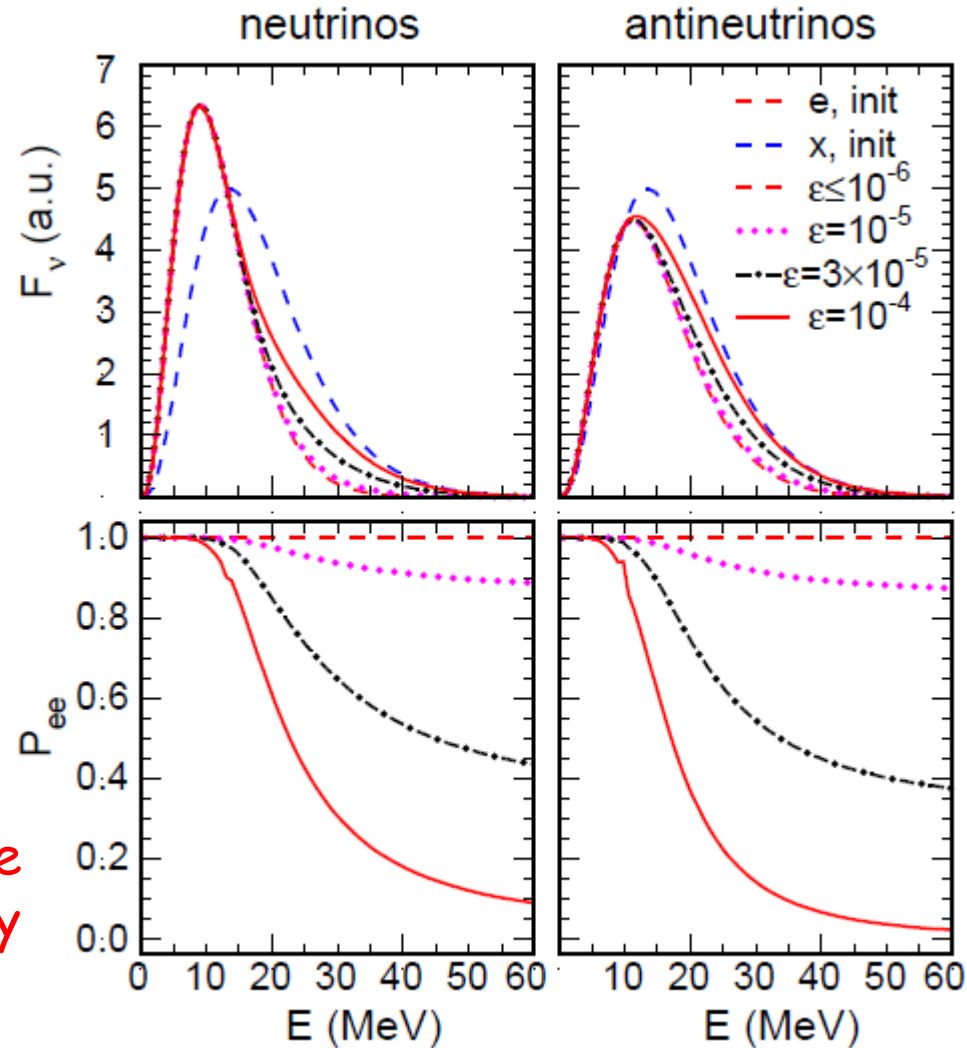
Two reasons:

- **insufficient growth of instability**  
[ $\tau \approx (\omega\mu)^{-1/2}$ ]
- **lack of adiabaticity**:  $\omega < d \ln \mu / dr$

Some initial disturbance helps to kick-start swaps

Strength of instability given by the off-diagonal terms in the density matrix. We put as initial seeds

$$\rho_{ex} = \varepsilon |\rho_{ee} + \rho_{xx}|$$





# FLAVOR DYNAMICS IN 3 GENERATIONS

We work in the rotated basis

$$(v_e, v_x, v_y) = R^T(\theta_{23})(v_e, v_\mu, v_\tau)$$

Let's split the density matrix and the Hamiltonian in (0) +(1) parts

$$\rho^{(0)} = \begin{pmatrix} \rho_{ee} & \rho_{ex} & 0 \\ \rho_{ex}^* & \rho_{xx} & 0 \\ 0 & 0 & \rho_{yy} \end{pmatrix}$$

**e-x block. Oscillations in the L =  $(\Delta m^2_{sol}, \theta_{12})$  sector**

$$\rho^{(1)} = \begin{pmatrix} 0 & 0 & \rho_{ey} \\ 0 & 0 & \rho_{xy} \\ \rho_{ey}^* & \rho_{xy}^* & 0 \end{pmatrix}$$

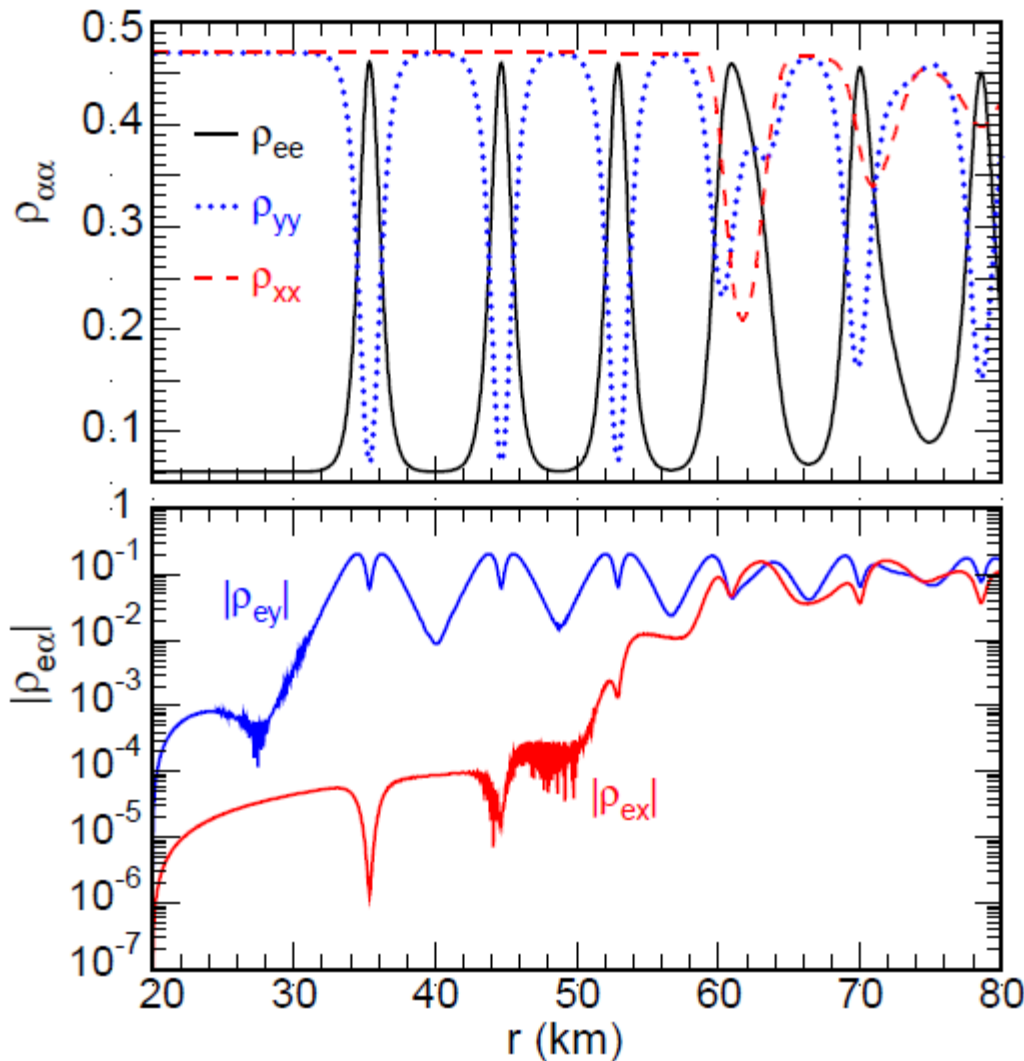
**e-y block. Oscillations in the H =  $(\Delta m^2_{atm}, \theta_{13})$  sector**

For  $\theta_{13} \neq 0$ , H and L sectors are coupled

$H^{(1)}$  produces the off-diagonal components first in  $\rho^{(1)}$  in and subsequently in  $\rho^{(0)}$  by  $\theta_{13}$  effects.

The growth is speeded up by  $H^{(1)}$  which induces oscillations  $\Delta m^2_{atm}$ -dependent at the leading order.

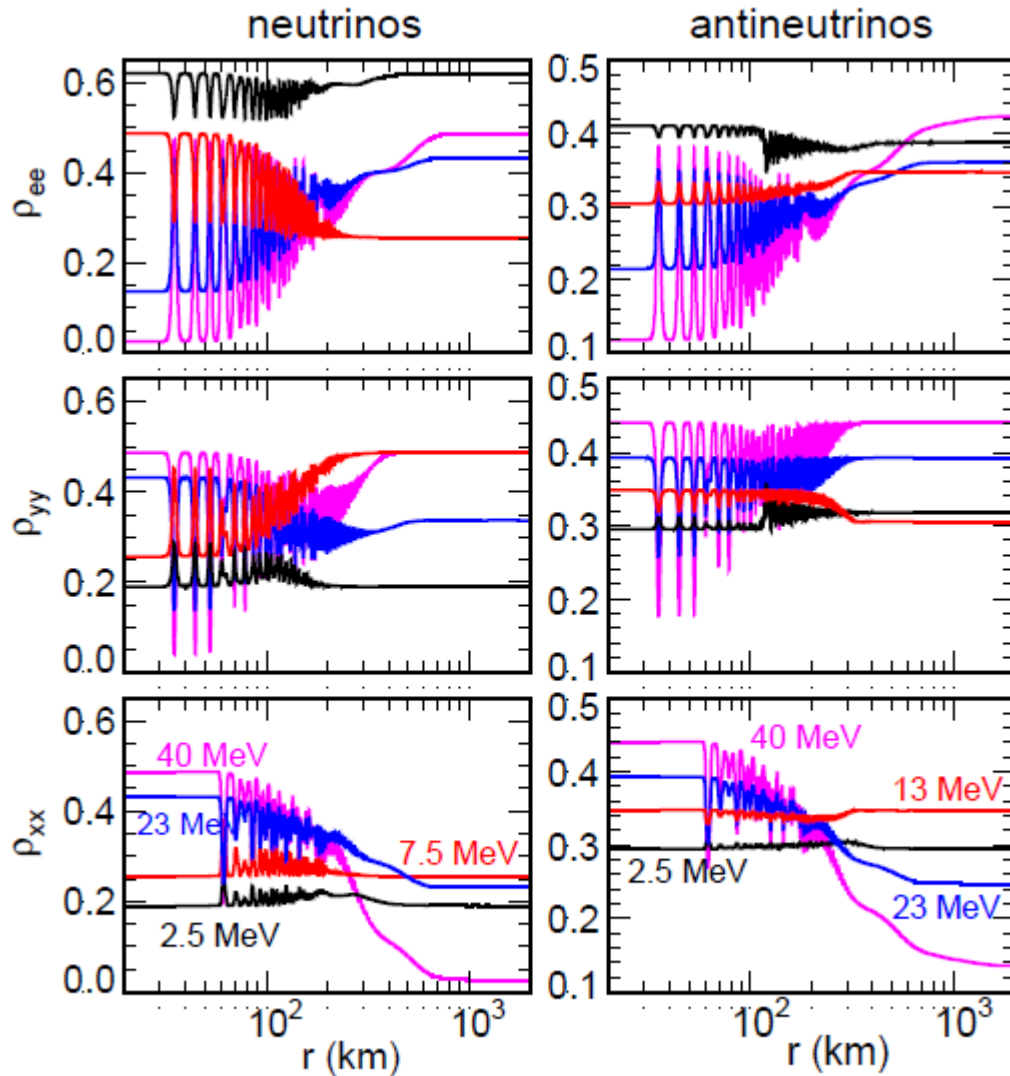
# GROWTH OF THE INSTABILITIES IN $3\nu$



The coupling between the L and H sectors induces  $3\nu$  effects.

The initial kick, associated with  $\Delta m^2_{\text{atm}}$ , is necessary to trigger the instability in the L system.

# THREE-FLAVOR EVOLUTION IN INVERTED HIERARCHY



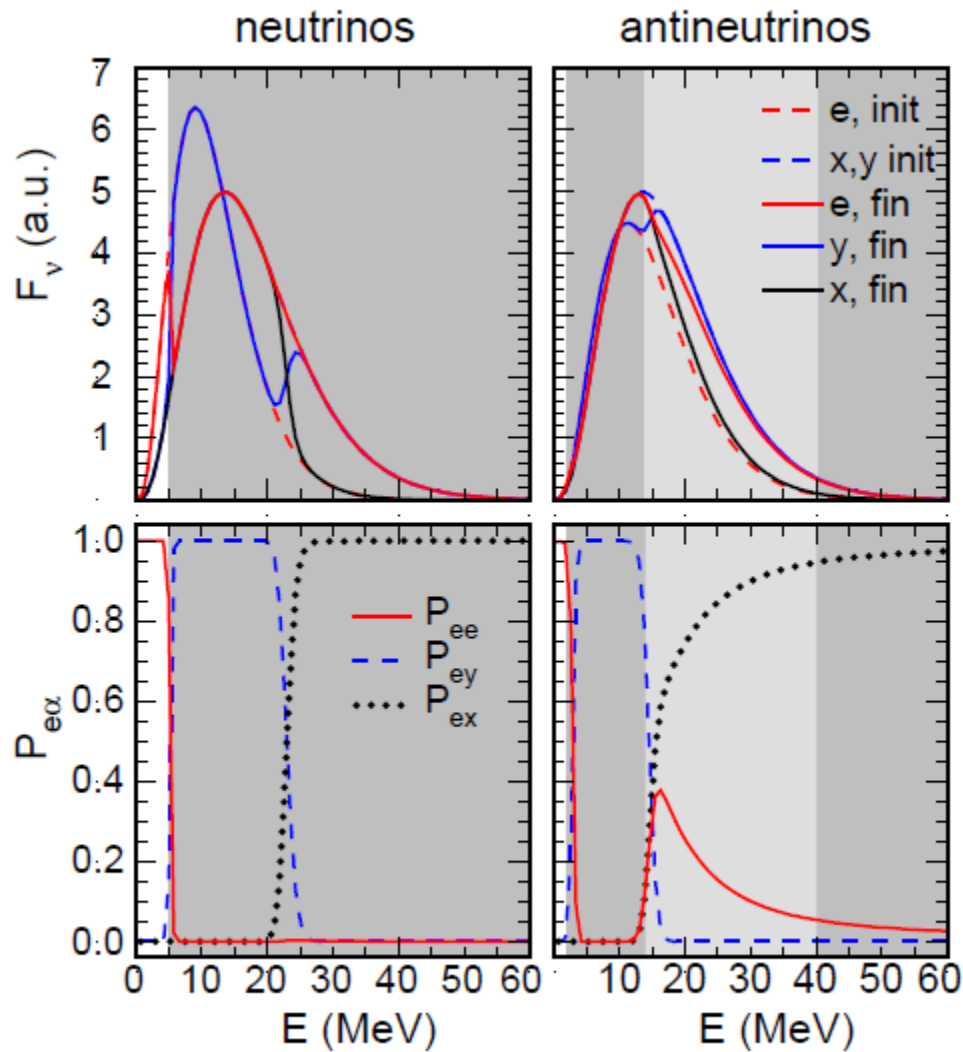
The two mass differences  $\Delta m^2_{\text{atm}} < 0$  and  $\Delta m^2_{\text{sol}} > 0$  process complementary parts of the  $\nu$  spectra.

Their effects do not interfere in the same energy range.

$\nu_e$  and  $\nu_y$  swap part of their spectra *unstable* under the action of  $\Delta m^2_{\text{atm}}$ .

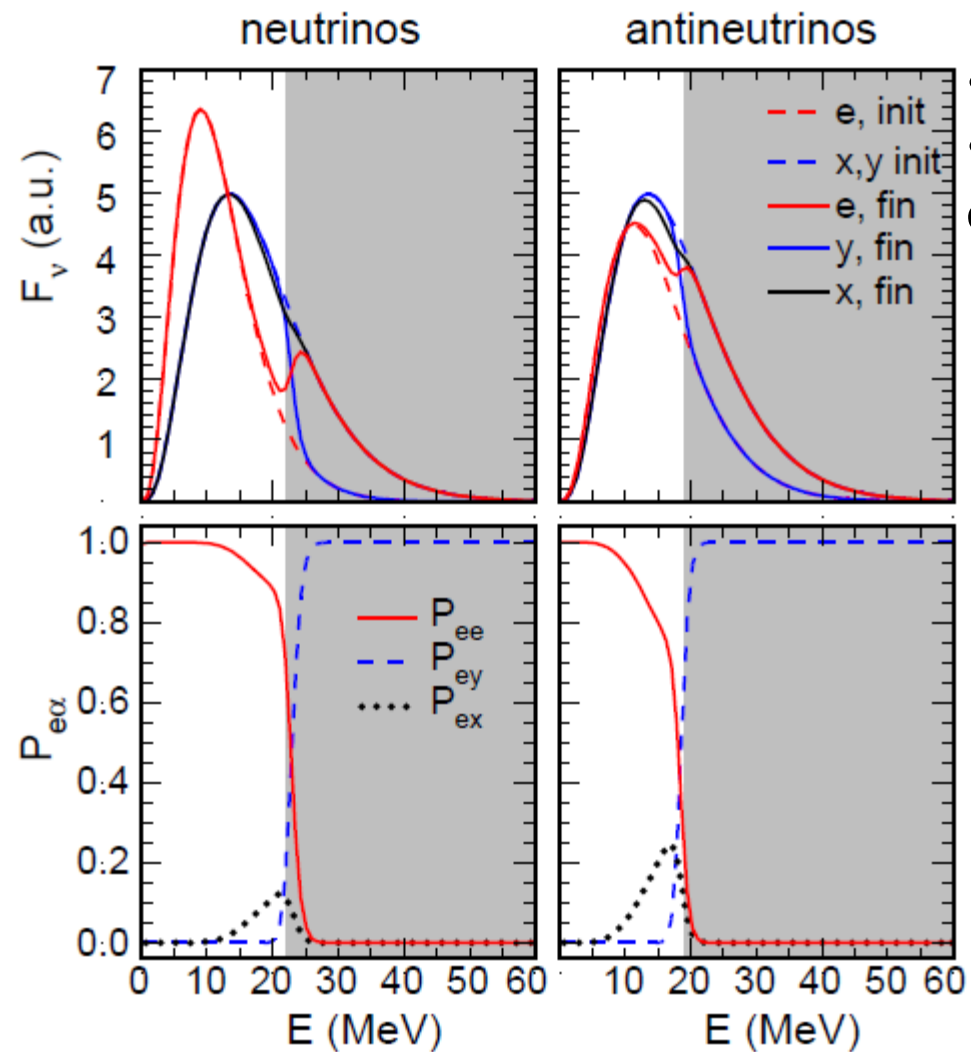
$\nu_e$  and  $\nu_x$  swap part of their spectra *unstable* under the action of  $\Delta m^2_{\text{sol}}$ .

# 3 $\nu$ SPECTRAL SPLITS IN INVERTED HIERARCHY



- Atmospheric swaps ( $e, y$ )
- Solar swaps ( $e, x$ )
- Higher energy split is transferred from  $e$  to  $x$
- Non-adiabatic effects (especially for anti- $\nu$ )

# 3 $\nu$ SPECTRAL SPLITS IN NORMAL HIERARCHY



- Almost same as 2-flavors (e-y).
- Solar driven conversions are too slow to compete.

In this case 3 and 2 $\nu$  instabilities act in the same regions of the  $\nu$  spectrum leading only to minor departures from the 2 $\nu$  evolution.

# CONCLUSIONS

- Neutrino-neutrino interactions induce peculiar spectral swaps among different neutrino flavors.
- The development of these features is associated with instabilities in the flavor space.
- **In inverted hierarchy**, during the cooling phase, the presence of  $\Delta m_{\text{sol}}^2 > 0$  gives rise to instabilities in regions of the neutrino energy spectra that were stable under the two-flavor evolution governed by  $\Delta m_{\text{atm}}^2 < 0$  and  $\theta_{13}$ . **The combinations of these two instabilities would produce a wash-out of the high-energy splitting spectral features in  $\nu_e$**
- **In normal hierarchy** the three-flavor instabilities and the two-flavor ones act in the same regions of the neutrino energy spectrum leading only **to minor departures from the two-flavor evolution.**

# SPECTRAL SPLITS IN THE ACCRETION PHASE

$$F_{\nu_e} : F_{\bar{\nu}_e} : F_{\nu_x} = 2.4 : 1.6 : 1.0$$

We should have seen 4 splits, but we see 2 only, because the inner swap is exponentially narrower.

