Models for neutrino mass with discrete symmetries

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NEUTRINO-OSCILLATION-WORKSHOP

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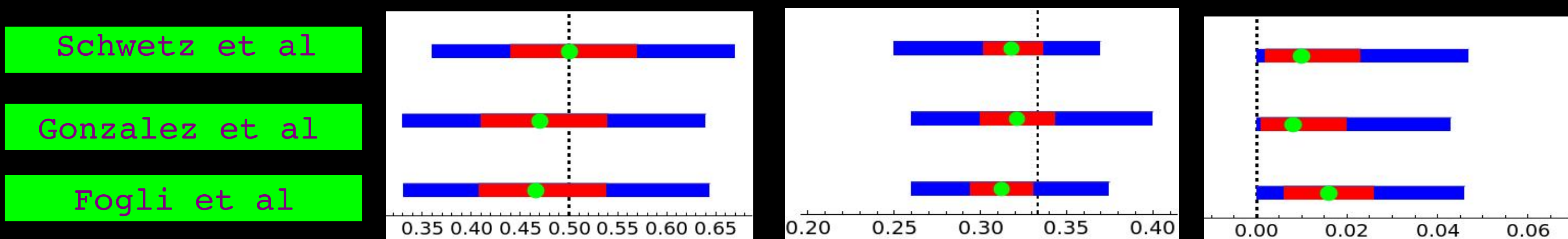
TRI-BIMAXIMAL MIXING

Harrison, Perkins & Scott

$$U_{\rm HPS} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

$$\sin^2\theta_{23} = 0.5 \qquad \sin^2\theta_{12} = 1/3 \qquad \sin^2\theta_{13} = 0$$



Different ansatz has been studied: mu-tau, trimaximal, tetramaximal, symmetric mixing, hexagon mixing, bimaximal, golden, quark-lepton complementarity...

Albright,Dueck,Rodejohann 1004.2798

TBM NEUTRINO MASS MATRIX

$$m_\nu = U_{TB} \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} U_{TB}^T \equiv \begin{pmatrix} x & y & y \\ y & x+z & y-z \\ y & y-z & x+z \end{pmatrix}$$

mu-tau invariant

$$m_{\nu,\,22} = m_{\nu,\,33}$$

$$m_{\nu,\,12} = m_{\nu,\,13}$$

$$O = \begin{pmatrix} -\cos\theta & \sin\theta & 0 \\ \frac{1}{\sqrt{2}}\sin\theta & \frac{1}{\sqrt{2}}\cos\theta & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}}\sin\theta & \frac{1}{\sqrt{2}}\cos\theta & \frac{1}{\sqrt{2}} \end{pmatrix}$$

reactor = 0

Atm = maximal

$$m_{\nu,\,11} + m_{\nu,\,13} = m_{\nu,\,22} + m_{\nu,\,23}$$

sol=trimaximal

DISCRETE GROUPS & TBM

Frampton and Kephart, PRD64 (01)

order	groups
6	$S_3 \equiv D_3$
8	$D_4, Q = Q_4$
10	D_5
12	$D_6, Q_6, T \equiv A_4$
14	D_7
16	$D_8, Q_8, Z_2 \times D_4, Z_2 \times Q$
18	$D_9, Z_3 \times D_3$
20	D_{10}, Q_{10}
22	D_{11}
24	$D_{12}, Q_{12}, Z_2 \times D_6, Z_2 \times Q_6, Z_2 \times T, Z_3 \times D_4, Z_3 \times Q, Z_4 \times D_3, S_4$
26	D_{13}
28	D_{14}, Q_{14}
30	$D_{15}, D_5 \times Z_3, D_3 \times Z_5$

A4 Babu, Ma, Valle PLB552
Altarelli, Feruglio NPB720

S3 Grimus, Lavoura, JHEP0904
Mohapatra,Nasri,Yu PLB639

S4 Lam PRL101
Bazzocchi, Morisi PRD80

T' Feruglio,Hagedorn,Lin,Merlo NPB775
Car, Frampton
Aranda,Carone,Lebed PLB474

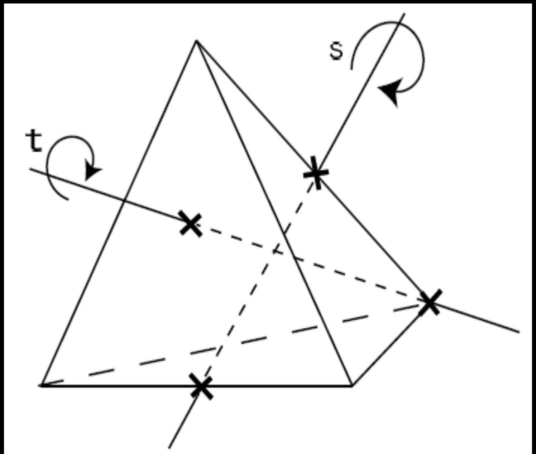
D(27) Medeiros,King,Ross PLB648

for a review of the properties of these groups see
Ishimori,Kobayashi,Ohki,Okada, Shimizu,Tanimoto 1003.3552

ORIGIN OF TBM

A4

discrete group
even permutations
four objects



tetrahedron

Irrep: three singlets 1, 1', 1'' and one triplet 3

smallest discrete group
with triplet irrep

ORIGIN OF TBM

A4

Z_3

$\langle \varphi_T \rangle \;=\; (v_T, 0, 0)$

CHARGED LEPTON

Z_2

$\langle \varphi_S \rangle \;=\; (v_S, v_S, v_S)$

NEUTRINO

$$U_{lep}^{\dagger} U_{\nu} = V_{TBM}$$

large lepton mixing
from the misalignment
of the two sectors

ORIGIN OF TBM

A4

Z_3

$\langle \varphi_T \rangle = (v_T, 0, 0)$

CHARGED LEPTON

Z_2

$\langle \varphi_S \rangle = (v_S, v_S, v_S)$

NEUTRINO

The two sector are separated by means of extra Abelian *Zn*

A PROTOTYPE A4 MODEL (SUSY)

Altarelli, Feruglio NPB741(06)

Field	l	e^c	μ^c	τ^c	$h_{u,d}$	φ_T	φ_S	ξ	$\tilde{\xi}$	φ_0^T	φ_0^S	ξ_0
A_4	3	1	$1'$	$1''$	1	3	3	1	1	3	3	1
Z_3	ω	ω^2	ω^2	ω^2	1	1	ω	ω	ω	1	ω	ω
$U(1)_R$	1	1	1	1	0	0	0	0	0	2	2	2

lepton assignment

flavon A4-singlet

two flavon A4-triplets with different vevs alignments

$$\langle \varphi_T \rangle = (v_T, 0, 0)$$

$$\langle \varphi_S \rangle = (v_S, v_S, v_S)$$

auxiliary fields needed for the alignments

A PROTOTYPE A4 MODEL (SUSY)

$$w_l = y_e e^c(\varphi_T l) + y_\mu \mu^c(\varphi_T l)' + y_\tau \tau^c(\varphi_T l)'' + y(\nu^c l) + (x_A \xi + \tilde{x}_A \tilde{\xi})(\nu^c \nu^c) + x_B(\varphi_S \nu^c \nu^c) + h.c.$$

$$m_l = v_d \frac{v_T}{\Lambda} \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix}$$

$$m_\nu^D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} y v_u$$

permutation of mu-tau

$$\langle \varphi_T \rangle = (v_T, 0, 0)$$

A PROTOTYPE A4 MODEL (SUSY)

$$w_l = y_e e^c(\varphi_T l) + y_\mu \mu^c(\varphi_T l)' + y_\tau \tau^c(\varphi_T l)'' + y(\nu^c l) + (x_A \xi + \tilde{x}_A \tilde{\xi})(\nu^c \nu^c) + x_B(\varphi_S \nu^c \nu^c) + h.c.$$

$$\langle \varphi_S \rangle = (v_S, v_S, v_S)$$

$$m_l = v_d \frac{v_T}{\Lambda} \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix}$$

$$M = \begin{pmatrix} A + 2B/3 & -B/3 & -B/3 \\ -B/3 & 2B/3 & A - B/3 \\ -B/3 & A - B/3 & 2B/3 \end{pmatrix} u$$

TBM mass matrix

with only two complex parameters

DEVIATION FROM TBM

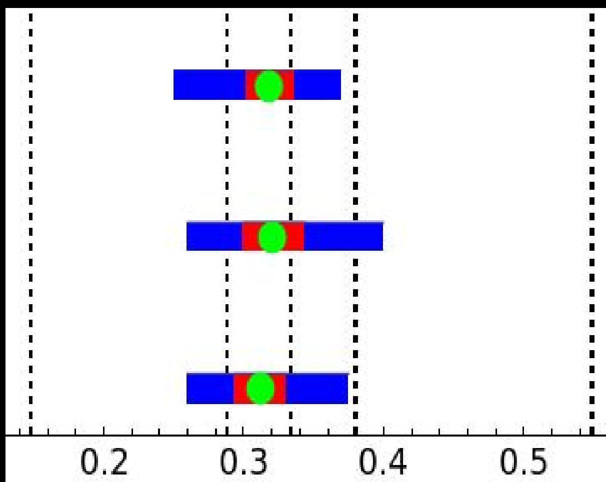
$$\sin^2\theta_{12} = \frac{1}{3} + \mathcal{O}(\varepsilon)$$
$$\sin^2\theta_{23} = \frac{1}{2} + \mathcal{O}(\varepsilon)$$
$$\sin\theta_{13} = \mathcal{O}(\varepsilon)$$

Altarelli, Feruglio 1002.0211

In general the three mixing angles receive corrections of the same order from higher order effects → ε

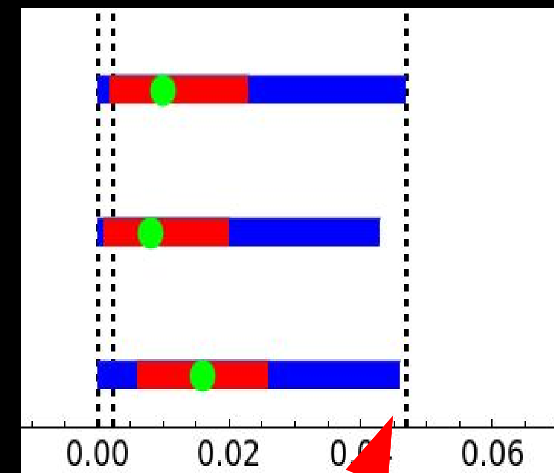
DEVIATION FROM TBM

solar



$$\sin^2\theta_{12} = \frac{1}{3} + \mathcal{O}(\varepsilon)$$
$$\sin^2\theta_{23} = \frac{1}{2} + \mathcal{O}(\varepsilon)$$
$$\sin\theta_{13} = \mathcal{O}(\varepsilon)$$

reactor



$$\varepsilon \sim \lambda_C^2$$

$$\varepsilon \sim \lambda_C$$

$$\sin\theta_{13} \sim \lambda_C^2$$

This is a limitation for most of the models with TBM at leading order (LO)

See also Albright, Rodejohann PLB665 (08)

small corrections to TBM neutrino mass texture → **small deviations of TBM mixing**

Perturbation	Results	
	NH	IH ($\lvert A\rvert \simeq \lvert B/2\rvert$)
$\begin{pmatrix} A & B & -B(1+\epsilon) \\ \cdot & \frac{1}{2}(A+B+D) & \frac{1}{2}(D-A-B) \\ \cdot & \cdot & \frac{1}{2}(A+B+D) \end{pmatrix}$	$\lvert U_{e3}\rvert = \mathcal{O}(\epsilon\sqrt{R})$ $\sin^2\theta_{23} \simeq \frac{1}{2} + \mathcal{O}(\epsilon^2)$ $\sin^2\theta_{12} \simeq \frac{1}{3}(1 + \frac{\lvert U_{e3}\rvert}{\sqrt{R}})$	$\lvert U_{e3}\rvert \simeq \frac{\epsilon}{4\sqrt{2}}$ $\sin^2\theta_{23} \simeq \frac{1}{2}(1+\epsilon)$ $\sin^2\theta_{12} \simeq \frac{1}{3} + \frac{8\sqrt{2}}{27}\lvert U_{e3}\rvert$
$\begin{pmatrix} A & B & -B \\ \cdot & \frac{1}{2}(A+B+D)(1+\epsilon) & \frac{1}{2}(D-A-B) \\ \cdot & \cdot & \frac{1}{2}(A+B+D) \end{pmatrix}$	$\lvert U_{e3}\rvert = \mathcal{O}(\epsilon\sqrt{R})$ $\sin^2\theta_{23} \simeq \frac{1}{2}(1+\epsilon/2)$ $\tan 2\theta_{12} \simeq \frac{2\sqrt{2}}{1+\mathcal{O}(\epsilon/\sqrt{R})}$	$\lvert U_{e3}\rvert \simeq \frac{\epsilon}{8\sqrt{2}}$ $\sin^2\theta_{23} = \frac{1}{2}(1+\lvert U_{e3}\rvert)$ $\sin^2\theta_{12} \simeq \frac{1}{3}(1-\mathcal{O}(\epsilon))$
$\begin{pmatrix} A(1+\epsilon) & B & -B \\ \cdot & \frac{1}{2}(A+B+D) & \frac{1}{2}(D-A-B) \\ \cdot & \cdot & \frac{1}{2}(A+B+D) \end{pmatrix}$	$\lvert U_{e3}\rvert = 0$ $\sin^2\theta_{23} = \frac{1}{2}$ $\sin^2\theta_{12} \simeq \frac{1}{3} + \frac{4}{27}\epsilon$	$\lvert U_{e3}\rvert = 0$ $\sin^2\theta_{23} = \frac{1}{2}$ $\sin^2\theta_{12} \simeq \frac{1}{3} - \frac{2}{27}\epsilon$
$\begin{pmatrix} A & B & -B \\ \cdot & \frac{1}{2}(A+B+D) & \frac{1}{2}(D-A-B)(1+\epsilon) \\ \cdot & \cdot & \frac{1}{2}(A+B+D) \end{pmatrix}$	$\lvert U_{e3}\rvert = 0$ $\sin^2\theta_{23} = \frac{1}{2}$ $\sin^2\theta_{12} \simeq \frac{1}{3} - \frac{2}{27}\epsilon/\sqrt{R}$	$\lvert U_{e3}\rvert = 0$ $\sin^2\theta_{23} = \frac{1}{2}$ $\sin^2\theta_{12} \simeq \frac{1}{3} - \frac{1}{27}\epsilon$

from Plentinger, Rodejohann PLB625

approximatively
TBM neutrino mass texture ?

⇐

small deviations
of TBM mixing

approximate TBM neutrino mass: *a model independent analysis*

$$s_{13} = \frac{r}{\sqrt{2}}, \quad s_{12} = \frac{1}{\sqrt{3}}(1+s), \quad s_{23} = \frac{1}{\sqrt{2}}(1+a)$$

King, PLB659 (2008)

$$U \approx \begin{pmatrix} \sqrt{\frac{2}{3}}(1-\frac{1}{2}s) & \frac{1}{\sqrt{3}}(1+s) & \frac{1}{\sqrt{2}}re^{-i\delta} \\ -\frac{1}{\sqrt{6}}(1+s-a+re^{i\delta}) & \frac{1}{\sqrt{3}}(1-\frac{1}{2}s-a-\frac{1}{2}re^{i\delta}) & \frac{1}{\sqrt{2}}(1+a) \\ \frac{1}{\sqrt{6}}(1+s+a-re^{i\delta}) & -\frac{1}{\sqrt{3}}(1-\frac{1}{2}s+a+\frac{1}{2}re^{i\delta}) & \frac{1}{\sqrt{2}}(1-a) \end{pmatrix}$$

$$m_\nu = U \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} U^T$$

is approximatively TBM for small *a, s, r* ?

PARAMETRIZE THE DEVIATION OF THE TBM TEXTURE

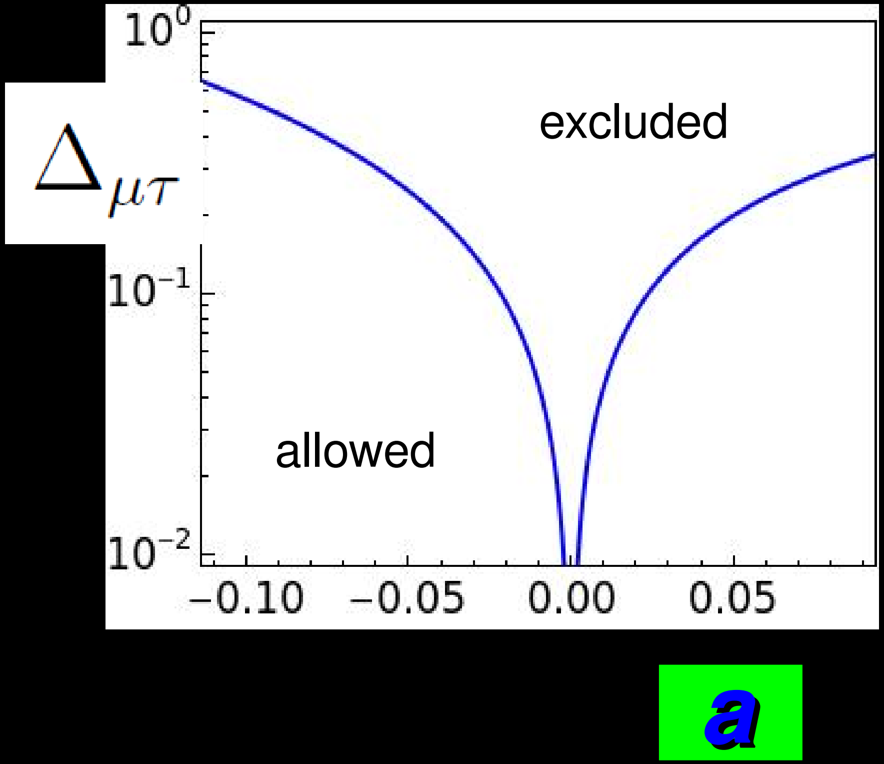
Abbas, Smirnov 1004.0099

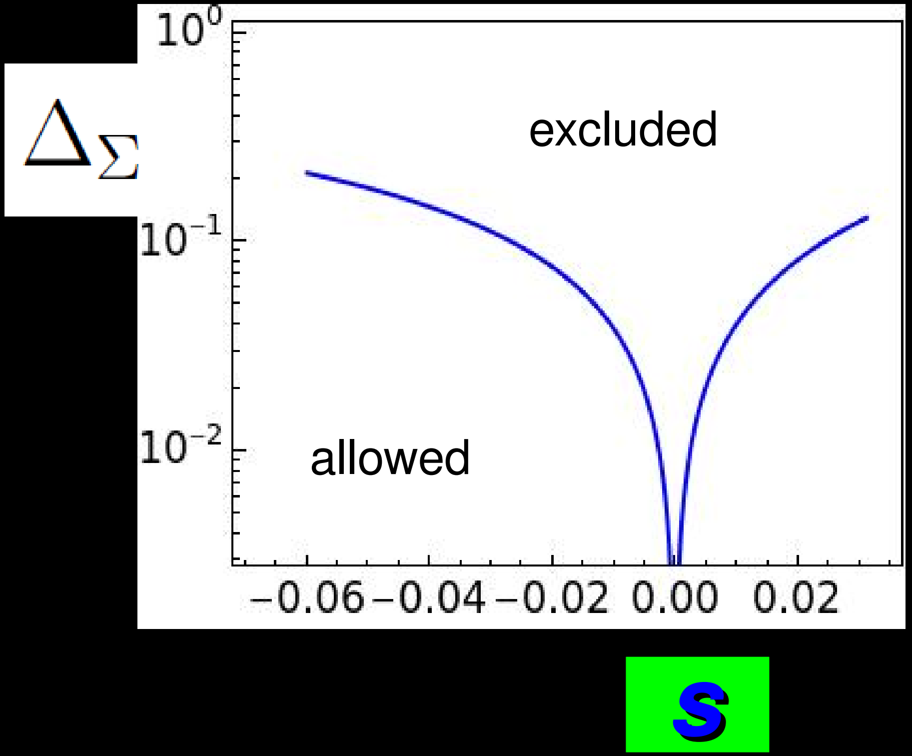
$m_{\nu,\,12} = m_{\nu,\,13}$ → $\Delta_e \equiv \dfrac{m_{e\mu} - m_{e\tau}}{m_{e\mu}}$

$m_{\nu,\,22} = m_{\nu,\,33}$ → $\Delta_{\mu\tau} \equiv \dfrac{m_{\mu\mu} - m_{\tau\tau}}{m_{\tau\tau}}$

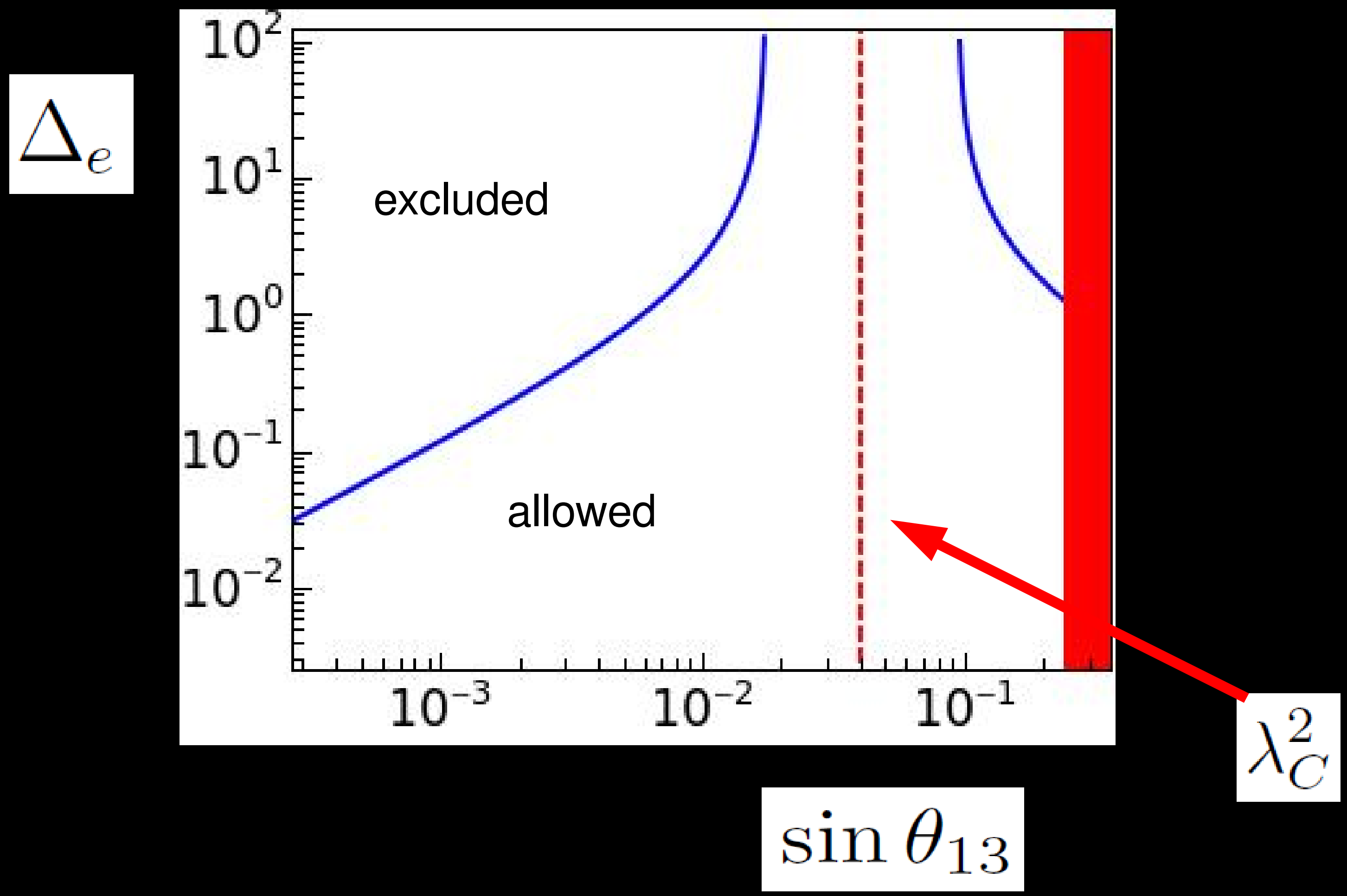
$m_{\nu,\,11} + m_{\nu,\,13} = m_{\nu,\,22} + m_{\nu,\,23}$ → $\Delta_\Sigma \equiv \dfrac{\Sigma_L - \Sigma_R}{\Sigma_R}$

$$\Sigma_L \equiv m_{ee} + \frac{m_{e\mu} + m_{e\tau}}{2}, \quad \Sigma_R \equiv m_{\mu\tau} + \frac{m_{\mu\mu} + m_{\tau\tau}}{2}$$

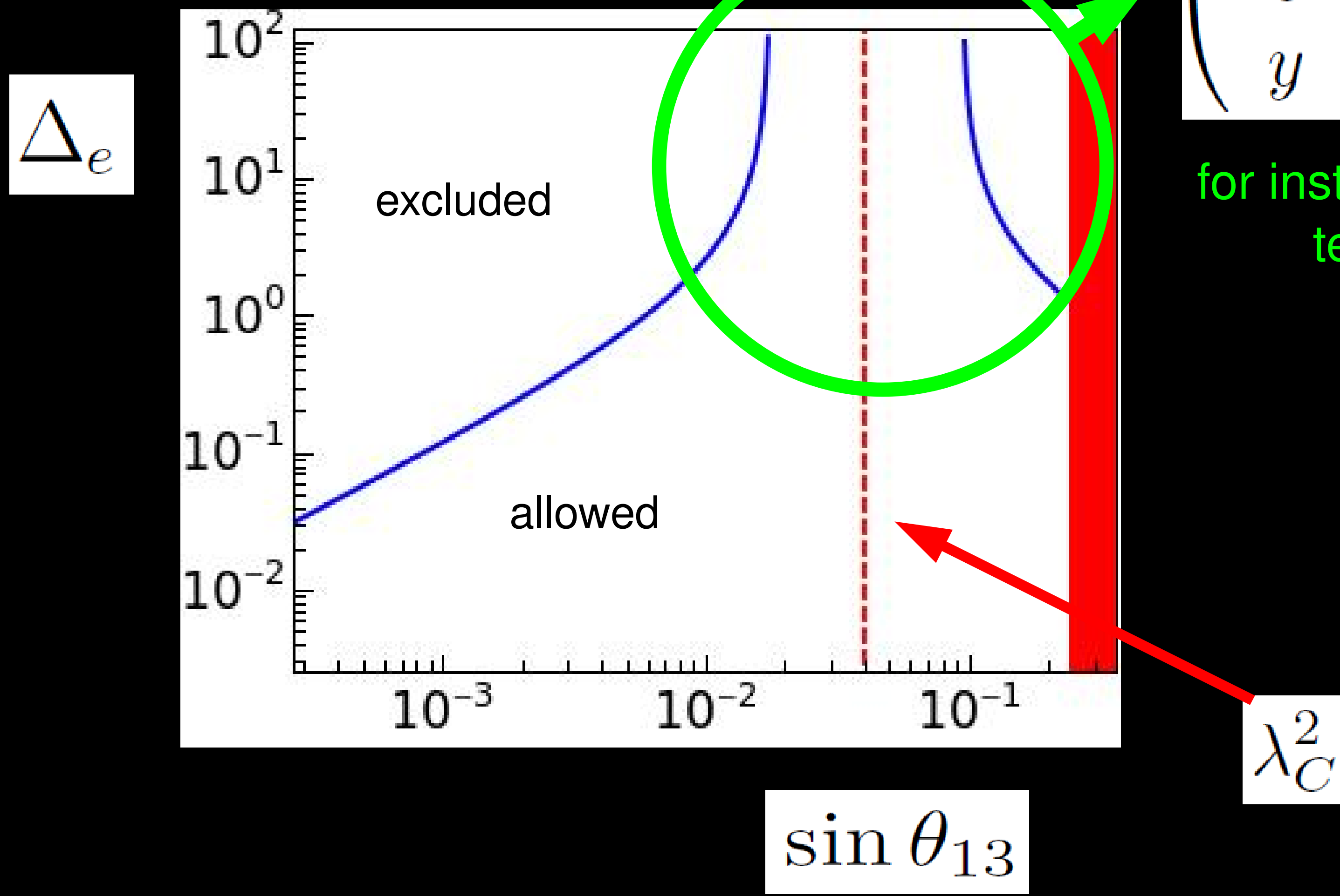




small deviation of atmospheric and solar angles give small variation of TBM neutrino mass texture



If reactor angle is **small** neutrino mass texture can be very **different from TBM** (depending from the Dirac phase)



$$\begin{pmatrix} x & 0 & y \\ 0 & c & d \\ y & d & c \end{pmatrix}$$

for instance zero texture

If reactor angle is **small** neutrino mass texture can be very **different from TBM** (depending from the Dirac phase)

$$S_3 \otimes Z_5 \otimes Z_2$$

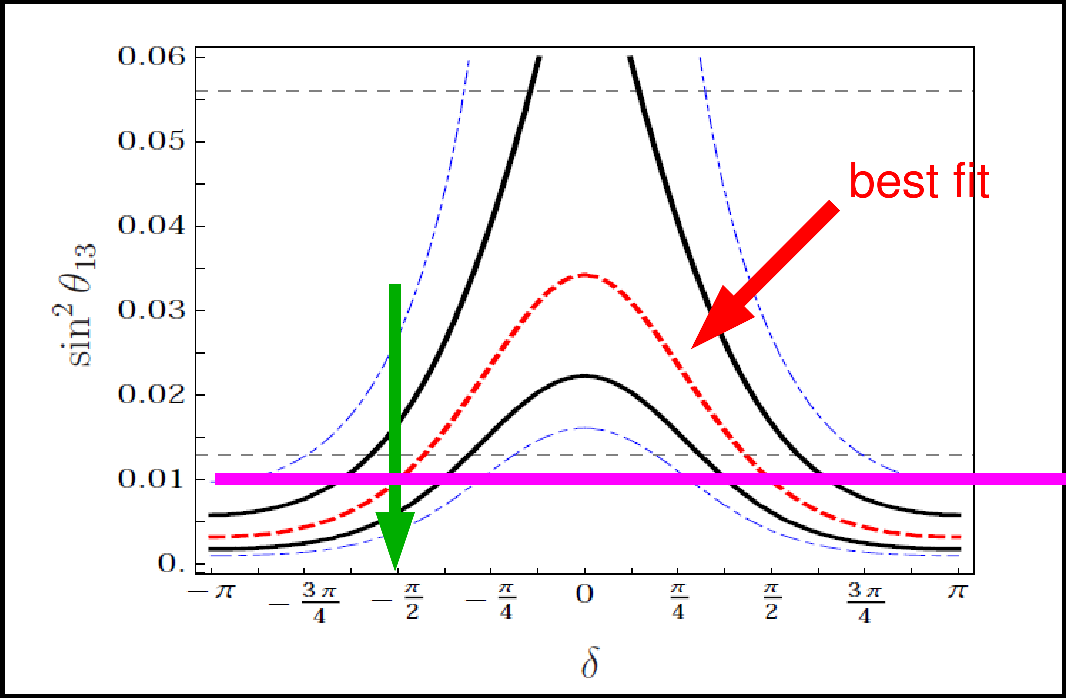
Meloni, Morisi, Peinado 1005.3482

fields	$L_D = L_{1,2}$	L_3	$l_{R_D} = l_{R_{1,2}}$	l_{R_3}	H_D	H_S	H'_S	χ
$SU_L(2)$	2	2	1	1	2	2	2	1
S_3	2	1	2	1	2	1	1	1
Z_2	+	−	+	−	+	+	−	+
Z_5	ω^2	ω	ω	ω^2	ω^3	ω^4	ω^4	ω^2

$$M_l = \begin{pmatrix} \sqrt{m_e m_\mu} & 0 & 0 \\ -m_\mu(1-\frac{m_e}{m_\mu}) & \sqrt{m_e m_\mu} & 0 \\ 0 & 0 & m_\tau \end{pmatrix}$$

$$M_\nu = \begin{pmatrix} 0 & b & 0 \\ b & a & c \\ 0 & c & d \end{pmatrix}$$

Fritzsch neutrino mass matrix



$$\sin^2\theta_{13} \approx 0.01$$

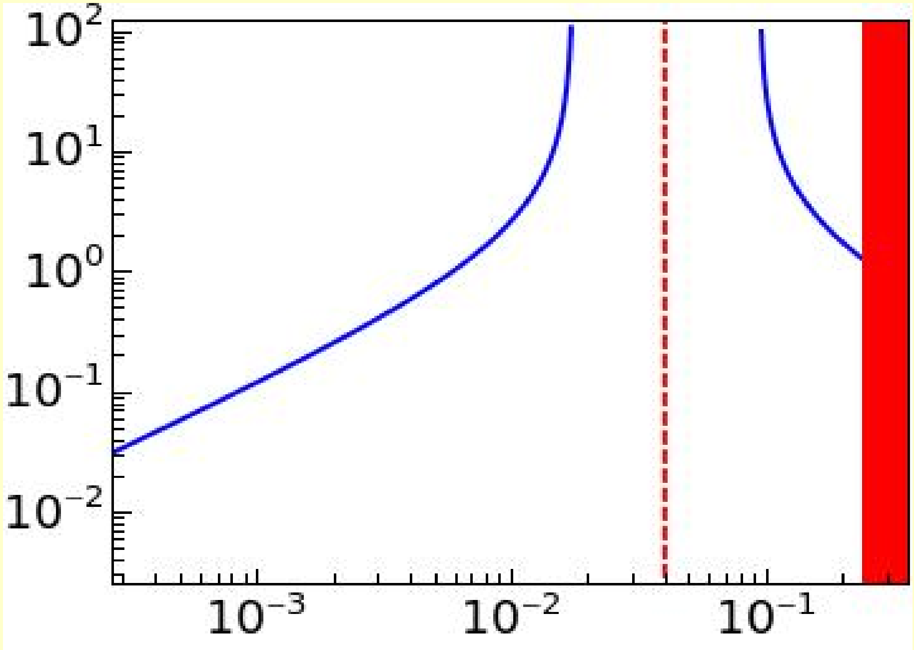
$$\delta \sim \pm\pi/2$$

CONCLUSIONS

- In general small deviations of TBM mixing *(within the sensitivity of future experiments)* do not imply TBM neutrino mass matrix

- Neutrino mass texture different from TBM are also interesting, new approach must be studied

- However discrete non-Abelian flavor symmetries simply give TBM mixing (*or deviation*) that is in very good agreement with data



$$\Delta_e = \frac{2\sqrt{2}(3\sqrt{\Delta_{atm} + m_1^2} - e^{2i\delta}(2m_1 + \sqrt{\Delta_{sol} + m_1^2}))s_{13}}{-2e^{i\delta}(-m_1 + \sqrt{\Delta_{sol} + m_1^2}) + 3\sqrt{2}\sqrt{\Delta_{atm} + m_1^2}s_{13}\sqrt{2}e^{2i\delta}(2m_1 + \sqrt{\Delta_{sol} + m_1^2})}$$

$m_1 = 0, \qquad \delta = 0$

pole

$$s_{13} = \frac{\sqrt{2\Delta_{sol}}}{3\sqrt{\Delta_{atm}}} \sim 10^{-2} \div 10^{-1}$$