Flavor symmetries and GUTs

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0802.0090 (JHEP 0803:052,2008), 1003.4249 (JHEP 1006:048,2010)

Outline

- Observations: fermion masses and mixings
- Overview of GUTs with flavor symmetries
- Two examples
  - $SU(5) \times A_4$ model

    (Altarelli et al. ('08))
  - $SU(5) \times S_4$ model

    (H et al. ('10))
- Conclusions & Outlook

Observations: Fermion Masses and Mixings

(Jamin ('06), Xing et al. ('07))

	Mass at M_Z	in units of $m_t(M_Z)$
u	$(1.7 \pm 0.4)\,\mathrm{MeV}$	λ^8
c	$(0.62 \pm 0.03)\,\mathrm{GeV}$	λ^4
t	$(171 \pm 3)\,\mathrm{GeV}$	1

$\lambda \equiv \theta_C \approx 0.22$

	Mass at M_Z	in units of $m_b(M_Z)$
d	$(3.0 \pm 0.6)\,\mathrm{MeV}$	λ^4
s	$(54 \pm 8)\,\mathrm{MeV}$	λ^2
b	$(2.87 \pm 0.03)\,\mathrm{GeV}$	1

	Mass at M_Z	in units of $m_\tau(M_Z)$
e	$(0.486570161 \pm 0.000000042)\,\mathrm{MeV}$	$\lambda^{4\div 5}$
μ	$(102.7181359 \pm 0.0000092)\,\mathrm{MeV}$	λ^2
τ	$1.74624^{+0.00020}_{-0.00019}\,\mathrm{GeV}$	1

Observations: Fermion Masses and Mixings

- Mild hierarchy among light neutrino masses
  - Two mass squared differences Δm^2_{21} and $|\Delta m^2_{31}|$ are known $(2\,\sigma)$

    (Schwetz et al. ('08, '10 update))

$$\Delta m^2_{21} = \left(7.59^{+0.44}_{-0.37}\right) \cdot 10^{-5}\ \mathrm{eV}^2 \quad \text{and} \quad |\Delta m^2_{31}| = \left(2.40^{+0.24}_{-0.22}\right) \cdot 10^{-3}\ \mathrm{eV}^2$$

  - Cosmological data give upper bound on m_0 ***(Fogli et al. ('08))***

$$\sum m_i \lesssim 0.7\ \mathrm{eV} \qquad (2\,\sigma)$$

  - The bounds on m_β and $|m_{ee}|$ also constrain m_0

    (Kraus et al. ('04), Lobashev ('03); Klapdor et al. ('01), Aalseth et al. ('02), Arnaboldi et al. ('05), Arnold et al. ('05))

$$m_\beta \leq 2.2\,\mathrm{eV} \quad \text{and} \quad |m_{ee}| \leq (0.2 \ldots 1)\,\mathrm{eV}$$

- Normal (NH) & inverted hierarchy (IH) still allowed

Observations: Fermion Masses and Mixings

- The lepton mixing pattern is very peculiar ***(Schwetz et al. ('08, '10 update))***

$$\sin^2(\theta^l_{12}) = 0.318^{+0.042}_{-0.028}\ , \quad \sin^2(\theta^l_{23}) = 0.50^{+0.13}_{-0.11} \quad \text{and} \quad \sin^2(\theta^l_{13}) \leq 0.039$$

$$\theta^l_{12} = (34.3^{+2.5}_{-1.7})^\circ\ , \quad \theta^l_{23} = (45.0^{+7.5}_{-6.4})^\circ \quad \text{and} \quad \theta^l_{13} \leq 11.4^\circ \qquad (2\,\sigma)$$

compare to quark sector ***(PDG ('08))***

$$\theta^q_{12} \approx 13^\circ\ , \quad \theta^q_{23} \approx 2.4^\circ \quad \text{and} \quad \theta^q_{13} \approx 0.21^\circ$$

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- Special mixing patterns
  - $\mu\tau$ symmetry ***(Fukuyama/Nishiura ('97), Mohapatra/Nussinov ('99), Lam ('01))***

$$\sin^2(\theta^l_{23}) = \frac{1}{2} \; , \quad \sin^2(\theta^l_{13}) = 0$$

$$\Rightarrow U_{MNS} = \begin{pmatrix} \cos(\theta^l_{12}) & \sin(\theta^l_{12}) & 0 \\ -\frac{\sin(\theta^l_{12})}{\sqrt{2}} & \frac{\cos(\theta^l_{12})}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{\sin(\theta^l_{12})}{\sqrt{2}} & -\frac{\cos(\theta^l_{12})}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Observations: Fermion Masses and Mixings

- The lepton mixing pattern is very peculiar ***(Schwetz et al. ('08, '10 update))***

$$\sin^2(\theta^l_{12}) = 0.318^{+0.042}_{-0.028}\ , \quad \sin^2(\theta^l_{23}) = 0.50^{+0.13}_{-0.11} \quad \text{and} \quad \sin^2(\theta^l_{13}) \leq 0.039$$

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  compare to quark sector ***(PDG ('08))***

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- Special mixing patterns
  - Tri-bimaximal (TB) mixing ***(Harrison et al. ('02), Xing ('02))***

$$\sin^2(\theta^l_{12}) = \frac{1}{3}\ , \quad \sin^2(\theta^l_{23}) = \frac{1}{2}\ , \quad \sin^2(\theta^l_{13}) = 0$$

$$\Rightarrow U_{MNS} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Overview of GUTs with Flavor Symmetries

GUT	G_f	SB
$SU(5)$, $SO(10)$, E_6	(non-)Abelian: $U(1)$, Z_N vs $SU(3)$, A_4; discrete S_4 vs continuous $SU(2)$	spontaneous vs explicit; GUT + $G_f \rightarrow H \sim R_f$ vs GUT $\rightarrow H$ & $G_f \rightarrow \Phi$

GUT

- $SU(5)$
- $SO(10)$
- E_6

G_f

- (non-)Abelian
  $U(1)$, Z_N vs $SU(3)$, A_4
- discrete S_4
  vs
  continuous $SU(2)$

SB

- spontaneous
  vs
  explicit
- GUT + $G_f \rightarrow H \sim R_f$
  vs
  GUT $\rightarrow H$ & $G_f \rightarrow \Phi$

Additional features: extra-dimensional context, ...

e.g.
$SU(5) \times A_4$ *(Altarelli et al. ('08), Ciafaloni et al. ('09), Burrows/King ('09))*
$SU(5) \times S_4$ *(Ishimori et al. ('08, '10), H et al. ('10), Ding ('10))*
$SU(5) \times T'$ *(Chen/Mahanthappa ('07))*

Overview of GUTs with Flavor Symmetries

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SB

- spontaneous
  vs
  explicit
- GUT + $G_f \to H \sim R_f$
  vs
  GUT $\to H$ & $G_f \to \Phi$

Additional features: extra-dimensional context, ...

e.g.

$SO(10) \times A_4$ *(King/Malinsky ('06), Morisi et al. ('07), Bazzocchi et al. ('08))*

$SO(10) \times S_4$ *(Lee/Mohapatra ('94), Mohapatra et al. ('03), Cai/Yu ('06), Parida ('08))*

$SO(10) \times \Delta(27)$ *(de Medeiros Varzielas [dMV] et al. ('05, '06), Bazzocchi/dMV ('09))*

Overview of GUTs with Flavor Symmetries

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G_f

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  $U(1)$, Z_N vs $SU(3)$, A_4
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SB

- spontaneous
  vs
  explicit
- GUT + $G_f \longrightarrow H \sim R_f$
  vs
  GUT $\longrightarrow H$ & $G_f \longrightarrow \Phi$

Additional features: extra-dimensional context, ...

e.g.
GUT $\times U(2)_f$ ***(Barbieri et al. ('95), Barbieri/Hall ('96), Barbieri et al. ('96, '98))***
$SO(10) \times SO(3)_f$ ***(King ('05), King/Malinsky ('06))***
$SO(10) \times SU(3)_f$ ***(King/Ross ('01, '03), de Medeiros Varzielas/Ross ('05))***

Overview of GUTs with Flavor Symmetries

GUT

- $SU(5)$
- $SO(10)$
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G_f

- (non-)Abelian
  $U(1)$, Z_N vs $SU(3)$, A_4
- discrete S_4
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SB

- spontaneous
  vs
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- GUT + $G_f \to H \sim R_f$
  vs
  GUT $\to H$ & $G_f \to \Phi$

Additional features: extra-dimensional context, ...

Presented here: $SU(5) \times A_4$ ***(Altarelli et al. ('08))***
$SU(5) \times S_4$ ***(H et al. ('10))***

$SU(5) \times A_4$ Model: Basic Setup

(Altarelli et al. ('08))

- SUSY $SU(5)$ GUT in 4+1 dimensions ($1/R \sim$ GUT scale)
- UV-cutoff Λ
- Flavor symmetry is A_4
- $F \sim (\overline{\mathbf{5}}, \mathbf{3})$ and $N \sim (\mathbf{1}, \mathbf{3})$ for TB mixing
- T_i are singlets of A_4 and $T_{1,2}$ are bulk fields
- Flavons (gauge singlets) break A_4 spontaneously on $SU(5)$ brane
- Additional (local) $U(1)$ for charged fermion mass hierarchy
- GUT Higgs fields $H_5 \sim (\mathbf{5}, \mathbf{1})$, $H_{\overline{5}} \sim (\overline{\mathbf{5}}, \mathbf{1}')$ are in bulk
- Flavor-independent Z_3 symmetry for alignment of flavon VEVs

$SU(5) \times A_4$ Model: Main Results

- TB mixing in neutrino sector
- Sum rule among (complex) neutrino masses

$$\frac{2}{m_2} = \frac{1}{m_1} - \frac{1}{m_3}$$

  which is characteristic for a certain class of A_4 models

  (Altarelli/Feruglio ('05), Altarelli/Meloni ('09))

- Fermion mass hierarchies are correctly reproduced
- Bottom-tau unification is achieved
  Mass relations among the first two generations are relaxed
- Quark mixings are reproduced
- GUT symmetry is broken by extra dimensions
  DT-splitting problem is solved

Group Theory of A_4

- A_4 is the group of even permutations of four distinct objects, isomorphic to the symmetry group T of a tetrahedron, with order 12
- Irred. reps. are $\mathbf{1}$, $\mathbf{1}'$, $\mathbf{1}''$ and $\mathbf{3}$
- Generators S and T fulfill

$$S^2 = \mathbb{1} \; , \quad T^3 = \mathbb{1} \; , \quad (ST)^3 = \mathbb{1}$$

Group Theory of A_4

- A_4 is the group of even permutations of four distinct objects, isomorphic to the symmetry group T of a tetrahedron, with order 12
- Irred. reps. are $\mathbf{1}$, $\mathbf{1}'$, $\mathbf{1}''$ and $\mathbf{3}$
- Choice of S and T ***(Altarelli/Feruglio ('05))*** ($\omega = e^{2\pi i/3}$)

$$\begin{aligned}
\mathbf{1}: &\quad S = 1\,, && T = 1\,,\\
\mathbf{1}': &\quad S = 1\,, && T = \omega^2\,,\\
\mathbf{1}'': &\quad S = 1\,, && T = \omega\,,\\
\mathbf{3}: &\quad S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}\,, && T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}
\end{aligned}$$

$SU(5) \times A_4$ Model: Neutrino Sector

Field	F	N	H_5	φ_S	$\xi,\ \tilde{\xi}$
$SU(5)$	$\bar{\mathbf{5}}$	$\mathbf{1}$	$\mathbf{5}$	$\mathbf{1}$	$\mathbf{1}$
A_4	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{1}$
Z_3	ω	ω	ω	ω	ω

Superpotential in 4 dimensions

$$\frac{y^D}{\Lambda^{1/2}} H_5 F N + (x_a \xi + \tilde{x}_a \tilde{\xi}) N N + x_b \varphi_S N N$$

with vacuum $\langle \xi \rangle \neq 0\,,\ \langle \tilde{\xi} \rangle = 0\,,\ \langle \varphi_S \rangle \propto (1,1,1)^t$ leads to $\left(s = \frac{1}{\sqrt{\pi R \Lambda}} \approx \lambda\right)$

$$M_\nu = \frac{1}{3a(a+b)} \begin{pmatrix} 3a+b & b & b \\ b & \frac{2ab+b^2}{b-a} & \frac{b^2-ab-3a^2}{b-a} \\ b & \frac{b^2-ab-3a^2}{b-a} & \frac{2ab+b^2}{b-a} \end{pmatrix} \frac{s^2 (v_u^0)^2}{\Lambda}$$

$SU(5) \times A_4$ Model: Up Quarks

Field	T_1	T_2	T_3	H_5	θ	θ''
$SU(5)$	$\mathbf{10}$	$\mathbf{10}$	$\mathbf{10}$	$\mathbf{5}$	$\mathbf{1}$	$\mathbf{1}$
A_4	$\mathbf{1}''$	$\mathbf{1}'$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}''$
$U(1)$	3	1	0	0	-1	-1
Z_3	ω	ω	ω	ω	1	1

Superpotential in 4 dimensions

$$\frac{1}{\Lambda^{1/2}} H_5 T_3 T_3 + \frac{\theta''}{\Lambda^2} H_5 T_2 T_3 + \frac{\theta''^2}{\Lambda^{7/2}} H_5 T_2 T_2 + \frac{\theta\theta''^2}{\Lambda^4} H_5 T_1 T_3$$
$$+ \frac{\theta^4}{\Lambda^{11/2}} H_5 T_1 T_2 + \frac{\theta\theta''^3}{\Lambda^{11/2}} H_5 T_1 T_2 + \frac{\theta^5\theta''}{\Lambda^{15/2}} H_5 T_1 T_1 + \frac{\theta^2\theta''^4}{\Lambda^{15/2}} H_5 T_1 T_1$$

with VEVs of FN fields and bulk suppression factor

$$\langle\theta\rangle/\Lambda \approx \lambda\,, \; \langle\theta''\rangle/\Lambda \approx \lambda \quad \text{and} \quad \frac{1}{\sqrt{\pi R \Lambda}} \approx \lambda$$

$SU(5) \times A_4$ Model: Up Quarks

Field	T_1	T_2	T_3	H_5	θ	θ''
$SU(5)$	$\mathbf{10}$	$\mathbf{10}$	$\mathbf{10}$	$\mathbf{5}$	$\mathbf{1}$	$\mathbf{1}$
A_4	$\mathbf{1}''$	$\mathbf{1}'$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}''$
$U(1)$	3	1	0	0	-1	-1
Z_3	ω	ω	ω	ω	1	1

... we get for the up quark mass matrix M_u

$$M_u \approx \begin{pmatrix} \lambda^8 & \lambda^6 & \lambda^4 \\ \lambda^6 & \lambda^4 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix} \lambda v_u^0$$

$SU(5) \times A_4$ Model: Down Quarks & Charged Leptons

Field	T_1	T_2	T_3	F	$H_{\bar{5}}$	φ_T	θ	θ''
$SU(5)$	$\mathbf{10}$	$\mathbf{10}$	$\mathbf{10}$	$\bar{\mathbf{5}}$	$\bar{\mathbf{5}}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
A_4	$\mathbf{1}''$	$\mathbf{1}'$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{1}'$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}''$
$U(1)$	3	1	0	0	0	0	-1	-1
Z_3	ω	ω	ω	ω	ω	1	1	1

Superpotential in 4 dimensions

$$\frac{1}{\Lambda^{3/2}} H_{\bar{5}} \varphi_T F T_3 + \frac{\theta}{\Lambda^3} H_{\bar{5}} \varphi_T F T_2 + \frac{\theta^3}{\Lambda^5} H_{\bar{5}} \varphi_T F T_1 + \frac{\theta''^3}{\Lambda^5} H_{\bar{5}} \varphi_T F T_1$$
$$+ \frac{\theta''}{\Lambda^3} H_{\bar{5}} \varphi_T F T_2 + \frac{\theta^2 \theta''}{\Lambda^5} H_{\bar{5}} \varphi_T F T_1 + \frac{\theta \theta''^2}{\Lambda^5} H_{\bar{5}} \varphi_T F T_1 + \ldots$$

with the VEV alignment $\langle \varphi_T \rangle \propto (1,0,0)^t$, the bulk suppression $\frac{1}{\sqrt{\pi R \Lambda}} \approx \lambda$

and the VEV sizes $\langle \theta \rangle / \Lambda \approx \lambda\,,\ \langle \theta'' \rangle / \Lambda \approx \lambda\,,\ \langle \varphi_T \rangle / \Lambda \approx \lambda^2$

$SU(5) \times A_4$ Model: Down Quarks & Charged Leptons

Field	T_1	T_2	T_3	F	$H_{\overline{5}}$	φ_T	θ	θ''
$SU(5)$	$\mathbf{10}$	$\mathbf{10}$	$\mathbf{10}$	$\overline{\mathbf{5}}$	$\overline{\mathbf{5}}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
A_4	$\mathbf{1}''$	$\mathbf{1}'$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{1}'$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}''$
$U(1)$	3	1	0	0	0	0	-1	-1
Z_3	ω	ω	ω	ω	ω	1	1	1

... we get for down quarks and charged leptons at LO

$$M_d = \begin{pmatrix} \lambda^4 & \lambda^4 & \lambda^4 \\ \dots & \lambda^2 & \lambda^2 \\ \dots & \dots & 1 \end{pmatrix} \lambda^3 v_d^0 \quad \text{and} \quad M_l = \begin{pmatrix} \lambda^4 & \dots & \dots \\ \lambda^4 & \lambda^2 & \dots \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix} \lambda^3 v_d^0$$

which leads to $m_b \approx m_\tau$, but relaxes the mass relations among the first two generations, since T_1 and T_2 are in the bulk.

$SU(5) \times A_4$ Model: Down Quarks & Charged Leptons

Field	T_1	T_2	T_3	F	$H_{\overline{5}}$	φ_T	θ	θ''
$SU(5)$	$\mathbf{10}$	$\mathbf{10}$	$\mathbf{10}$	$\overline{\mathbf{5}}$	$\overline{\mathbf{5}}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
A_4	$\mathbf{1}''$	$\mathbf{1}'$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{1}'$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}''$
$U(1)$	3	1	0	0	0	0	-1	-1
Z_3	ω	ω	ω	ω	ω	1	1	1

... and if we also include NLO corrections

$$M_d = \begin{pmatrix} \lambda^4 & \lambda^4 & \lambda^4 \\ \lambda^4 & \lambda^2 & \lambda^2 \\ \lambda^2 & \lambda^2 & 1 \end{pmatrix} \lambda^3 v_d^0 \quad \text{and} \quad M_l = \begin{pmatrix} \lambda^4 & \lambda^4 & \lambda^2 \\ \lambda^4 & \lambda^2 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix} \lambda^3 v_d^0$$

These lead, together with NLO corrections to M_ν, to corrections of $O(\lambda^2)$ to TB lepton mixing.

$SU(5) \times S_4$ Model: Basic Setup

(H et al. ('10))

- SUSY $SU(5)$ GUT in 4 dimensions
- Effective theory with cutoff M for flavor dynamics
- Flavor symmetry is S_4
- $F \sim (\overline{\mathbf{5}}, \mathbf{3})$ and $N \sim (\mathbf{1}, \mathbf{3})$ for TB mixing
- $T_3 \sim (\mathbf{10}, \mathbf{1})$ and $T \sim (\mathbf{10}, \mathbf{2})$ for top mass
- Flavons (gauge singlets) break S_4 spontaneously
- Additional (global) $U(1)$ symmetry for forbidding unwanted operators
- GUT Higgs fields are $H_5 \sim (\mathbf{5}, \mathbf{1})$, $H_{\overline{5}} \sim (\overline{\mathbf{5}}, \mathbf{1})$ and $H_{\overline{45}} \sim (\overline{\mathbf{45}}, \mathbf{1})$

$SU(5) \times S_4$ Model: Main Results

- TB mixing encoded in right-handed neutrino Majorana mass terms
- Large top quark mass through $T + T_3 \sim \mathbf{2} + \mathbf{1}$ under S_4
- Quark mixings arise from down quark sector
- GST and GJ relations are achieved through specific high energy completion
- Corrections from charged lepton sector relevant for lepton mixings, lead to sum rules

Group Theory of S_4

- S_4 is the permutation group of four distinct objects, isomorphic to the symmetry group O of a regular octahedron, with order 24
- Irred. reps. are $\mathbf{1}$, $\mathbf{1}'$, $\mathbf{2}$, $\mathbf{3}$ and $\mathbf{3}'$
- Generators S, T and U fulfill

$$S^2 = \mathbb{1} \; , \;\; T^3 = \mathbb{1} \; , \;\; U^2 = \mathbb{1} \; ,$$
$$(ST)^3 = \mathbb{1} \; , \;\; (SU)^2 = \mathbb{1} \; , \;\; (TU)^2 = \mathbb{1} \; , \;\; (STU)^4 = \mathbb{1}$$

Group Theory of S_4

- S_4 is the permutation group of four distinct objects, isomorphic to the symmetry group O of a regular octahedron, with order 24
- Irred. reps. are $\mathbf{1}$, $\mathbf{1}'$, $\mathbf{2}$, $\mathbf{3}$ and $\mathbf{3}'$
- Choice of S, T and U ***(Bazzocchi et al. ('09))*** ($\omega = e^{2\pi i/3}$)

$$\mathbf{1}: \quad S = 1\,, \qquad T = 1\,, \qquad U = 1\,,$$

$$\mathbf{1}': \quad S = 1\,, \qquad T = 1\,, \qquad U = -1\,,$$

$$\mathbf{2}: \quad S = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\,, \qquad T = \begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix}\,, \qquad U = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\,,$$

$$\mathbf{3}: \quad S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}\,, \quad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}\,, \quad U = -\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}\,,$$

$$\mathbf{3}': \quad S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}\,, \quad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}\,, \quad U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$SU(5) \times S_4$ Model: TB Mixing in Neutrino Sector

Field	F	N	H_5	$\Phi^\nu_{3'}$	Φ^ν_2	Φ^ν_1
$SU(5)$	$\bar{\mathbf{5}}$	$\mathbf{1}$	$\mathbf{5}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
S_4	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{3'}$	$\mathbf{2}$	$\mathbf{1}$
$U(1)$	y	$-y$	0	$2y$	$2y$	$2y$
$U(1)$	4	-4	0	8	8	8

Yukawas at leading order

$$y_D F N H_5 + \alpha N N \Phi^\nu_1 + \beta N N \Phi^\nu_2 + \gamma N N \Phi^\nu_{3'}$$

For vacuum

$$\langle \Phi^\nu_{3'} \rangle \propto (1,1,1)^t \, , \qquad \langle \Phi^\nu_2 \rangle \propto (1,1)^t \, , \qquad \langle \Phi^\nu_1 \rangle \neq 0$$

which leaves invariant the group generated by S and U in the neutrino sector ...

$SU(5) \times S_4$ Model: TB Mixing in Neutrino Sector

Field	F	N	H_5	$\Phi^\nu_{3'}$	Φ^ν_2	Φ^ν_1
$SU(5)$	$\bar{\mathbf{5}}$	$\mathbf{1}$	$\mathbf{5}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
S_4	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{3'}$	$\mathbf{2}$	$\mathbf{1}$
$U(1)$	y	$-y$	0	$2y$	$2y$	$2y$
$U(1)$	4	-4	0	8	8	8

Yukawas at leading order

$$y_D F N H_5 + \alpha N N \Phi^\nu_1 + \beta N N \Phi^\nu_2 + \gamma N N \Phi^\nu_{3'}$$

... we get

$$M_D = y_D \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} v_u \quad \text{and} \quad M_R = \begin{pmatrix} \alpha\varphi^\nu_1 + 2\gamma\varphi^\nu_{3'} & \beta\varphi^\nu_2 - \gamma\varphi^\nu_{3'} & \beta\varphi^\nu_2 - \gamma\varphi^\nu_{3'} \\ \beta\varphi^\nu_2 - \gamma\varphi^\nu_{3'} & \beta\varphi^\nu_2 + 2\gamma\varphi^\nu_{3'} & \alpha\varphi^\nu_1 - \gamma\varphi^\nu_{3'} \\ \beta\varphi^\nu_2 - \gamma\varphi^\nu_{3'} & \alpha\varphi^\nu_1 - \gamma\varphi^\nu_{3'} & \beta\varphi^\nu_2 + 2\gamma\varphi^\nu_{3'} \end{pmatrix}$$

$SU(5) \times S_4$ Model: Up Quarks

Field	T_3	T	H_5	Φ_2^u	$\widetilde{\Phi}_2^u$
$SU(5)$	$\mathbf{10}$	$\mathbf{10}$	$\mathbf{5}$	$\mathbf{1}$	$\mathbf{1}$
S_4	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{2}$
$U(1)$	0	x	0	$-2x$	0
$U(1)$	0	5	0	-10	0

Yukawas at leading order

$$T_3 T_3 H_5 + \frac{1}{M} T T \Phi_2^u H_5 + \frac{1}{M^2} T T \Phi_2^u \widetilde{\Phi}_2^u H_5$$

For

$$\langle \Phi_2^u \rangle \ , \ \langle \widetilde{\Phi}_2^u \rangle \ \propto (0,1)^t \ \text{ and } \ \langle \Phi_2^u \rangle / M \ , \ \langle \widetilde{\Phi}_2^u \rangle / M \approx \lambda^4$$

we get

$$M_u \approx \text{diag}(\varphi_2^u \widetilde{\varphi}_2^u / M^2, \varphi_2^u / M, 1) v_u \approx \text{diag}(\lambda^8, \lambda^4, 1) v_u$$

$SU(5) \times S_4$ Model: Down Quarks & Charged Leptons

Field	T_3	T	F	$H_{\overline{5}}$	$H_{\overline{45}}$	Φ^d_3	$\widetilde{\Phi}^d_3$	Φ^d_2
$SU(5)$	$\mathbf{10}$	$\mathbf{10}$	$\overline{\mathbf{5}}$	$\overline{\mathbf{5}}$	$\overline{\mathbf{45}}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
S_4	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{2}$
$U(1)$	0	x	y	0	z	$-y$	$-x-y-2z$	z
$U(1)$	0	5	4	0	1	-4	-11	1

Yukawas at leading order

$$\frac{1}{M} F T_3 \Phi^d_3 H_{\overline{5}} + \frac{1}{M^2} (F \widetilde{\Phi}^d_3)_1 (T \Phi^d_2)_1 H_{\overline{45}} + \frac{1}{M^3} (F \Phi^d_2 \Phi^d_2)_3 (T \widetilde{\Phi}^d_3)_3 H_{\overline{5}}$$

For vacuum alignment

$$\langle \Phi^d_3 \rangle \propto (0,1,0)^t \,, \quad \langle \widetilde{\Phi}^d_3 \rangle \propto (0,-1,1)^t \,, \quad \langle \Phi^d_2 \rangle \propto (1,0)^t$$

and

$$\langle \Phi^d_3 \rangle / M \approx \lambda^2 \,, \; \langle \widetilde{\Phi}^d_3 \rangle / M \approx \lambda^3 \,, \; \langle \Phi^d_2 \rangle / M \approx \lambda$$

$SU(5) \times S_4$ Model: GJ and GST Relations

... we find GJ relations

$$m_d \approx 3\, m_e \quad \text{and} \quad m_s \approx m_\mu/3 \quad \text{and} \quad m_b \approx m_\tau$$

and hierarchies

$$m_d : m_s : m_b \approx \lambda^4 : \lambda^2 : 1 \quad \text{with} \quad m_b \approx m_\tau \approx \lambda^2\, v_d$$

Since $M_u \propto$ diag we get also the GST relation

$$\tan\theta^q_{12} \approx \sqrt{m_d/m_s}$$

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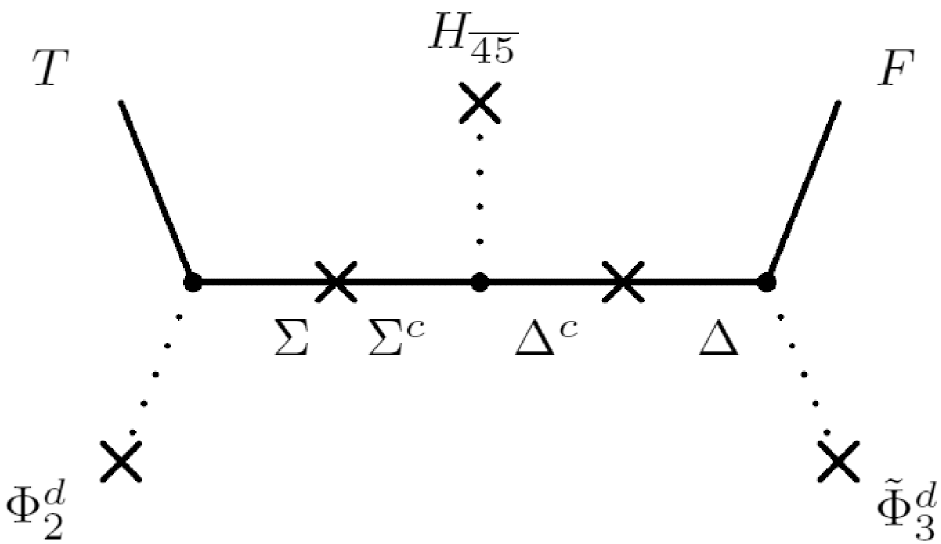
Note for $\frac{1}{M^2}(F\widetilde{\Phi}^d_3)_1(T\Phi^d_2)_1 H_{\overline{45}}$ specified high energy completion is needed



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Since $M_u \propto$ diag we get also the GST relation

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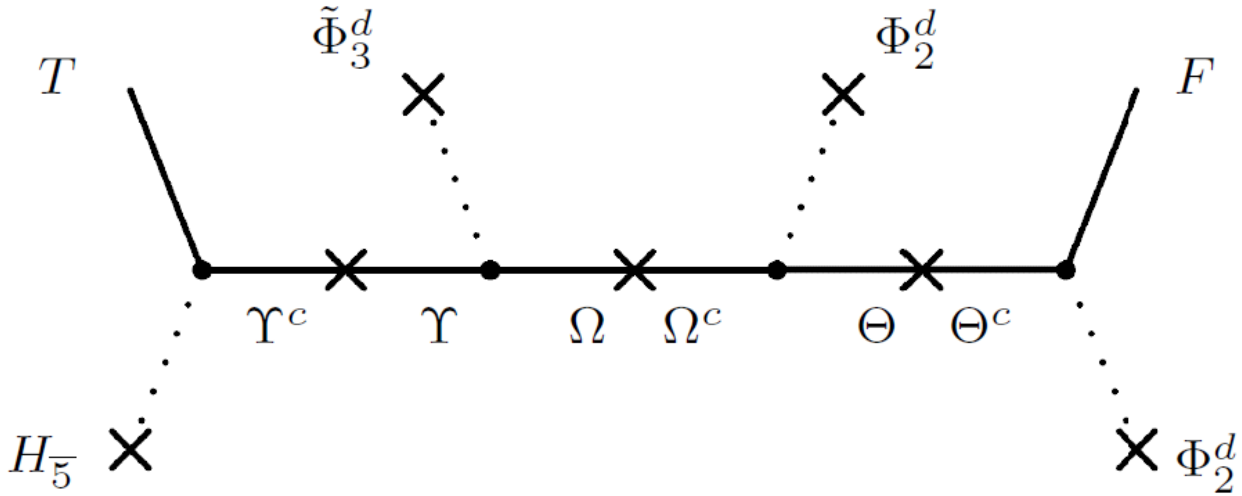

$SU(5) \times S_4$ Model: Lepton Mixing Sum Rules

Mixing in neutrino sector (up to corrections of order λ^4)

$$\sin^2(\theta_{12}^\nu) \approx \frac{1}{3} , \quad \sin^2(\theta_{23}^\nu) \approx \frac{1}{2} , \quad \sin^2(\theta_{13}^\nu) \approx 0$$

Mixing in charged lepton sector

$$\sin(\theta_{13}^e) \approx \lambda^4 , \quad \tan(\theta_{12}^e) \approx \tan(\theta_{12}^q)/3 , \quad \tan(\theta_{23}^e) \approx \lambda^4$$

$$\Downarrow$$

Lepton mixings obey sum rules ***(King ('05), Masina ('05), Antusch/King ('05))***

$$\sin^2(\theta_{12}^l) \approx \frac{1}{3}\left(1 + 2\sqrt{2}\sin(\theta_{13}^l)\cos(\delta^l)\right) , \quad \sin(\theta_{13}^l) \approx \tan(\theta_{12}^q)/(3\sqrt{2})$$

and for $\sin(\theta_{13}^l) \equiv \frac{r}{\sqrt{2}}$ and $\sin(\theta_{23}^l) \equiv \frac{1}{\sqrt{2}}(1+a)$ holds: $\quad a \approx -r^2/4$.

Conclusions & Outlook

- GUTs and flavor symmetries can lead to interesting synergies, but satisfying all constraints is non-trivial
- $SU(5) \times A_4$ and $SU(5) \times S_4$ are two successful models
- TB mixing is achieved in neutrino sector
- Charged fermion mass hierarchies and quark mixings are reproduced
- Alignment of flavon VEVs requires elaborate (super)potential
- Study of NLO corrections is mandatory

To be considered ...

- ... RG and threshold effects on fermion masses and mixings
- ... phenomenology related to sfermions

Thank you.

Back up

$SU(5) \times A_4$ Model : Flavon Potential

The flavon potential consists of two parts, F-term and D-term part

$$V = V_F + V_D$$

The F-term part is derived from

$$M\varphi_0^T \varphi_T + g\varphi_0^T \varphi_T \varphi_T$$
$$+g_1\varphi_0^S \varphi_S \varphi_S + g_2\tilde{\xi}\varphi_0^S \varphi_S + g_3\xi_0\varphi_S\varphi_S + g_4\xi_0\xi^2 + g_5\xi_0\xi\tilde{\xi} + g_6\xi_0\tilde{\xi}^2$$

where we introduced the driving fields

Driving field	φ_0^T	φ_0^S	ξ_0
$SU(5)$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
A_4	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$
Z_3	1	ω	ω

$SU(5) \times A_4$ Model : Flavon Potential

The F-term part is responsible for the alignment

$$\langle\varphi_T\rangle \propto (1,0,0)^t\,,\ \langle\varphi_S\rangle \propto (1,1,1)^t\,,\ \langle\xi\rangle \neq 0\,,\ \langle\tilde{\xi}\rangle = 0$$

while the D-term part

$$V_D = \frac{1}{2}(M_{FI}^2 - g_{FN}|\theta|^2 - g_{FN}|\theta''|^2 + \ldots)^2$$

gives VEVs to the FN fields θ and θ''.

The different origins of the VEVs naturally lead to different orders of magnitude

$$\langle\varphi_T\rangle\,,\ \langle\varphi_S\rangle\,,\ \langle\xi\rangle \approx \lambda^2\,\Lambda \qquad \text{vs} \qquad \langle\theta\rangle\,,\ \langle\theta''\rangle \approx \lambda\,\Lambda$$

$SU(5) \times S_4$ Model : Messengers

Field	Σ	Σ^c	Δ	Δ^c	Υ	Υ^c	Ω	Ω^c	Θ	Θ^c
$SU(5)$	$\overline{\mathbf{10}}$	$\mathbf{10}$	$\mathbf{5}$	$\overline{\mathbf{5}}$	$\mathbf{5}$	$\overline{\mathbf{5}}$	$\overline{\mathbf{5}}$	$\mathbf{5}$	$\overline{\mathbf{5}}$	$\mathbf{5}$
S_4	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$
$U(1)$	$-x-z$	$x+z$	$x+2z$	$-x-2z$	x	$-x$	$y+2z$	$-y-2z$	$y+z$	$-y-z$
$U(1)$	-6	6	7	-7	5	-5	6	-6	5	-5

Renormalizable terms involving these messengers are

$$\alpha_1 T\Phi_2^d\Sigma + \alpha_2 H_{\overline{45}}\Delta^c\Sigma^c + \alpha_3\Delta F\widetilde{\Phi}_3^d$$
$$+\beta_1 T\Upsilon^c H_{\overline{5}} + \beta_2\Upsilon\Omega\widetilde{\Phi}_3^d + \beta_3\Omega^c\Theta\Phi_2^d + \beta_4 F\Theta^c\Phi_2^d$$
$$+M_\Sigma\Sigma^c\Sigma + M_\Delta\Delta^c\Delta + M_\Upsilon\Upsilon^c\Upsilon + M_\Omega\Omega^c\Omega + M_\Theta\Theta^c\Theta$$
$$+\gamma_1\widetilde{\Phi}_2^u\Upsilon\Upsilon^c + \gamma_2\widetilde{\Phi}_2^u\Omega\Omega^c + \gamma_3\widetilde{\Phi}_2^u\Theta\Theta^c$$

$SU(5) \times S_4$ Model : Flavon Superpotential

Driving field	X_1^d	Y_2^d	$Z_{3'}^\nu$	Y_2^ν	$\overline{X}_1^d$	$X_{1'}^{\nu d}$	Y_2^{du}	X_1^u
$SU(5)$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
S_4	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{3}'$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}'$	$\mathbf{2}$	$\mathbf{1}$
$U(1)$	$-2z$	$2y-2z$	$-4y$	$-4y$	$x+2y+z$	$x-y+2z$	$2x-z$	$2x$
$U(1)$	-2	6	-16	-16	14	3	9	10

Superpotential

$$X_1^d(\Phi_2^d)^2 + Y_2^d(\Phi_2^d)^2(\Phi_3^d)^2/M^2$$
$$+Y_2^\nu\Phi_1^\nu\Phi_2^\nu + Y_2^\nu(\Phi_2^\nu)^2 + Y_2^\nu(\Phi_{3'}^\nu)^2 + Z_{3'}^\nu\Phi_1^\nu\Phi_{3'}^\nu + Z_{3'}^\nu\Phi_2^\nu\Phi_{3'}^\nu + Z_{3'}^\nu(\Phi_{3'}^\nu)^2$$
$$+\overline{X}_1^d\Phi_2^d\Phi_3^d\widetilde{\Phi}_3^d/M + X_{1'}^{\nu d}\widetilde{\Phi}_3^d\Phi_{3'}^\nu + Y_2^{du}\Phi_2^d\Phi_2^u + X_1^u\Phi_2^u\widetilde{\Phi}_2^u$$

$SU(5) \times S_4$ Model : Flavon Superpotential

In order to reduce the number of free parameters among the flavon VEVs we introduce further fields

$$X_1^{\rm new} \sim (\mathbf{1},\mathbf{1},18) \quad \text{and} \quad \widetilde{X}_{1'}^{\rm new} \sim (\mathbf{1},\mathbf{1'},15)$$

which couple to flavons through

$$X_1^{\rm new}\Phi_2^u(\Phi_3^d)^2/M + X_1^{\rm new}\Phi_2^d\widetilde{\Phi}_3^d(\Phi_3^d)^2/M^2$$
$$+\widetilde{X}_{1'}^{\rm new}\widetilde{\Phi}_2^u\Phi_3^d\widetilde{\Phi}_3^d/M + \widetilde{X}_{1'}^{\rm new}\Phi_2^d(\Phi_3^d)^4/M^3$$

With the fields

$$V_0 \sim (\mathbf{1},\mathbf{1},0) \quad \text{and} \quad V_2 \sim (\mathbf{1},\mathbf{2},-8)$$

also spontaneous S_4 symmetry breaking can be ensured and correlations can be maximised. The leading terms in the superpotential read

$$V_0 M_{V_0}^2 + V_0(\widetilde{\Phi}_2^u)^2 + V_0(\Phi_3^d)^2\Phi_1^\nu/M + V_0(\Phi_3^d)^2\Phi_2^\nu/M + V_0(\Phi_3^d)^2\Phi_{3'}^\nu/M$$
$$+M_{V_2}V_2\Phi_2^\nu + V_2\widetilde{\Phi}_2^u\Phi_1^\nu + V_2\widetilde{\Phi}_2^u\Phi_2^\nu + V_2(\Phi_2^d)^8/M^6$$