ADVANCES IN THE THEORY OF $0\nu\beta\beta$ DECAY

Francesco Iachello

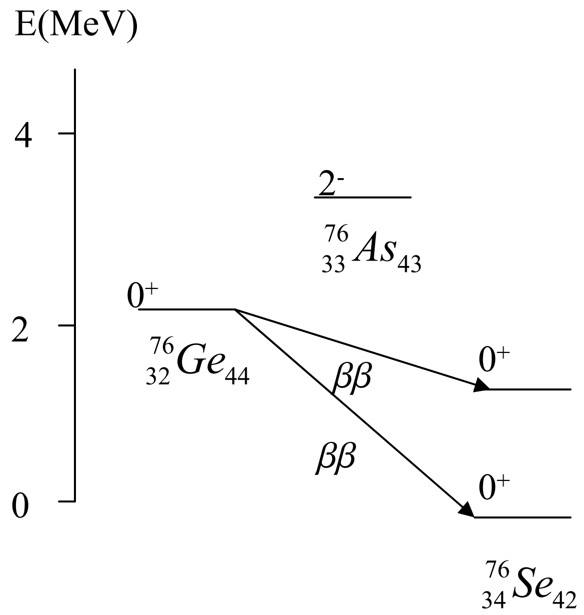
Yale University

Neutrino Oscillation Workshop
Otranto, September 5, 2010

INTRODUCTION

Fundamental Process $0\nu\beta\beta$:

$$^{A}_{Z}X_{N} \rightarrow {}^{A}_{Z+2}Y_{N-2} + 2e^{-}$$



Half-life for the process:

$$\left[T_{1/2}^{0\nu\beta\beta}(0^{+} \rightarrow 0^{+})\right]^{-1} = G_{0\nu}\left|M^{(0\nu)}\right|^{2}\left|f_{b}(m,\eta)\right|^{2}$$

Beyond the standard model
(Particle physics)

Matrix elements
(Nuclear physics)

Phase-space factor
(Atomic physics)

Difficult calculation: Three different scales

1. Particle physics
   Weak Lagrangean, $\mathscr{L}$
   Transition operator inducing the decay

$$T(p) = H(p) f_b(m, \eta)$$

coupling constants

masses

2. Nuclear physics
   Matrix elements

$$M^{(0\nu)} = \langle f | H(p) | i \rangle$$

3. Atomic physics
   Kinematical factor

charge

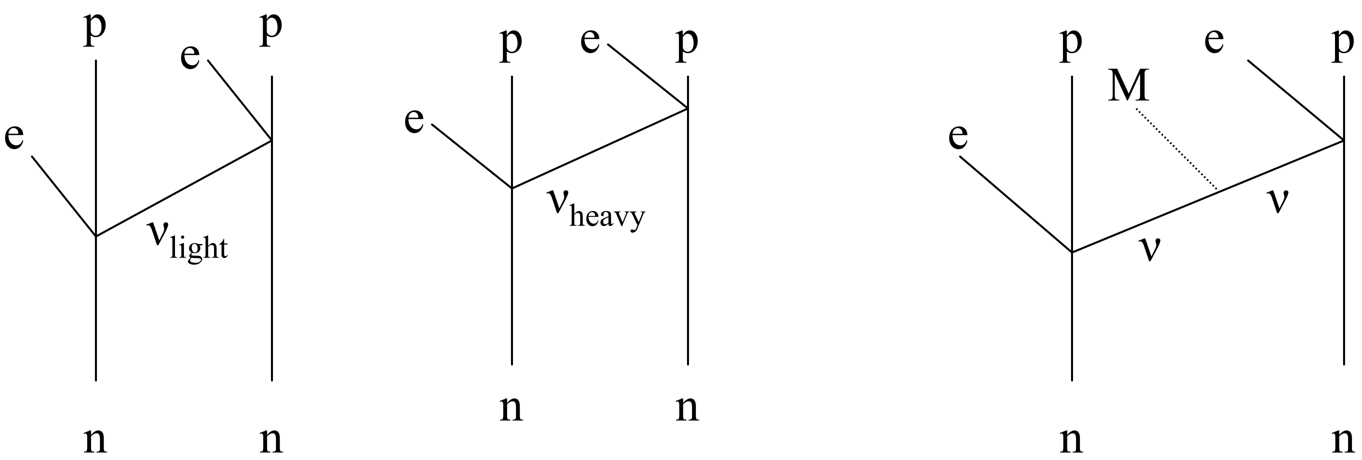
$$G_{0\nu} = G_{0\nu}(Q_{\beta\beta}, Z)$$

Q-value

$$Q_{\beta\beta} = E_i - E_f - 2m_e c^2$$

1. PARTICLE PHYSICS

The transition operator *T(p)* depends on the model of 0νββ decay.
Three scenarios have been considered [¶,§].



$m_{\nu light}$<1MeV $m_{\nu heavy}$>1GeV

¶ T.Tomoda, Rep. Prog. Phys. 54, 53 (1991).

§ Šimkovic *et al.*, Phys. Rev. C60, 055502 (1999).

After the discovery of neutrino oscillations, attention has been focused on the first scenario. [In our calculations, we have also considered the other two.]

Brief review of theory of T(p)

Weak interaction Hamiltonian

$$H^{\beta} = \frac{G_F}{\sqrt{2}}\left[\overline{e}\gamma_{\mu}\left(1-\gamma_5\right)\nu_{eL}\right]J_L^{\mu\dagger} + h.c.$$

Nucleon current [§]

$$J_L^{\mu\dagger} = \overline{\Psi}\tau^{+}\left[g_V(q^2)\gamma^{\mu} - ig_M(q^2)\frac{\sigma^{\mu\nu}}{2m_p}q_{\nu} - g_A(q^2)\gamma^{\mu}\gamma_5 + g_P(q^2)q^{\mu}\gamma_5\right]\Psi$$

vector   weak-magnetism   axial vector   induced pseudo-scalar

HOC

q^{μ} = momentum transferred from hadrons to leptons

[§] F. Šimkovic *et al.*, loc.cit.

From the weak interaction Hamiltonian, $\mathscr{H}$, and the weak nucleon current, J^{μ}, one finds the transition operator, *T(p)*, which, for scenario 1, can be written as

$$T(p) = H(p)\frac{\langle m_{\nu}\rangle}{m_e}$$

with

$$\langle m_{\nu}\rangle = \sum_{k=ligth} |U_{\nu k}|^2 m_k$$

and $p = |\vec{q}|$

To lowest order and in momentum space, *H(p)*, can be written as

$$H(p) = \tau_n^+ \tau_{n'}^+ [-h^F(p) + h^{GT}(p)\vec{\sigma}_n \bullet \vec{\sigma}_{n'}]$$

Higher order corrections (HOC) induce a tensor term, and modify the Fermi and Gamow-Teller terms, producing an operator §

$$H(p) = \tau_n^+ \tau_{n'}^+ [-h^F(p) + h^{GT}(p)\vec{\sigma}_n \bullet \vec{\sigma}_{n'} + h^T(p) S^p_{nn'}]$$

[The general formulation of Tomoda ¶ includes more terms, nine in all, 3GT, 3F, 1T, one pseudoscalar (P) and one recoil (R).]

§ F. Šimkovic *et al.*, Phys. Rev. C60, 055502 (1999).

¶ T. Tomoda, Rep. Prog. Phys. 54, 53 (1991).

The form factors $h^{F,GT,T}(p)$ are given by:

$$h^{F,GT,T}(p) = v(p)\tilde{h}^{F,GT,T}(p)$$

with

$$v(p) = \frac{2}{\pi}\frac{1}{p\left(p+\tilde{A}\right)}$$

Ã=closure energy=1.12A$^{1/2}$(MeV)

called neutrino "potential", and $\tilde{h}(p)$ listed by Šimkovic *et al.* [§]

The finite nucleon size (FNS) is taken into account by taking the coupling constants, g_V and g_A, momentum dependent

$$g_V(p^2) = g_V \frac{1}{\left(1+\frac{p^2}{M_V^2}\right)^2}$$

$$g_A(p^2) = g_A \frac{1}{\left(1+\frac{p^2}{M_A^2}\right)^2}$$

$$g_V = 1; M_V^2 = 0.71\left(GeV/c^2\right)^2$$

$$g_A = 1.25; M_A^2 = 1.09\left(GeV/c^2\right)^2$$

Short range correlations (SRC) are taken into account by convoluting the "potential" *v(p)* with the Jastrow function *j(p)*

$$u(p) = \int v(p-p')j(p')dp'$$

[š] F. Šimkovic, *loc.cit.*

[Note: Tomoda's form factors are slightly different from Šimkovic. His formulation is in coordinate space, i.e. the form factors are the Fourier transform of those given above.]

2. NUCLEAR PHYSICS

Calculation of the “nuclear matrix elements” $M^{(0\nu)}$

$$M^{(0\nu)} = g_A^2 \tilde{M}^{(0\nu)} \equiv M_{GT}^{(0\nu)} - \left(\frac{g_V}{g_A}\right)^2 M_F^{(0\nu)} + M_T^{(0\nu)}$$

Calculations up to 2008:

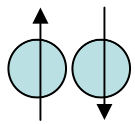
1. Quasi-particle random phase approximation (QRPA).
   Limitations: Cannot address strongly deformed nuclei, for example ^{150}Nd, due to the instability of the QRPA equations for large deformations.
2. Shell model (SM).
   Limitations: Cannot address nuclei with many particles in the valence shells, for example ^{150}Nd, due to the exploding size of the Hamiltonian matrices ($>10^9$).

Recent advances >2009:

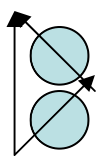
3. Development of a program to compute 0νββ (and 2νββ) nuclear matrix elements in the closure approximation within the framework of the microscopic Interacting Boson Model (IBM-2). This approach can be used for any nucleus with mass A≥70.

THE INTERACTING BOSON MODEL/ THE INTERACTING BOSON FERMION MODEL

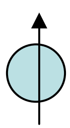
A model of even-even nuclei in terms of correlated pairs of protons and neutrons with angular momentum J=0,2 treated as bosons (s_π,d_π and s_ν,d_ν), called IBM-2 [¶].
A model of odd-even or odd-odd nuclei in terms of correlated pairs (bosons) and unpaired particles, $a_{j\pi}$ and $a_{j\nu}$, (fermions), called IBFM-2 [§].



J=0 s-boson



J=2 d-boson



Unpaired fermions

¶ F. Iachello and A. Arima, *The Interacting Boson Model*, Cambridge University Press, 1987.

§ F.Iachello and P. Van Isacker, *The Interacting Boson Fermion Model*, Cambridge University Press, 1991.

EVALUATION OF MATRIX ELEMENTS IN IBM-2 ¶

All matrix elements, F, GT and T, can be calculated at once using the compact expression:

$$V^{(\lambda)}_{s_1,s_2} = \frac{1}{2}\sum_{n,n'} \tau_n^+ \tau_{n'}^+ \left[\Sigma_n^{(s_1)} \times \Sigma_{n'}^{(s_2)}\right]^{(\lambda)} \cdot V(r_{nn'}) C^{(\lambda)}(\Omega_{nn'})$$

$$\lambda = 0, s_1 = s_2 = 0 (F)$$
$$\lambda = 0, s_1 = s_2 = 1 (GT)$$
$$\lambda = 2, s_1 = s_2 = 1 (T)$$

In second quantized form:

$$V^{(\lambda)}_{s_1,s_2} = -\frac{1}{4}\sum_{j_1 j_2} \sum_{j'_1 j'_2} \sum_{J} (-1)^J \sqrt{1+(-1)^J \delta_{j_1 j_2}} \sqrt{1+(-1)^J \delta_{j'_1 j'_2}}$$
$$\times G^{(\lambda)}_{s_1 s_2}(j_1 j_2 j'_1 j'_2; J) \left[\left(\pi^\dagger_{j_1} \times \pi^\dagger_{j_2}\right)^{(J)} \cdot \left(\tilde{\nu}_{j'_1} \times \tilde{\nu}_{j'_2}\right)^{(J)}\right]$$

Creates a pair of protons with angular momentum J

Annihilates a pair of neutrons with angular momentum J

¶ J. Barea and F. Iachello, Phys. Rev. C79, 044301 (2009).

The fermion operator V is then mapped onto the boson space by using:

$$\left(\pi_j^\dagger \times \pi_j^\dagger\right)^{(0)} \mapsto A_\pi(j) s_\pi^\dagger$$

$$\left(\pi_j^\dagger \times \pi_{j'}^\dagger\right)_M^{(2)} \mapsto B_\pi(j, j') d_{\pi,M}^\dagger$$

$$V_{s_1 s_2}^{(\lambda)} \mapsto -\frac{1}{2} \sum_{j_1} \sum_{j'_1} G_{s_1 s_2}^{(\lambda)} \left(j_1 j_1 j'_1\, j'_1; 0\right) A_\pi(j_1) A_\nu(j'_1) s_\pi^\dagger \cdot \tilde{s}_\nu$$

$$-\frac{1}{4} \sum_{j_1 j_2} \sum_{j'_1 j'_2} \sqrt{1+\delta_{j_1 j_2}} \sqrt{1+\delta_{j'_1 j'_2}}\, G_{s_1 s_2}^{(\lambda)} (j_1 j_2 j'_1\, j'_2; 2) B_\pi(j_1, j_2) B_\nu(j'_1, j'_2) d_\pi^\dagger \cdot \tilde{d}_\nu$$

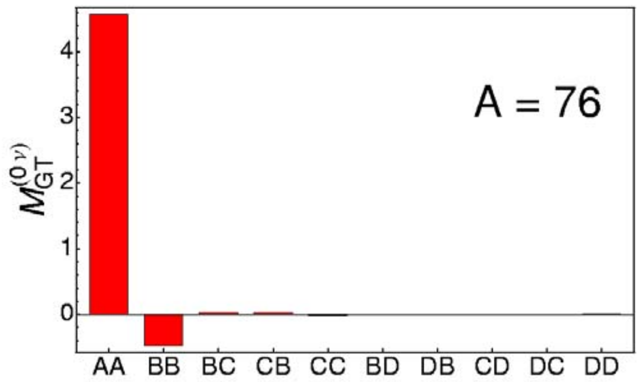
The coefficients A, B are obtained by means of the so-called OAI mapping procedure §

§ T. Otsuka, A. Arima and F. Iachello, Nucl. Phys. A309, 1 (1978).

[We have carried out the mapping to next to leading order (NLO)]

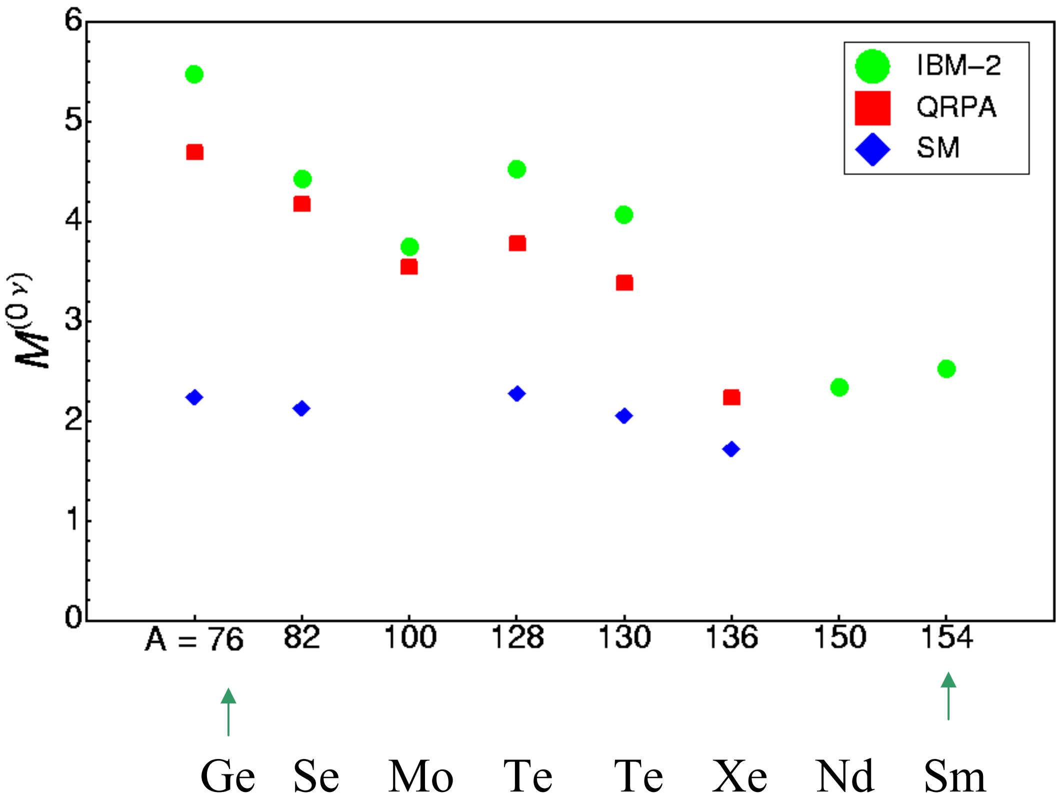
$$(\pi_j^\dagger \times \pi_{j'}^\dagger)_M^{(2)} \mapsto B_\pi(j,j')\left(d_\pi^\dagger\right)_M + C_\pi(j,j')s_\pi^\dagger\left(s_\pi^\dagger \tilde{d}_\pi\right)_M^{(2)} + D_\pi(j,j')s_\pi^\dagger\left(d_\pi^\dagger \tilde{d}_\pi\right)_M^{(2)}$$

However, the contribution of the additional terms appears to be rather small and henceforth neglected.]



Matrix elements of the mapped operators are then evaluated with realistic wave functions of the initial and final nuclei either taken from the literature, when available, or obtained from a fit to the observed energies and other properties.

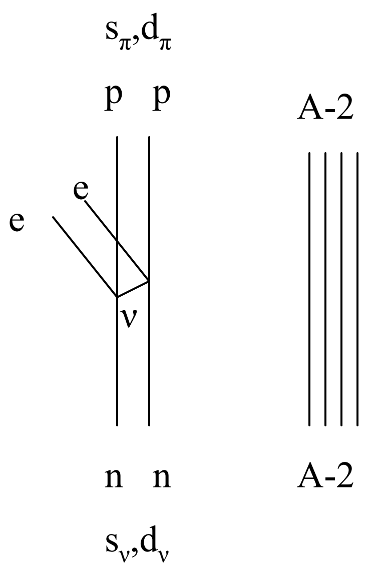
RESULTS FOR THE MATRIX ELEMENTS (2009)



IBM-2 from J. Barea and F. Iachello, Phys. Rev. C79, 044301 (2009), g_A=1.25, Jastrow SRC.
QRPA from F. Šimkovic, A Faessler, V. Rodin, P. Vogel, and J. Engel, Phys. Rev. C77, 045503 (2008), with g_A=1.25, Jastrow SRC.
SM from E. Caurier, J. Menendez, F. Nowacki, and A. Poves, Phys. Rev. Lett. 100, 052503 (2008).

Matrix elements in dimensionless units.

Enhancement in IBM-2 due to pairing correlations (the same correlations that make double beta decay at all possible).



[Also note that since the neutrino is almost massless, the "potential" $H(r_{12})$ is a long range potential, almost Coulomb-like. For this reason, it is convenient to calculate the matrix elements in momentum space, Horie method §.]

$0\nu\beta\beta$ (F) and (GT)

$$v^{(0\nu)}(p) = \frac{2}{\pi}\frac{1}{p(p+\tilde{A})}$$

§ H. Horie and K. Sasaki, Prog. Theor. Phys. 25, 475 (1961).

ERROR ANALYSIS

Estimated sensitivity to input parameter changes:

1.	Single-particle energies [¶,§]	10%
2.	Strength of surface delta interaction	5%
3.	Oscillator parameter	5%
4.	Closure energy	5%

Estimated sensitivity to model assumptions:

1.	Truncation to S, D space	1% (spherical nuclei)-10% (deformed nuclei)
2.	Isospin purity	1%(GT)-20%(F)-1%(T)

Estimated sensitivity to operator assumptions:

1.	Form of the operator	5%
2.	Finite nuclear size (FNS)	2%
3.	Short range correlations (SRC)	2%

[¶] This point has been emphasized by J. Suhonen and O. Civitarese, Phys. Lett. B668, 277 (2008).

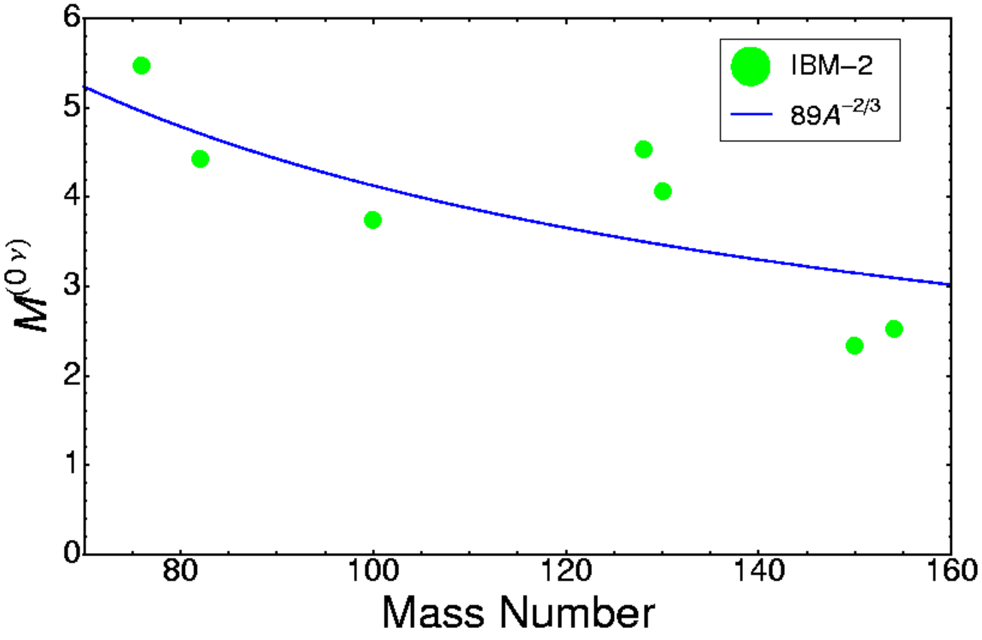
[§] New experiments are being done to check the single particle levels in Ge, Se and Te, J.P. Schiffer *et al.*, Phys. Rev. Lett. **100**, 112501 (2008).

SIMPLE FEATURES OF IBM-2 CALCULATIONS

1. Mass dependence

$$M^{(0\nu)} \cong 89A^{-2/3}$$

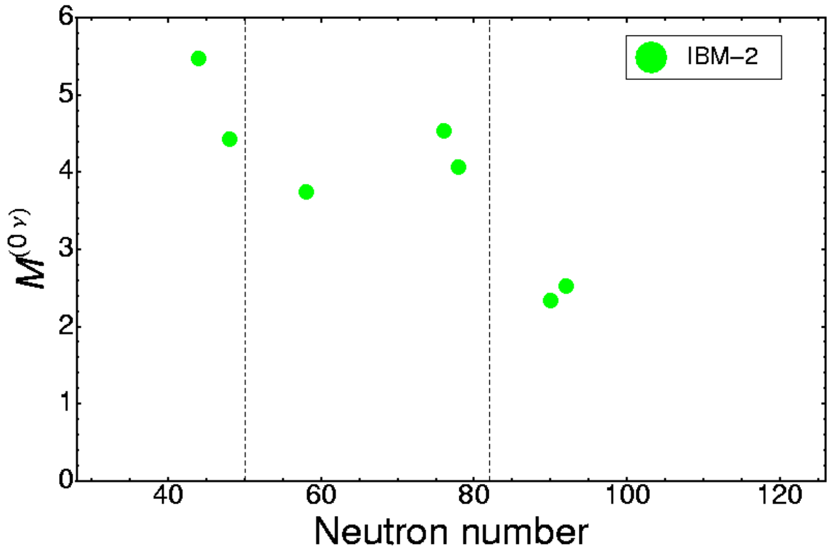
Very mild dependence



Mass dependence of the input parameters:

Strength of the Surface Delta Interaction	$A_1 = 25A^{-1}(MeV)$
Oscillator size	$\nu = 0.994A^{-1/3}(fm^{-2})$
Radius	$R = 1.2A^{1/3}(fm)$
Closure energy	$\tilde{A} = 1.12A^{1/2}(MeV)$

2. Shell effects

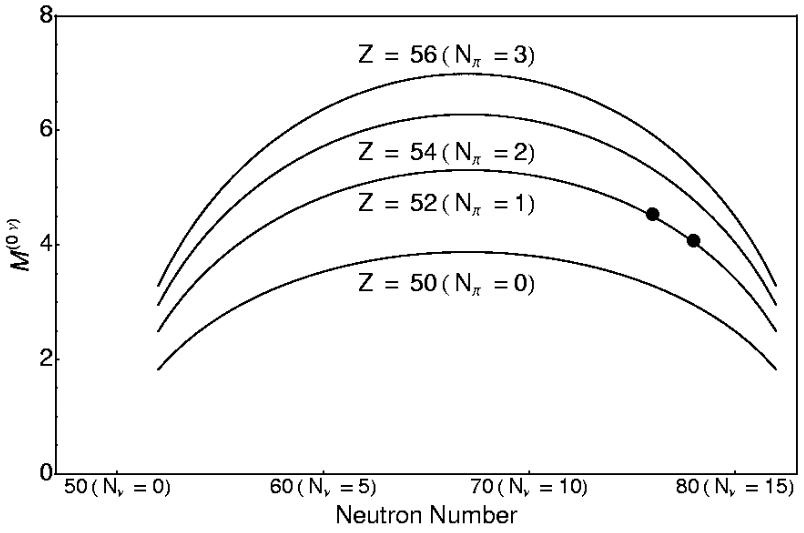


Neutron number dependence

This is a major effect: The matrix elements are small at the closed shells

Simple formula to estimate shell effects:

$$M^{(0\nu)} \cong \alpha_\pi \alpha_\nu \sqrt{N_\pi + 1}\sqrt{N_\nu}\sqrt{\Omega_\pi - N_\pi}\sqrt{\Omega_\nu - N_\nu + 1}$$



28-50 shell $\quad \alpha_\pi \alpha_\nu = 0.186$

50-82 shell $\quad \alpha_\pi \alpha_\nu = 0.114$

$$\frac{M^{(0\nu)}(^{128}Te)}{M^{(0\nu)}(^{130}Te)} = 1.11$$

IBM-2: 1.11

QRPA: 1.13

3. Deformation effects

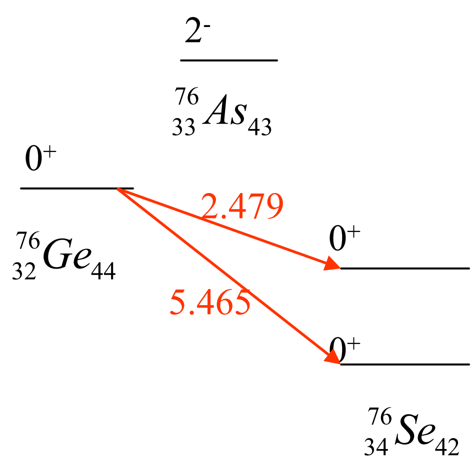
Estimated from comparison between a GS calculation (only S pairs) and a full IBM-2 calculation (S and D pairs).

Deformation effects always decrease the matrix elements:

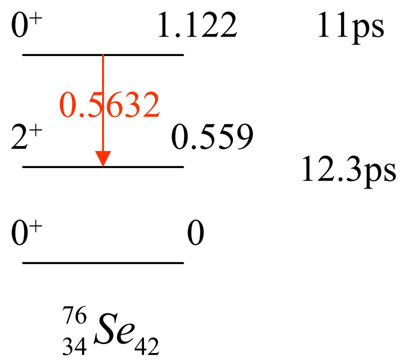
^{76}Ge	-19%
^{128}Te	-26%
^{154}Sm	-32%

MATRIX ELEMENTS TO FIRST EXCITED 0^+ STATE

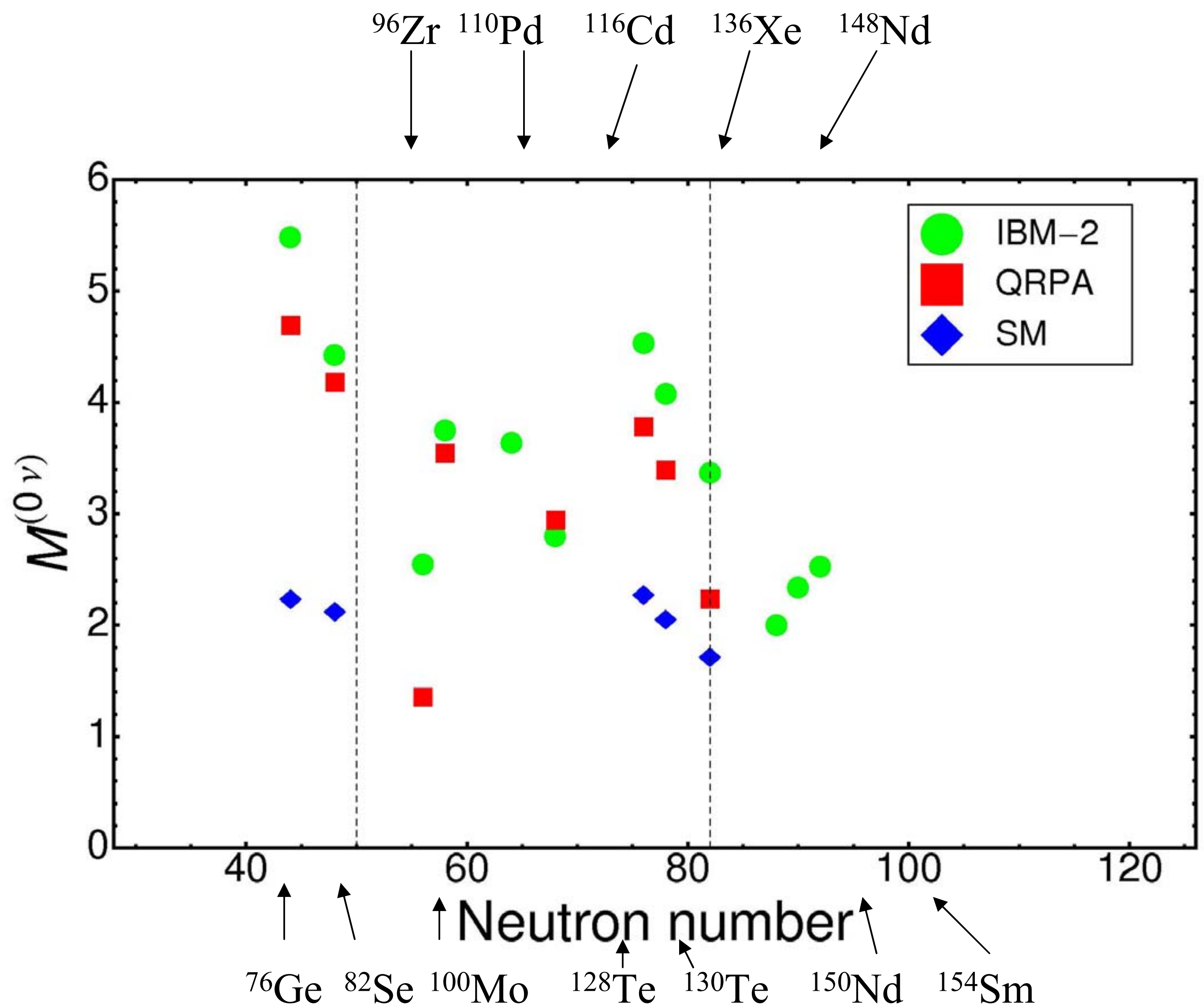
^{76}Ge	2.479
^{82}Se	1.247
^{100}Mo	0.419
^{128}Te	3.243
^{130}Te	3.090
^{150}Nd	0.395
^{154}Sm	0.021



[In some cases, the matrix elements are large. Although the kinematical factor hinders the decay to the excited state, large matrix elements offer the possibility of a direct detection, by looking at the γ-ray de-exciting the 0^+ level.]



RECENT RESULTS (2010)



COMMENTS

(i) While QRPA calculations are within our estimated error, 25%, SM calculations are not.
Understanding why there is this discrepancy is of crucial importance for extracting the average neutrino mass (in case $0\nu\beta\beta$ would be seen) [§].
A study is under way to find the origin of the discrepancy.

(ii) Although we cannot be absolutely confident that the absolute scale is correct, we are very confident that the relative values are correct. It is very important therefore to do experiments in several nuclei [¶].

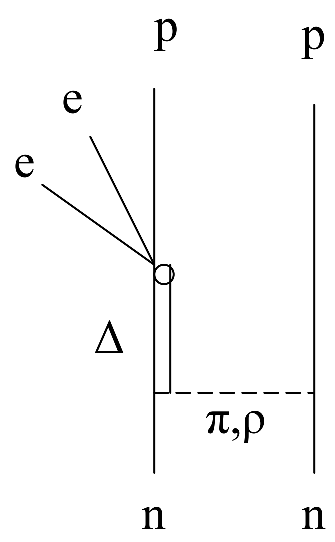
[§] This point has been emphasized by J. N. Bahcall, H. Murayama, and C. Peña-Garay, Phys. Rev. D70, 033012 (2004).
[¶] This point has been emphasized by E. Fiorini, in *Proc. Int. School "Enrico Fermi", Course CLXIX,* ed. by A. Covello *et al.*(IOS Press, Amsterdam, 2008), p.477.

THE NEXT IMPORTANT PROBLEM: RENORMALIZATION OF G_A

After agreeing on the nuclear matrix elements, one should consider the next important problem, i.e.,
Renormalization of the axial vector coupling constant g_A in nuclei.

A well know problem for single β decay where $g_{A,\,eff}$~$0.7g_A$

A crucial problem for extraction of the neutrino mass.
g_A appears to the fourth power in the half-life!

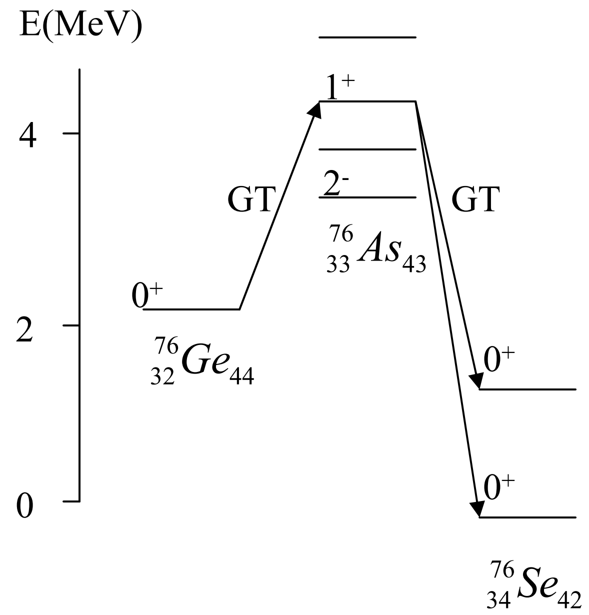


Origin of the renormalization:
1. Limited model space
2. Missing hadronic degrees of freedom, Δ,…

This is a difficult problem to solve: for case 1 we are limited by the size of the matrices (>10^9); for case 2 we are limited by a detailed knowledge of the decay process.
It can only be solved indirectly (by studying 2νββ).

Fundamental Process $2\nu\beta\beta$:

$$ {}^{A}_{Z}X_{N} \rightarrow {}^{A}_{Z+2}Y_{N-2} + 2e^{-} + 2\bar{\nu} $$



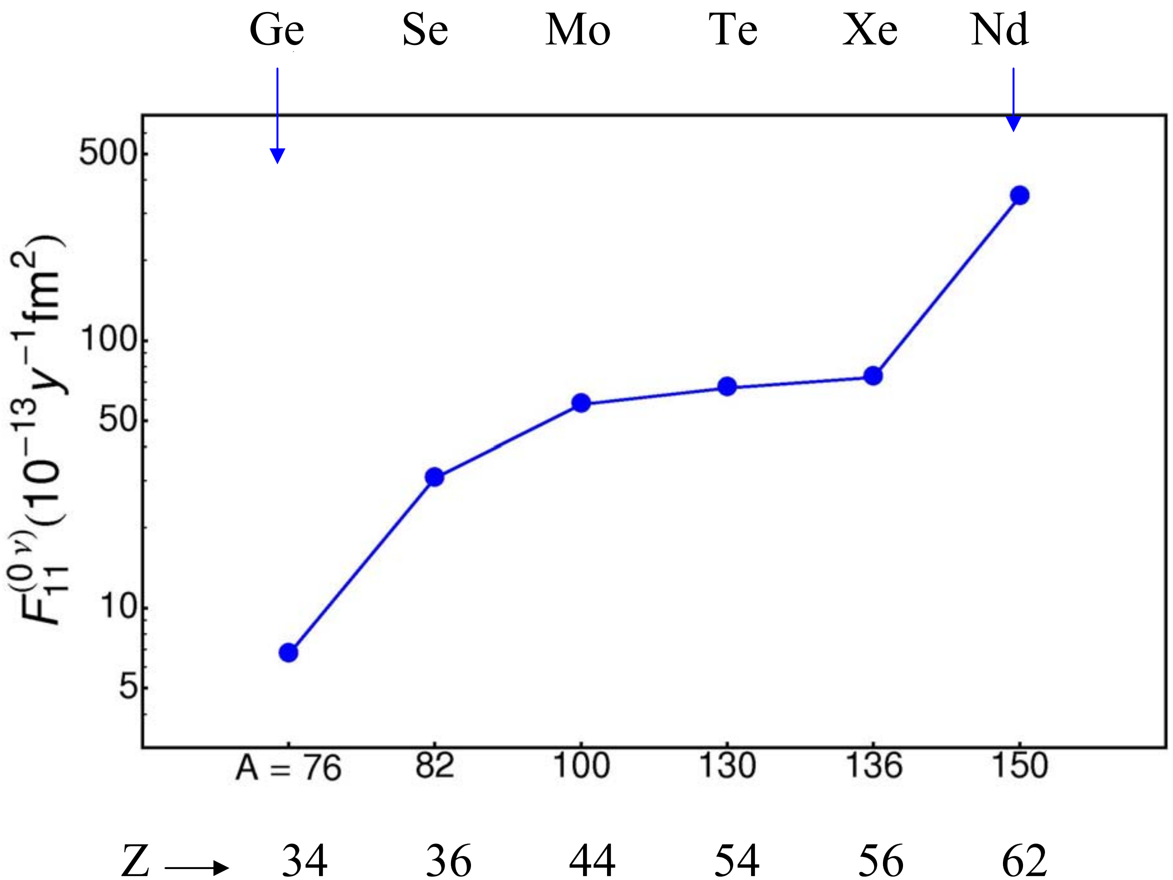
We have done a calculation of 2νββ in the closure approximation and find a renormalization of $g_{A,eff}$~$0.7g_A$.
However, the closure approximation may not be good for 2νββ (only a selected number of states contributes to the decay). The average neutrino momentum is of the order of 10MeV. We have therefore started a full scale calculation.
Also, the renormalization effects could be different in 0νββ than in 2νββ.

[The calculation is similar to that of 0νββ except that the neutrino "potential" is different.]

$$ v^{(2\nu)}(p) = \frac{\delta(p)}{p^2} $$

3. ATOMIC PHYSICS

For an extraction of the neutrino mass and for estimates of the half-life we also need the phase-space factor $G_{0\nu}$. A general relativistic formulation was given by Tomoda ¶ and results for selected cases tabulated.



$$G_{0\nu} = F_{11}^{(0)} / 4R^2$$

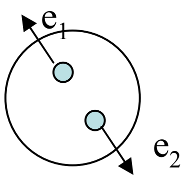
$$R = 1.2A^{1/3}(fm)$$

$$F_{11}^{(0)} = F_{11}^{(0)}\left(Q_{\beta\beta}, Z\right)$$

Charge of daughter nucleus

¶ T. Tomoda, *loc.cit.*

Brief review of theory of $F_{11}^{(0)}$



$$F_{11}^{(0)} \propto \left|\psi_{e_1}(0)\psi_{e_2}(0)\right|^2$$

Scattering electron wave functions at the nucleus

Non-relativistic: $\left|\psi(0)\right|^2 = \dfrac{2\pi y}{1-e^{-2\pi y}}$ $\qquad y = \dfrac{(Z\alpha)}{(v/c)}$

Relativistic: $\left|\psi(0)\right|^2$ diverges

Regularization: uniform charge distribution with $R = 1.2 A^{1/3} (fm)$

Dependence on Z: $\approx (Z\alpha)^{\beta}, \beta \geq 3$

Tomoda ¶ solved the Dirac equation numerically for a uniform distribution

Simple parametrization of Tomoda's results

$$F_{11}^{(0)} = C\left(\frac{Q_{\beta\beta}}{Q_{\beta\beta(^{130}Te)}}\right)^3 \left(\frac{Z}{54}\right)^{\beta}$$

$C = 66(10^{-13} y^{-1} fm^2)$

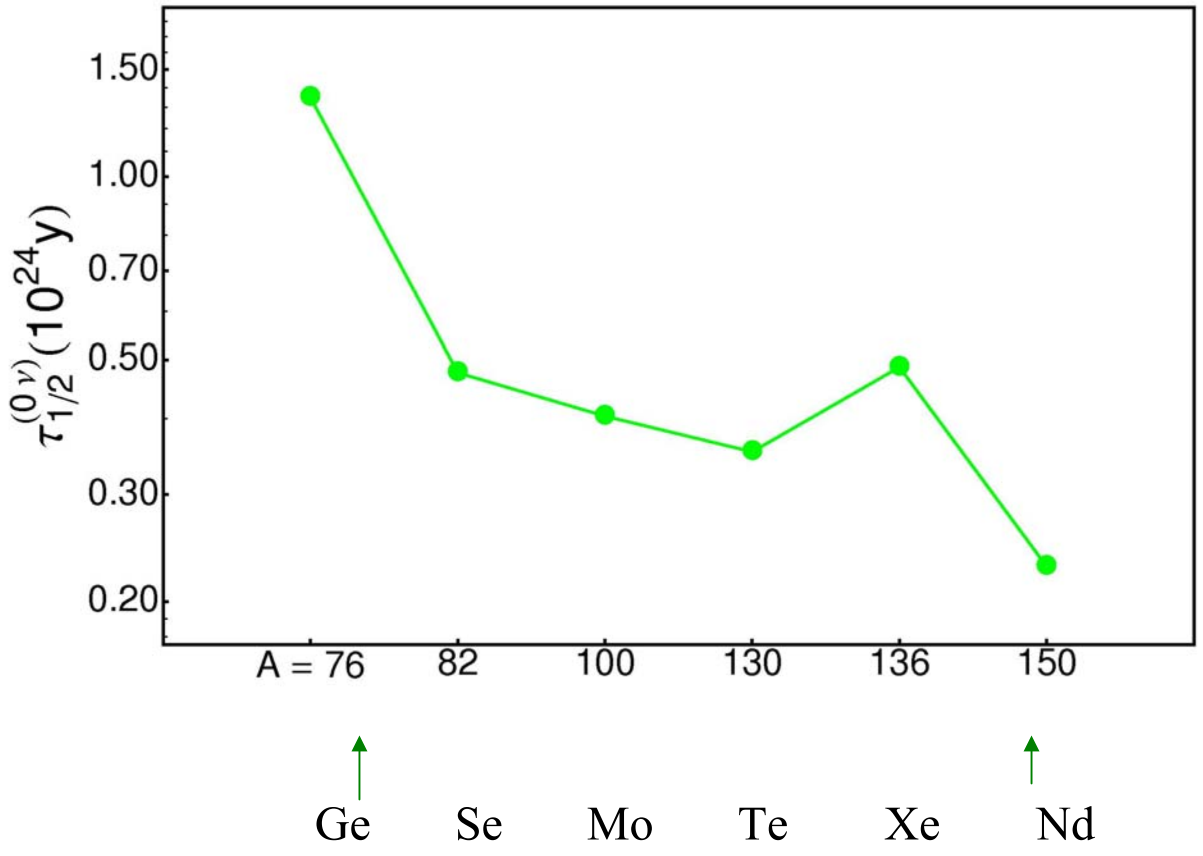
$\beta = 3 - 5$

[For ^{150}Nd decay, the wave function is already highly relativistic, 62/137~0.45]

[Because of the complex nature of this calculation and of the resulting strong dependence on *Z* we are planning to do a new and independent calculation of $F_{11}^{(0)}$.]

¶ T. Tomoda, Rep. Prog. Phys. 54, 53 (1991).

FINAL RESULTS FOR HALF-LIFE



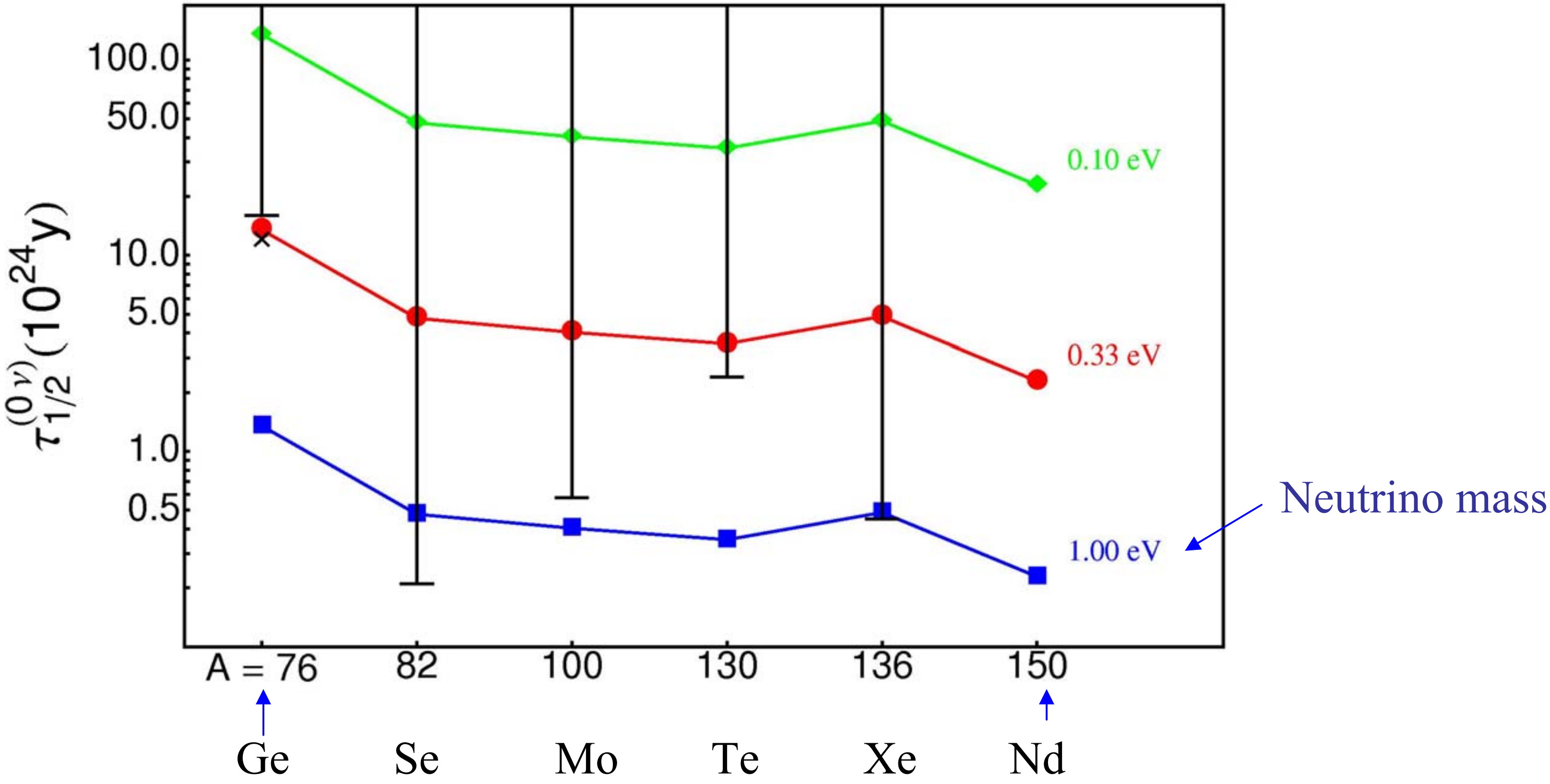
Neutrino mass
m_ν= 1 eV

g_A=1.25

Nuclear matrix elements from J. Barea and F. Iachello, Phys. Rev. C79, 044301 (2009) and to be published.
Phase space factors from T. Tomoda, Rep. Prog. Phys. 54, 553 (1991).

LIMITS ON NEUTRINO MASS

$$\left[\tau_{1/2}^{(0\nu)}\right]^{-1} = \frac{F_{11}^{(0)}}{4R^2}\left|M^{(0\nu)}\right|^2 \left(\frac{\langle m_\nu \rangle}{m_e}\right)^2$$



Theory: Nuclear matrix elements from J. Barea and F. Iachello, Phys. Rev. C79, 044301 (2009).
Phase space factors from T. Tomoda, Rep. Prog. Phys. 54, 53 (1991).
Experimental upper limits: from a compilation of A. Barabash, arXiv:hep-ex/0608054v1 23 Aug 2006.
Ge [IGEX], Se and Mo [NEMO-3], Te [CUORICINO], Xe [DAMA].
× : from H.V. Klapdor-Kleingrothaus *et al.*, Phys. Lett. B586, 198 (2004).

CONCLUSIONS

• A new program (IBM-2) has been developed to calculate $0\nu\beta\beta$ (and $2\nu\beta\beta$, and $0\nu\beta\beta M$) nuclear matrix elements $M^{(0\nu)}$ in nuclei with mass A>70 in the closure approximation. Results have been published in 2009 for ^{76}Ge, ^{82}Se, ^{100}Mo, ^{128}Te, ^{130}Te, ^{150}Nd and ^{154}Sm. Several new results have been obtained recently (2010) for ^{96}Zr, ^{110}Pd, ^{116}Cd, ^{136}Xe and ^{148}Nd. [A table of results obtained so far is available for distribution.]

• Matrix elements to first excited 0^+ states have been also calculated.

• Attempts are being made to reconcile different calculations within 25%.

• Other effects such as the renormalization of g_A are being considered.

• The calculation of the phase-space factors is being revisited.

TABLE OF NUCLEAR MATRIX ELEMENTS (SEP 2010)

Table I: Neutrinoless nuclear matrix elements including HOC calculated in the Microscopic Interacting Boson Model (IBM-2), Shell Model (SM) and Quasiparticle Random Phase Approximation (QRPA). All matrix elements in dimensionless units.

	IBM-2			IBM-2[a]	QRPA[b]	SM[c]
	$M_F^{(0\nu)}$	$M_{GT}^{(0\nu)}$	$M_T^{(0\nu)}$		$M^{(0\nu)}$	
$^{76}Ge \rightarrow {}^{76}Se$	-2.529	4.096	-0.250	5.465	4.680	2.220
$^{82}Se \rightarrow {}^{82}Kr$	-2.197	3.260	-0.254	4.412	4.170	2.110
$^{96}Zr \rightarrow {}^{96}Mo$	-0.235	2.255	0.125	2.530	1.340	
$^{100}Mo \rightarrow {}^{100}Ru$	-0.327	3.318	0.204	3.732	3.530	
$^{110}Pd \rightarrow {}^{110}Cd$	-0.262	3.220	0.235	3.623		
$^{116}Cd \rightarrow {}^{116}Sn$	-0.225	2.485	0.152	2.782	2.930	
$^{128}Te \rightarrow {}^{128}Xe$	-1.897	3.463	-0.161	4.517	3.770	2.260
$^{130}Te \rightarrow {}^{130}Xe$	-1.693	3.119	-0.144	4.059	3.380	2.040
$^{136}Xe \rightarrow {}^{136}Ba$	-1.367	2.586	-0.109	3.352	2.220	1.700
$^{148}Nd \rightarrow {}^{148}Sm$	-0.276	1.726	0.082	1.985		
$^{150}Nd \rightarrow {}^{150}Sm$	-0.279	2.034	0.108	2.321		
$^{154}Sm \rightarrow {}^{154}Gd$	-0.255	2.226	0.118	2.507		

[a] From J. Barea and F. Iachello, Phys. Rev. C **79**, 044301 (2009) and paper in preparation.

[b] From F. Šimkovic, A Faessler, V. Rodin, P. Vogel, and J. Engel, Phys. Rev. C **77**, 045503 (2008).

[c] From E. Caurier, J. Menéndez, F. Nowacki, and A. Poves, Phys. Rev. Lett. **100**, 052503 (2008).

Table II: Neutrinoless matrix elements to first excited 0^+ state including HOC calculated in IBM-2. All matrix elements in dimensionless units.

	$M_F^{(0\nu)}$	$M_{GT}^{(0\nu)}$	$M_T^{(0\nu)}$	$M^{(0\nu)}$
$^{76}Ge \rightarrow {}^{76}Se$	-1.212	1.805	-0.102	2.479
$^{82}Se \rightarrow {}^{82}Kr$	-0.688	0.860	-0.053	1.247
$^{96}Zr \rightarrow {}^{96}Mo$	-0.004	0.039	0.002	0.044
$^{100}Mo \rightarrow {}^{100}Ru$	-0.034	0.380	0.017	0.419
$^{110}Pd \rightarrow {}^{110}Cd$	-0.100	1.448	0.086	1.599
$^{116}Cd \rightarrow {}^{116}Sn$	-0.086	0.934	0.058	1.047
$^{128}Te \rightarrow {}^{128}Xe$	-1.402	2.444	-0.099	3.243
$^{130}Te \rightarrow {}^{130}Xe$	-1.321	2.332	-0.088	3.090
$^{136}Xe \rightarrow {}^{136}Ba$	-0.745	1.398	-0.038	1.837
$^{148}Nd \rightarrow {}^{148}Sm$	-0.037	0.220	0.010	0.254
$^{150}Nd \rightarrow {}^{150}Sm$	-0.046	0.349	0.016	0.395
$^{154}Sm \rightarrow {}^{154}Gd$	-0.009	0.010	0.006	0.021

EFFECT OF HIGHER ORDER CORRECTIONS

TABLE III. Evolution of the different HOC in the ^{76}Ge → ^{76}Sc neutrinoless matrix elements (in fm^{-1}) calculated in IBM-2 as we add the FNS and the SRC corrections.

	$AA+VV$	AP	PP	MM	Sum
Fermi matrix elements $[M_F^{(0\nu)}]$					
Bare	−0.2845	0.0000	0.0000	0.0000	−0.2845
+FNS	−0.2640	0.0000	0.0000	0.0000	−0.2640
+SRC	−0.2557	0.0000	0.0000	0.0000	−0.2557
+FNS+SRC	−0.2487	0.0000	0.0000	0.0000	−0.2487
Gamow-Teller matrix elements $[M_{GT}^{(0\nu)}]$					
Bare	0.5418	−0.1164	0.0346	0.0362	0.4962
+FNS	0.5032	−0.0959	0.0262	0.0221	0.4557
+SRC	0.4548	−0.0734	0.0166	−0.0008	0.3973
+FNS+SRC	0.4464	−0.0714	0.0168	0.0111	0.4029
Tensor matrix elements $[M_T^{(0\nu)}]$					
Bare	0.0000	−0.0367	0.0120	−0.0061	−0.0308
+FNS	0.0000	−0.0296	0.0090	−0.0038	−0.0243
+SRC	0.0000	−0.0367	0.0119	−0.0053	−0.0300
+FNS+SRC	0.0000	−0.0299	0.0092	−0.0038	−0.0246
$M^{(0\nu)} = -(\frac{g_V}{g_A})^2 M_F^{(0\nu)} + M_{GT}^{(0\nu)} + M_T^{(0\nu)}$					
Bare	0.7239	−0.1531	0.0466	0.0301	0.6475
+FNS	0.6722	−0.1255	0.0353	0.0184	0.6004
+SRC	0.6185	−0.1101	0.0286	−0.0061	0.5309
+FNS+SRC	0.6056	−0.1014	0.0260	0.0073	0.5376

WEAK INTERACTION HAMILTONIAN

β-decay Hamiltonian

$$H^{\beta} = \frac{G_F}{\sqrt{2}} \left[\bar{e} \gamma_{\mu} \left(1 - \gamma_5 \right) \nu_{eL} \right] J_L^{\mu\dagger} + h.c.$$

Nucleon current

$$J_L^{\mu\dagger} = \bar{\Psi} \tau^{+} \left[g_V(q^2)\gamma^{\mu} - i g_M(q^2) \frac{\sigma^{\mu\nu}}{2m_p} q_{\nu} - g_A(q^2)\gamma^{\mu}\gamma_5 + g_P(q^2) q^{\mu}\gamma_5 \right] \Psi$$

Form factors $p = |\vec{q}|$

$$g_V(p^2) = \frac{g_V}{\left(1 + \frac{p^2}{\Lambda_V^2} \right)^2}$$

$$g_M(p^2) = \left(\mu_p - \mu_n \right) g_V(p^2)$$

$$g_A(p^2) = \frac{g_A}{\left(1 + \frac{p^2}{\Lambda_A^2} \right)^2}$$

$$g_P(p^2) = 2 m_p g_A(p^2) \frac{\left(1 - \frac{m_\pi^2}{\Lambda_A^2} \right)}{\left(p^2 + m_\pi^2 \right)}$$