Charged Lepton Flavour Violation

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\mathcal{III} . Discrete symmetries and LFV

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Introduction

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Motivation

Lepton flavor is violated:

Parameter	Best fit	3σ c.l.	
Δm_\odot^2 ($10^{-5}~{ m eV^2}$)	$7.59_{-0.18}^{+0.23}$	7.03 - 8.27	
$\Delta m^2_{ m Atm}$ ($10^{-3}~ m eV^2$)	$2.40^{+0.12}_{-0.11}$	2.07 - 2.75	
$\sin^2 heta_{\odot}$	$0.318\substack{+0.019\\-0.016}$	0.27 - 0.38	
$\sin^2 heta_{ m Atm}$	$0.50\substack{+0.07 \\ -0.06}$	0.36 - 0.67	
$\sin^2 heta_{13}$	$0.013\substack{+0.013\\-0.009}$	≤ 0.053	

Data from updated global fit:

Schwetz, Tórtola & Valle, New J Phys 10:113011, 2008; arXiv:0808.2016 (hep-ph) updated V3: 11 Feb 2010

Hint for no-zero θ_{13} at 1.5 σ ? - Fogli et al., 2008

Experimental status: LFV

Decay	Current Limit
$ au o \mu \gamma$	$4.4 \cdot 10^{-8}$
$ au o e\gamma$	$3.3 \cdot 10^{-8}$
$\mu ightarrow e \gamma$	$1.2 \cdot 10^{-11}$
$ au o 3\mu$	$3.2 \cdot 10^{-8}$
$\tau^- \to e^- \mu^+ \mu^-$	$3.7 \cdot 10^{-8}$
$\tau^- \to e^+ \mu^- \mu^-$	$2.3 \cdot 10^{-8}$
$\tau^- \to \mu^- e^+ e^-$	$2.7 \cdot 10^{-8}$
$\tau^- \to \mu^+ e^- e^-$	$2.0 \cdot 10^{-8}$
$ au \to 3e$	$3.6 \cdot 10^{-8}$
$\mu ightarrow 3e$	$1 \cdot 10^{-12}$

Particle Data Group 2010

> Sensitivity MEG: Br($\mu \rightarrow e\gamma$)~ 10^{-13} see talk by: G. Cavoto

Experimental status: LFV

Capture	Current Limit
$\mu^{-32}S \to e^{-32}S$	$7 \cdot 10^{-11}$
$\mu^{-32}S \to e^{+32}Si$	$9 \cdot 10^{-10}$
$\mu^- T i \to e^- T i$	$4.3 \cdot 10^{-12}$
$\mu^- Ti \to e^+ Ca$	$3.6 \cdot 10^{-11}$
$\mu^- Pb \rightarrow e^- Pb$	$4.6 \cdot 10^{-11}$
$\mu^- A u \to e^- A u$	$7 \cdot 10^{-13}$

Particle Data Group 2010

Future sensitivity: $\sim 10^{-16}$ see talk by: Y. Kuno

Simplest LFV by m_{ν}

 \Rightarrow Extend the minimal SM by neutrino masses

 \Rightarrow LFV appears, such as $\mu \rightarrow e \gamma$:



$$Br(\mu \to e\gamma) \sim \frac{3\alpha}{32\pi} \sum_{i=2,3} U^*_{\mu i} U_{ei} \frac{\Delta m^2_{i1}}{m^2_W} \le 10^{-54}$$

 \Rightarrow GIM suppressed by small neutrino masses \Rightarrow any observation of charged LFV points to physics beyond (neutrino mass extended) SM

LFV beyond m_{ν}

Simple example: Heavy neutrinos (N) with masses order $\mathcal{O}(TeV)$ exist:

$$\begin{split} \mathrm{Br}(\mu \to e\gamma) &\sim \quad \frac{\alpha^3 s_W^2}{256\pi^2} \frac{m_{\mu}^5}{m_W^4 \Gamma_{\mu}} \Big(\sum_i K_{\mu i}^* K_{ei} G(\frac{m_{N_k}^2}{m_W^2}) \Big)^2 \\ &\leq \quad 7 \times 10^{-6} \Big(\sum_i K_{\mu i}^* K_{ei} G(\frac{m_{N_k}^2}{m_W^2}) \Big)^2 & \swarrow \bigvee_{W} \bigvee_$$

 $\Rightarrow K_{ik}$ heavy neutrino - lepton mixing \Rightarrow G(x) loop function, G(1) = 1/8

(C)LFV - Models

 \Rightarrow Many models produce sizeable CLFV

- RPC Supersymmetry
- RPV Supersymmetry
- Practically any extended Higgs sector: Little Higgs models, additional Higgs doublets, triplets, etc...
- Extra generations
- Extra (large) dimensions
- etc ...
- \Rightarrow "Flavour problem" of BSM

Theoretical description

see, for example review: Kuno & Okada, 2001

General Lagrangian:

$$\mathcal{L} = -\frac{4G_F}{\sqrt{2}} \quad (m_{\mu}A_R\bar{\mu}\sigma^{\mu\nu}P_LeF_{\mu\nu} + \frac{\text{photonic diagrams}}{m_{\mu}A_L\bar{\mu}\sigma^{\mu\nu}P_ReF_{\mu\nu}}) \\ -\frac{G_F}{\sqrt{2}}\sum_f \quad (g_{L,\alpha,f}\bar{e}\mathcal{O}^{\alpha}P_L\mu + \frac{\text{contact interaction}}{g_{R,\alpha,f}\bar{e}\mathcal{O}^{\alpha}P_R\mu})(\bar{f}\mathcal{O}_{\alpha}f) + h.c$$

where $\alpha = S, P, V, A, T$ and $f = l_i, q_i$

 $\Rightarrow A_L$, A_R and $g_{L/R,\alpha,q}$ depend on model $\Rightarrow \tau$ lepton same structure, A and g matrices

Diagramatically





 \Rightarrow If photonic diagram dominates:

$$Br(l_i \to l_j l_k l_k) \sim \alpha \times Br(e \to l_j + \gamma)$$
$$Cr(\mu \to eN) \sim \alpha \times Br(\mu \to e + \gamma)$$

Photon dominance?

From Buras et al., 2010: Different particle models predict different ratios for ...

ratio	LHT	MSSM (dipole)	MSSM (Higgs)	SM4
$\frac{\mathrm{Br}(\mu^- \to e^- e^+ e^-)}{\mathrm{Br}(\mu \to e\gamma)}$	0.02 1	$\sim 6 \cdot 10^{-3}$	$\sim 6 \cdot 10^{-3}$	$0.06 \dots 2.2$
$\frac{\operatorname{Br}(\tau \xrightarrow{-} e^{-} e^{+} e^{-})}{\operatorname{Br}(\tau \xrightarrow{-} e^{\gamma})}$	0.04 0.4	$\sim 1 \cdot 10^{-2}$	$\sim 1\cdot 10^{-2}$	$0.07 \dots 2.2$
$\frac{\mathrm{Br}(\tau^- \to \mu^- \mu^+ \mu^-)}{\mathrm{Br}(\tau \to \mu \gamma)}$	0.04 0.4	$\sim 2 \cdot 10^{-3}$	$0.06\ldots 0.1$	$0.06 \dots 2.2$
$\frac{\operatorname{Br}(\tau^- \to e^- \mu^+ \mu^-)}{\operatorname{Br}(\tau \to e\gamma)}$	0.04 0.3	$\sim 2 \cdot 10^{-3}$	$0.02 \dots 0.04$	$0.03 \dots 1.3$
$\frac{\operatorname{Br}(\tau^- \to \mu^- e^+ e^-)}{\operatorname{Br}(\tau \to \mu \gamma)}$	0.04 0.3	$\sim 1\cdot 10^{-2}$	$\sim 1\cdot 10^{-2}$	$0.04 \dots 1.4$
$\frac{\operatorname{Br}(\tau^- \to e^- e^+ e^-)}{\operatorname{Br}(\tau^- \to e^- \mu^+ \mu^-)}$	0.82	~ 5	0.30.5	$1.5 \dots 2.3$
$\frac{\operatorname{Br}(\tau^- \to \mu^- \mu^+ \mu^-)}{\operatorname{Br}(\tau^- \to \mu^- e^+ e^-)}$	0.71.6	~ 0.2	5 10	$1.4 \dots 1.7$
$\frac{\mathrm{R}(\mu\mathrm{Ti} \rightarrow e\mathrm{Ti})}{\mathrm{Br}(\mu \rightarrow e\gamma)}$	$10^{-3} \dots 10^2$	$\sim 5 \cdot 10^{-3}$	0.080.15	$10^{-12} \dots 26$

LHT: Little Higgs model with T-parity MSSM: Minimal supersymmetric model (with R_P) SM4: Standard model with 4th generation

Target dependence

Fig. from Cirigliano et al., 2009



Kitano et al., 2002

 \Rightarrow use different nuclear targets to distinguish different operators



SUSY LFV and Seesaw

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The MSSM: Superfields

Superfield	Bosons	Fermions	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
Gauge Multiplets					
\widehat{G}	g	\widetilde{g}	8	0	0
\widehat{V}	W^a	\widetilde{W}^a	1	3	0
\widehat{V}'	В	\widetilde{B}	1	1	0
Matter N	lultiplets				
\widehat{L}	$(\tilde{ u}, \tilde{e}_L^-)$	(u, e_L^-)	1	2	-1
\widehat{E}^C	$ ilde{e}^+_R$	e_L^+	1	1	2
\widehat{Q}	$(ilde{u}_L, ilde{d}_L)$	(u_L,d_L)	3	2	1/3
\widehat{U}^C	$ ilde{u}_R^*$	u_L^c	3^*	1	-4/3
\widehat{D}^C	$ ilde{d}_R^*$	d_L^c	3*	1	2/3
Higgs Multiplets					
\widehat{H}_d	(H^0_d,H^d)	$(\tilde{H}_d^0, \tilde{H}_d^-)$	1	2	-1
\widehat{H}_{u}	(H_u^+, H_u^0)	$(\tilde{H}_u^+, \tilde{H}_u^0)$	1	2	1



Soft SUSY breaking terms:

$$-\mathcal{L}_{soft} = (M_{\tilde{L}}^2)_{ij} (\tilde{e}_{L,i}^* \tilde{e}_{L,j} + \tilde{\nu}_{L,i}^* \tilde{\nu}_{L,j}) + \cdots$$

Off-diagonal elements lead to:



 \Rightarrow In general MSSM much too big: SUSY flavour problem

mSugra = CMSSM

At the GUT scale:

$$(m_{\tilde{L}}^2)_{ij} = (m_{\tilde{E}}^2)_{ij} = \cdots = m_0^2 \delta_{ij}$$

 $(A_l)_{ij} = A_0(Y_l)_{ij} , \quad (A_\nu)_{ij} = A_0(Y_\nu)_{ij}$
 $\cdots \cdots$

Imposing unification on all soft terms, one is left with only 5 parameters @ M_X :

$$m_0$$
, $M_{1/2}$, A_0 , t_{eta} , sgn(μ)

 \Rightarrow Essential

assumptions:

(i) SUSY breaking flavour blind (ii) $\Lambda_{seesaw} < \Lambda_{SUSY}$

`Classical' Seesaw

In the basis (ν_L , ν_R) write mass matrix:

$$\mathcal{M}_{
u} = \left(egin{array}{cc} 0 & m_D \ m_D & M_M \end{array}
ight)$$

Minkowski, 1977 Yanagida, 1979 Gell-Mann, Ramond & Slansky, 1979 Mohapatra & Senjanovic, 1980

If $m_D \ll M_M$:

$$m_{1/2}\simeq (-rac{m_D^2}{M_M},M_M)$$



⇒ For 3 ν_R 21 parameters ⇒ At low energy12 parameters measurable: 3 m_{l_i} , 3 m_{ν_i} , 3 angles & 3 phases ⇒ Predictive power: -9

Santamaria, 1993

.

mSugra and RGEs

Seesaw type-I:

Borzumati & Masiero, 1986

$$(\Delta M_{\tilde{L}}^2)_{ij} \sim -\frac{1}{8\pi^2} f(m_0, A_0, ...) (Y_{\nu}^{\dagger} L Y_{\nu})_{ij}$$

 $(\Delta M_{\tilde{E}}^2)_{ij} \simeq 0$

Note: $L_i = \log[M_G/M_i]$.

 \Rightarrow 9 independent parameters \Rightarrow 9+12=21!

Ellis et al., 2002

 \Rightarrow Rewrite Y_{ν}

Casas & Ibarra, 2001

$$Y_{\nu} = \sqrt{2} \frac{i}{v_U} \sqrt{\hat{M}_R} R \sqrt{\hat{m}_{\nu}} U^{\dagger}.$$

 $\Rightarrow \text{Measure } \hat{m}_{\nu} \& U \text{ at low-energy} \\\Rightarrow \text{Learn about } \hat{M}_{R} \text{ and } R \text{ from } (\Delta M_{\tilde{L}}^{2})_{ij} \dots ?$



$$(\Delta M_{\tilde{L}}^2)_{ij} \sim -\frac{1}{8\pi^2} g(m_0, A_0, M_{1/2}, ...) (Y_T^{\dagger} Y_T)_{ij} \log(M_G/M_T)$$
$$(\Delta M_{\tilde{E}}^2)_{ij} = 0$$

 \Rightarrow 9+12=21, but only 15 parameters

 $\Rightarrow \text{Measuring all entries in } (\Delta M_{\tilde{L}}^2)_{ij}$ "over-constrains" triplet seesaw ???

Analytical results



WARNING:

- Plot assumes right-handed neutrinos are degenerate
- Hierarchical right-handed neutrinos lead to very different results

Analytical results



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Numerical results: SPheno3

Example: Seesaw-I, SPS3

Calculated assuming: (i) Degenerate N^c , (ii) TBM angles, (iii) best fit $\Delta m_{\rm A}^2$ and

 Δm_\odot^2 , (iv) $m_{
u_1}\equiv 0$:



 \Rightarrow Ratios determined by seesaw parameters!

 \Rightarrow LHC can see LFV (if SPS3-like ...)



Numerical results: LHC

Left: SPS1a'



Discrete symmetries and LFV

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Discrete flavour symmetries

A very partial list:

S3: Kubo et al., 2003; Chen et al., 2004; Grimus and Lavoura, 2005;Lavoura and Ma, 2005; Teshima, 2006;Koide, 2006; Mohapatra et al., 2006; ···, ···

*S*₄: Ma, 2006; Hagedorn et al., 2006; Cai & Yu, 2006; Zhang, 2006; Koide, 2007; · · · , · · · Altarelli & Feruglio, 2010

*A*₄: Ma & Rajasekaran, 2001; Ma, 2002; Babu et al., 2003; Hirsch et al., 2003; Altarelli and Feruglio, 2005; Babu and He, 2005; Koide, 2007; · · · , · · ·

 Q_4 : Frigerio et al. 2005, · · ·

. . .

See also talks by: S Morisi C Hagedorn

arXiv:0705.0327

 D_4 : Grimus & Lavoura, 2003; Grimus et al., 2004; · · · , · · ·

A_4 : linear & inverse seesaw

Inverse seesaw, basis (ν, ν^c, S) :

$$M_{\nu} = \begin{pmatrix} 0 & m_D & 0 \\ m_D^T & 0 & M \\ 0 & M^T & \mu \end{pmatrix},$$

Mohapatra & Valle, 1986

After EWSB the effective light neutrino mass matrix is given by

$$M_{\nu} = m_D M^{T^{-1}} \mu M^{-1} m_D^T$$

Linear seesaw:

$$M_{\nu} = \begin{pmatrix} 0 & m_D & M_L \\ m_D^T & 0 & M \\ M_L^T & M^T & 0 \end{pmatrix}.$$

Akhmedov et al., 1995

Light neutrino mass:

$$M_{\nu} = m_D (M_L M^{-1})^T + (M_L M^{-1}) m_D^T$$

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If M_l is diagonalized on the left by the magic matrix U_ω

$$U_{\omega} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix},$$

(with $\omega \equiv \exp i\pi/3$) and

TBM in A_4

$$m_{\nu} = \left(\begin{array}{ccc} c & 0 & 0 \\ 0 & a & b \\ 0 & b & a \end{array} \right).$$

- \Rightarrow Lepton mixing is exactly TBM
- \Rightarrow If in addition: c = a, neutrino spectrum fixed

Mixing between light and heavy neutrinos is generically:

 $K = m_D . M^{-1}$

Use A_4 to fix charged lepton matrix and:

	(a	0	0	
$m_D =$		0	a	b	
		0	b	a)

Hirsch, Morisi & Valle, 2009

Then, $\sum_{j} K_{ik}^* K_{jk}$ fixed, example inverse seesaw:

$$\sum_{j} K_{ik}^{*} K_{jk} = \begin{pmatrix} a^{2} + \frac{4ab}{3} + \frac{2b^{2}}{3} & -\frac{1}{3}b(2a+b) & -\frac{1}{3}b(2a+b) \\ -\frac{1}{3}b(2a+b) & \frac{1}{3}b(4a-b) & a^{2} - \frac{2ab}{3} + \frac{2b^{2}}{3} \\ -\frac{1}{3}b(2a+b) & a^{2} - \frac{2ab}{3} + \frac{2b^{2}}{3} & \frac{1}{3}b(4a-b) \end{pmatrix}$$

Linear and inverse SS in A_4



 ${
m Br}(\mu
ightarrow e\gamma)$ for 3 different values of m_N for inverse and linear seesaw

Ratio:

 ${\rm Br}(\tau \to \mu \gamma)/{\rm Br}(\tau \to e \gamma)$ for inverse and linear seesaw assuming exact TBM mixing as function of

$$\alpha = \frac{\Delta m_{\odot}^2}{\Delta m_{\rm Atm}^2}$$





Exotic decay $\mu \rightarrow eJ$

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Experimental status

Limits on Majoron emission both very old and very weak. PDG 2010 gives ($X^0 =$ "familon"):

$$Br(\mu o eX^0) \le 2.6 imes 10^{-6}$$
 A. Jodidio ef al. PRD34 (1986)

 \Rightarrow Not a valid limit for Majoron, since experimental cuts to minimize backgrounds eliminated interesting (angular) region. Estimated limit from fig.(7) of this paper, very roughly: $Br(\mu \rightarrow eJ) \sim (\text{few}) \ 10^{-5}$

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Br(\tau \to \mu + J) \le 2.3\%Br(\tau \to e + J) \le 0.73\%
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MARK-III Collaboration PRL 55 (1985)

Theoretical status?

(i) Classical "Singlet" Majoron

Chikashige, Mohapatra and Peccei, 1981

(ii) "Doublet" Majoron

Aulakh & Mohapatra, 1982

(iii) Triplet Majoron

Gelmini & Roncadelli, 1981

(iv) "Singlet-doublet" (?) Majoron

Masiero & Valle, 1990

(v) · · ·

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Masiero & Valle, 1990

Alive, but Experimentally hopeless

DEAD - LEP

DEAD - LEP

ALIVE

$\mathcal{W} = h_U^{ij} \widehat{Q}_i \widehat{U}_j \widehat{H}_u + h_D^{ij} \widehat{Q}_i \widehat{D}_j \widehat{H}_d + h_E^{ij} \widehat{L}_i \widehat{E}_j \widehat{H}_d$ $+ \mu \widehat{H}_d \widehat{H}_u$

Spontaneous R/P

$$\mathcal{W} = h_U^{ij} \widehat{Q}_i \widehat{U}_j \widehat{H}_u + h_D^{ij} \widehat{Q}_i \widehat{D}_j \widehat{H}_d + h_E^{ij} \widehat{L}_i \widehat{E}_j \widehat{H}_d + h_\nu^{ij} \widehat{L}_i \widehat{\nu}_j^c \widehat{H}_u + \mu \widehat{H}_d \widehat{H}_u$$

 \Rightarrow Conserves L at level of $\mathcal W$

 \Rightarrow If scalar singlet gets vacuum expectation value:

 $\epsilon_i = h_i^{\nu} \langle \tilde{\nu}^c \rangle$

- \Rightarrow Spontaneous breaking of lepton number, Goldstone boson: Majoron
- \Rightarrow "Mostly" singlet Majoron of $\langle \tilde{\nu} \rangle \ll \langle \tilde{\nu}^c \rangle$
- \Rightarrow Neutrino data easily fitted

Spontaneous R/P

$\mathcal{W} = h_U^{ij} \widehat{Q}_i \widehat{U}_j \widehat{H}_u + h_D^{ij} \widehat{Q}_i \widehat{D}_j \widehat{H}_d + h_E^{ij} \widehat{L}_i \widehat{E}_j \widehat{H}_d$ $+ h_\nu^{ij} \widehat{L}_i \widehat{\nu}_j^c \widehat{H}_u - h_0 \widehat{H}_d \widehat{H}_u \widehat{\Phi} + h^{ij} \widehat{\Phi} \widehat{\nu}_i^c \widehat{S}_j$

As before, plus:

Masiero & Valle, 1990

- $\Rightarrow \widehat{\Phi}$ potentially solves μ -problem \widehat{a} la NMSSM
- \Rightarrow Dirac mass term for $\widehat{
 u}^c$ through v_{Φ}
- $\Rightarrow \nu^c$ light a la "inverse seesaw"
- \Rightarrow Many variants possible ...

Invisible neutralino decay



Lightest χ^0 decays to $J + \nu$ Decay channel large if $\mu - > eJ$ large

Hirsch et al., 2009

- \Rightarrow Large statistics necessary to improve limit
- \Rightarrow MEG experiment not sensitive, must search for $\mu > eJ + \gamma$ instead



- ⇒ observation of CLFV points to BSM beyond neutrino masses
- ⇒ distinguish between models by different operators (with same generation of leptons)
- \Rightarrow probe models of neutrino angles by comparing CLFV (using different generations of leptons)
- \Rightarrow If signs of SUSY at LHC, indirect insight into high energy world: Seesaw parameters (?)

Backup Slides

Neutrino angles

Very good first approximation: tri-bimaximal ansatz of Harrison, Perkins & Scott, 2002:

$$\mathcal{U}_{\nu}^{\text{HPS}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Corresponding to

$$\tan^2 \theta_{\rm Atm} = 1$$
 , $\tan^2 \theta_{\odot} = \frac{1}{2}$, $\sin^2 \theta_{\rm R} = 0$

Trilinear RPV SUSY



Contrary to RPC SUSY photon diagram not dominant

