Majorana CP-Violation, $(\beta\beta)_{0\nu}-$Decay and Leptogenesis

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Compelling Evidences for ν−Oscillations

$-\nu_{\text{atm}}$: SK UP-DOWN ASYMMETRY

θ_Z-, $L/E-$ **dependences of** $\mu-$**like events**

Dominant $\nu_{\mu} \longrightarrow \nu_{\tau}$ K2K; MINOS, CNGS (OPERA)

$-\nu_{\odot}$: **Homestake, Kamiokande, SAGE, GALLEX/GNO**

Super-Kamiokande, SNO; KamLAND

Dominant $\nu_{e} \longrightarrow \nu_{\mu,\tau}$ **BOREXINO,..., LowNu**

− LSND

Dominant $\bar{\nu}_{\mu} \longrightarrow \bar{\nu}_{e}$ **MiniBOONE**

$$\nu_{l\mathrm{L}} = \sum_{j=1} U_{lj}\, \nu_{j\mathrm{L}} \qquad l = e, \mu, \tau. \tag{1}$$

B. Pontecorvo, 1957; 1958; 1967;

Z. Maki, M. Nakagawa, S. Sakata, 1962;

Three Neutrino Mixing

$$\nu_{l\mathrm{L}} = \sum_{j=1}^{3} U_{lj}\, \nu_{j\mathrm{L}} \ . \tag{2}$$

U is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix,

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \tag{3}$$

- U - $n \times n$ unitary:

	n	2	3	4
mixing angles:	$\frac{1}{2}n(n-1)$	1	3	6
CP-violating phases:				
• ν_j- Dirac:	$\frac{1}{2}(n-1)(n-2)$	0	1	3
• ν_j- Majorana:	$\frac{1}{2}n(n-1)$	1	3	6

$n = 3$: 1 Dirac and
2 additional CP-violating phases, Majorana phases

S.M. Bilenky, J. Hosek, S.T.P.,1980;
J. Schechter, J.W.F. Valle,1980;
M. Doi, T. Kotani, E. Takasugi,1981

PMNS Matrix: Standard Parametrization

$$U = V \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_{31}}{2}} \end{pmatrix} \quad (4)$$

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13}e^{i\delta} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}e^{i\delta} \end{pmatrix} \quad (5)$$

- $s_{ij} \equiv \sin\theta_{ij}$, $c_{ij} \equiv \cos\theta_{ij}$, $\theta_{ij} = [0, \frac{\pi}{2}]$,
- δ - Dirac CP-violation phase, $\delta = [0, 2\pi]$,
- α_{21}, α_{31} - the two Majorana CP-violation phases.
- $\Delta m^2_{\odot} \equiv \Delta m^2_{21} \cong 8.0 \times 10^{-5}$ eV$^2 > 0$, $\sin^2\theta_{12} \cong 0.30$, $\cos 2\theta_{12} \gtrsim 0.28$ (2σ),
- $|\Delta m^2_{\rm atm}| \equiv |\Delta m^2_{31}| \cong 2.5 \times 10^{-3}$ eV^2, $\sin^2 2\theta_{23} \cong 1$,
- θ_{13} - the CHOOZ angle: $\sin^2\theta_{13} < 0.027$ (0.041) 2σ (3σ).

A.Bandyopadhyay, S.Choubey, S.Goswami, S.T.P., D.P.Roy, hep-ph/0406328 (updated)

T. Schwetz, hep-ph/0606060.

- $\sqrt{\Delta m^2_\odot} \sin^2\theta_{12} \cong 3.0 \times 10^{-3}$ eV $\;(\pm)\;$ $\sqrt{|\Delta m^2_{\rm atm}|} \sin^2\theta_{13} \lesssim 2.2 \times 10^{-3}$ eV;

- $\sqrt{|\Delta m^2_{\rm atm}|} \cong 5 \times 10^{-2}$ eV; $\sqrt{|\Delta m^2_{\rm atm}|} \cos 2\theta_{12} \gtrsim 1.4 \times 10^{-2}$ eV ($\cos 2\theta_{12} \gtrsim 0.28$)

- m_0: $m_0^2 \gg \Delta m^2_\odot, |\Delta m^2_{\rm atm}|$, $m_0 \gtrsim 0.1$ eV

- ${\rm sgn}(\Delta m^2_{\rm atm}) = {\rm sgn}(\Delta m^2_{31})$ not determined

$\Delta m^2_{\rm atm} \equiv \Delta m^2_{31} > 0$, normal mass ordering

$\Delta m^2_{\rm atm} \equiv \Delta m^2_{32} < 0$, inverted mass ordering

Convention: $m_1 < m_2 < m_3$ - NMO, $m_3 < m_1 < m_2$ - IMO

- **Majorana phases α_{21}, α_{31}:**
  - $\nu_l \leftrightarrow \nu_{l'}$, $\bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$ not sensitive;

S.M. Bilenky, J. Hosek, S.T.P.,1980;
P. Langacker, S.T.P., G. Steigman, S. Toshev, 1987

  - $|<m>|$ in $(\beta\beta)_{0\nu}$−decay depends on α_{21}, α_{31};
  - $\Gamma(\mu \to e + \gamma)$ etc. in SUSY theories depend on $\alpha_{21,31}$;
  - BAU, leptogenesis scenario: $\alpha_{21,31}$!

Future Progress

- Determination of the nature - Dirac or Majorana, of ν_j .

- Determination of $\text{sgn}(\Delta m^2_{\text{atm}})$, type of $\nu-$ mass spectrum

$$m_1 \ll m_2 \ll m_3, \qquad \text{NH},$$

$$m_3 \ll m_1 < m_2, \qquad \text{IH},$$

$$m_1 \cong m_2 \cong m_3,\ m^2_{1,2,3} >> \Delta m^2_{\text{atm}},\ \text{QD};\ m_j \gtrsim 0.10\ \text{eV}.$$

- Determining, or obtaining significant constraints on, the absolute scale of ν_j-masses, or $\min(m_j)$.

- Status of the CP-symmetry in the lepton sector: violated due to δ (Dirac), and/or due to α_{21}, α_{31} (Majorana)?

- High precision determination of $\Delta m^2_{\odot}$, $\theta_{\odot}$, Δm^2_{atm}, θ_{atm}.

- Measurement of, or improving by at least a factor of (5 - 10) the existing upper limit on, $\sin^2\theta_{13}$.

- Searching for possible manifestations, other than ν_l-oscillations, of the non-conservation of L_l, $l = e, \mu, \tau$, such as $\mu \to e + \gamma$, $\tau \to \mu + \gamma$, etc. decays.

• Understanding at fundamental level the mechanism giving rise to the $\nu-$ masses and mixing and to the L_l-non-conservation. Includes understanding

– the origin of the observed patterns of ν-mixing and ν-masses ;

– the physical origin of CPV phases in $U_{\rm PMNS}$;

– Are the observed patterns of ν-mixing and of $\Delta m^2_{21,31}$ related to the existence of a new symmetry?

– Is there any relations between $q-$mixing and $\nu-$ mixing? Is $\theta_{12} + \theta_c = \pi/4$?

– Is $\theta_{23} = \pi/4$, or $\theta_{23} > \pi/4$ or else $\theta_{23} < \pi/4$?

– Is there any correlation between the values of CPV phases and of mixing angles in $U_{\rm PMNS}$?

• Progress in the theory of ν-mixing might lead to a better understanding of the origin of the BAU.

– Can the Majorana and/or Dirac CPVP in $U_{\rm PMNS}$ be the leptogenesis CPV parameters at the origin of BAU?

$(\beta\beta)_{0\nu}-$Decay Experiments:

- Majorana nature of ν_j
- Type of $\nu-$mass spectrum (NH, IH, QD)
- Absolute neutrino mass scale

^{3}H β-decay , cosmology: m_ν (QD, IH)

- CPV due to Majorana CPV phases

ν_j- Dirac or Majorana particles, **fundamental problem**

ν_j-Dirac: **conserved lepton charge exists,** $L = L_e + L_\mu + L_\tau$**,** $\nu_j \neq \bar{\nu}_j$

ν_j-Majorana: **no lepton charge is exactly conserved,** $\nu_j \equiv \bar{\nu}_j$

The observed patterns of $\nu-$mixing and of Δm^2_{atm} and $\Delta m^2_{\odot}$ can be related to Majorana ν_j and an approximate symmetry:

$$L' = L_e - L_\mu - L_\tau$$

S.T.P., 1982

See-saw mechanism: ν_j- Majorana

Establishing that ν_j are Majorana particles would be as important as the discovery of $\nu-$ oscillations.

If ν_j- Majorana particles, U_{PMNS} **contains (3-ν mixing)**

δ-Dirac, α_{21}, α_{31} - Majorana **physical CPV phases**

ν-oscillations $\nu_l \leftrightarrow \nu_{l'}$, $\bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$, $l, l' = e, \mu, \tau$,

- **are not sensitive to the nature of ν_j,**

S.M. Bilenky et al.,1980;
P. Langacker et al., 1987

- **provide information on $\Delta m^2_{jk} = m^2_j - m^2_k$, but not on the absolute values of ν_j masses.**

The Majorana nature of ν_j can manifest itself in the existence of $\Delta L = \pm 2$ processes:

$$K^+ \to \pi^- + \mu^+ + \mu^+$$
$$\mu^- + (\text{A}, \text{Z}) \to \mu^+ + (\text{A}, \text{Z} - 2)$$

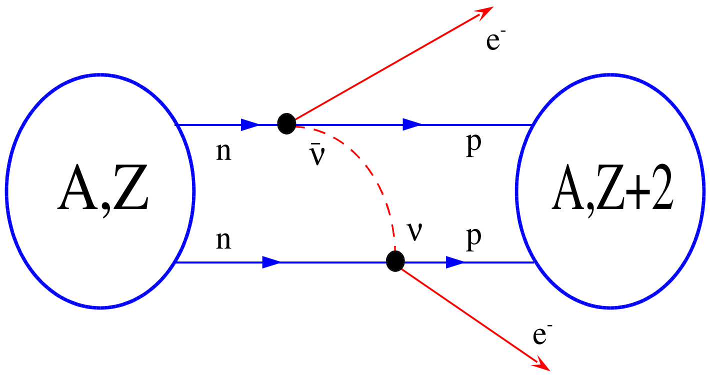
The process most sensitive to the possible Majorana nature of ν_j - $(\beta\beta)_{0\nu}$-decay

$$(\text{A}, \text{Z}) \to (\text{A}, \text{Z} + 2) + e^- + e^-$$

of even-even nuclei, ^{48}Ca, ^{76}Ge, ^{82}Se, ^{100}Mo, ^{116}Cd, ^{130}Te, ^{136}Xe, ^{150}Nd.

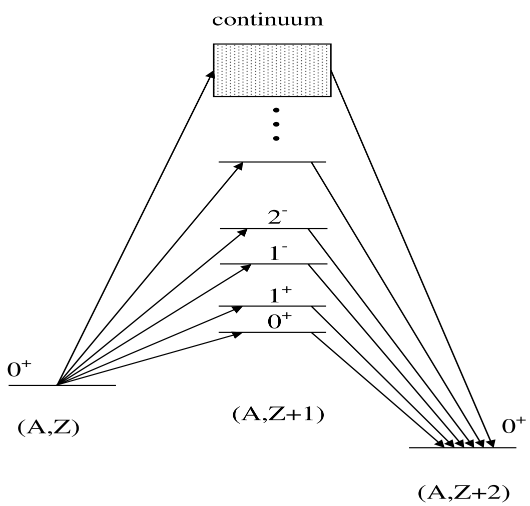
$2n$ from (A,Z) exchange a virtual Majorana ν_j (via the CC weak interaction) and transform into $2p$ of (A,Z+2) and two free e^-.

Nuclear $0\nu\beta\beta$-decay



strong in-medium modification of the basic process

$$dd \rightarrow uue^- e^- (\bar{\nu}_e \bar{\nu}_e)$$



virtual excitation
of states of all multipolarities
in (A,Z+1) nucleus

V. Rodin, talk at Gran Sasso, 2006

$A(\beta\beta)_{0\nu} \sim <m>$ M(A,Z), M(A,Z) - NME,

$$\left|<m>\right| = \left|m_1|U_{e1}|^2 + m_2|U_{e2}|^2\ e^{i\alpha_{21}} + m_3|U_{e3}|^2\ e^{i\alpha_{31}}\right|$$

$$= \left|m_1\ c_{12}^2\ c_{13}^2 + m_2\ s_{12}^2\ c_{13}^2\ e^{i\alpha_{21}} + m_3\ s_{13}^2\ e^{i\alpha_{31}}\right|,\ \ \theta_{12} \equiv \theta_{\odot},\ \theta_{13}\text{- CHOOZ}$$

α_{21}, α_{31} - the two Majorana CPVP of the PMNS matrix.

CP-invariance: $\alpha_{21} = 0, \pm\pi,\ \alpha_{31} = 0, \pm\pi;$

$$\eta_{21} \equiv e^{i\alpha_{21}} = \pm 1,\ \ \eta_{31} \equiv e^{i\alpha_{31}} = \pm 1$$

relative CP-parities of ν_1 and ν_2, and of ν_1 and ν_3 .

L. Wolfenstein, 1981;

S.M. Bilenky, N. Nedelcheva, S.T.P., 1984;

B. Kayser, 1984.

Best sensitivity: Heidelberg-Moscow ^{76}Ge experiment.

Claim for a positive signal at $> 3\sigma$:

H. Klapdor-Kleingrothaus et al., PL B586 (2004),

$|{<}\,m\,{>}| \; = (0.1 - 0.9)$ eV (99.73% C.L.).

IGEX ^{76}Ge: $|{<}\,m\,{>}| \; < (0.33 - 1.35)$ eV (90% C.L.).

Taking data - NEMO3 (^{100}Mo), CUORICINO (^{130}Te):

$|{<}\,m\,{>}| \; <$(0.7–1.2) eV, $|{<}\,m\,{>}| \; <$(0.18–0.90) eV (90% C.L.).

Large number of projects: $|{<}\,m\,{>}| \; \sim (0.01 - 0.05)$ eV

CUORE - ^{130}Te,
GERDA - ^{76}Ge,
SuperNEMO - ^{82}Se,
EXO - ^{136}Xe,
MAJORANA - ^{76}Ge,
MOON - ^{100}Mo,
CANDLES - ^{48}Ca,
XMASS - ^{136}Xe.

$|{<}\,m\,{>}|$: m_j, $\theta_\odot \equiv \theta_{12}$, θ_{13}, $\alpha_{21,31}$

$m_{1,2,3}$ - in terms of $\min(m_j)$, $\Delta m^2_{\rm atm}$, $\Delta m^2_\odot$

S.T.P., A.Yu. Smirnov, 1994

Convention: $m_1 < m_2 < m_3$ - NMO, $m_3 < m_1 < m_2$ - IMO

$$\Delta m^2_\odot \equiv \Delta m^2_{21}, \quad m_2 = \sqrt{m_1^2 + \Delta m^2_\odot},$$

while either

$$\Delta m^2_{\rm atm} \equiv \Delta m^2_{31} > 0, \quad m_3 = \sqrt{m_1^2 + \Delta m^2_{\rm atm}}, \quad \text{normal mass ordering,} \quad \text{or}$$

$$\Delta m^2_{\rm atm} \equiv \Delta m^2_{32} < 0, \quad m_1 = \sqrt{m_3^2 + |\Delta m^2_{\rm atm}| - \Delta m^2_\odot}, \quad \text{inverted mass ordering}$$

The neutrino mass spectrum –

Normal hierarchical (NH) if $m_1 << m_2 << m_3$,

Inverted hierarchical (IH) if $m_3 << m_1 \cong m_2$,

Quasi-degenerate (QD) if $m_1 \cong m_2 \cong m_3 = m$, $m_j^2 >> |\Delta m^2_{\rm atm}|$; $m_j \gtrsim 0.1$ eV

Given $|\Delta m^2_{\rm atm}|$, $\Delta m^2_\odot$, $\theta_\odot$, θ_{13},

$$|{<}\,m\,{>}| = |{<}\,m\,{>}|\ (\mathrm{m_{min}}, \alpha_{21}, \alpha_{31}; \mathrm{S}), \ \mathrm{S} = \mathrm{NO(NH), IO(IH)}.$$

$A(\beta\beta)_{0\nu} \sim <m>$ M(A,Z), M(A,Z) - NME,

$$|<m>| \cong \left|\sqrt{\Delta m^2_\odot}\sin^2\theta_{12}e^{i\alpha} + \sqrt{\Delta m^2_{31}}\sin^2\theta_{13}e^{i\beta_M}\right|, \; m_1 \ll m_2 \ll m_3 \text{ (NH)},$$

$$|<m>| \cong \sqrt{m_3^2 + \Delta m^2_{13}}\left|\cos^2\theta_{12} + e^{i\alpha}\ \sin^2\theta_{12}\right|, \; m_3 < (\ll) m_1 < m_2 \text{ (IH)},$$

$$|<m>| \cong m\left|\cos^2\theta_{12} + e^{i\alpha}\ \sin^2\theta_{12}\right|, \; m_{1,2,3} \cong m \gtrsim 0.10 \text{ eV} \text{ (QD)},$$

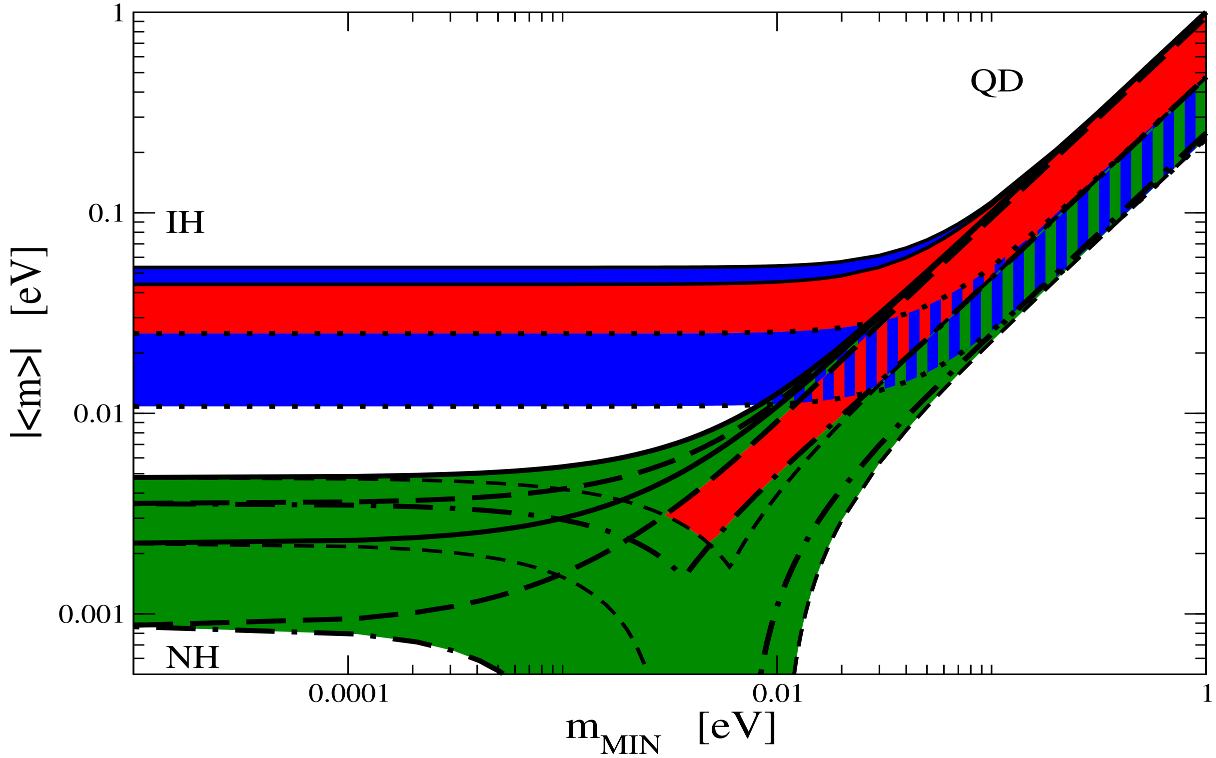
$\theta_{12} \equiv \theta_\odot$, θ_{13}-CHOOZ; $\alpha \equiv \alpha_{21}$, $\beta_M \equiv \alpha_{31}$.

CP-invariance: $\alpha = 0, \pm\pi$, $\beta_M = 0, \pm\pi$;

$$|<m>| \lesssim 5 \times 10^{-3} \text{ eV, NH};$$

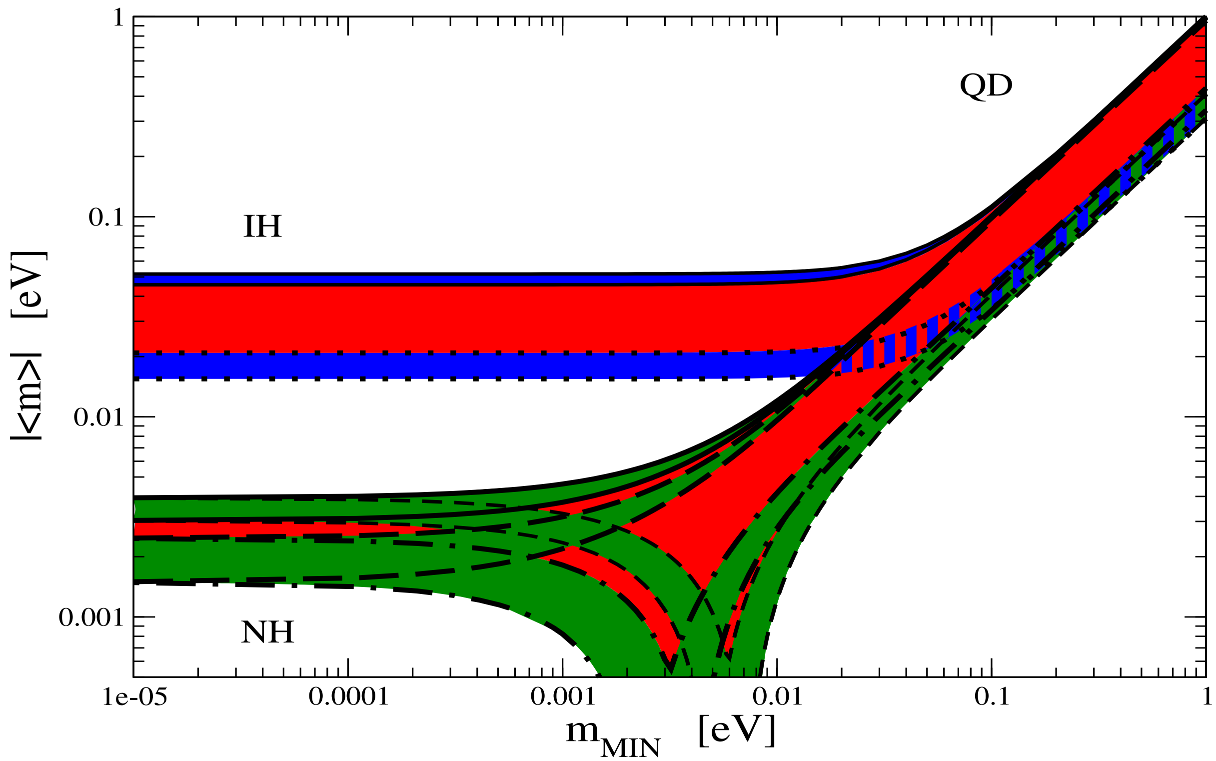
$$\sqrt{\Delta m^2_{13}}\cos 2\theta_{12} \cong 0.013 \text{ eV} \lesssim |<m>| \lesssim \sqrt{\Delta m^2_{13}} \cong 0.055 \text{ eV, IH};$$

$$m\cos 2\theta_{12} \lesssim |<m>| \lesssim m, \; m \gtrsim 0.10 \text{ eV, QD}.$$



S. Pascoli, S.T.P., 2006

The current 2σ ranges of values of the parameters used.



$\sin^2\theta_{13} = 0.015 \pm 0.006$; $1\sigma(\Delta m^2_{\odot}) = 4\%$, $1\sigma(\sin^2\theta_{\odot}) = 4\%$, $1\sigma(|\Delta m^2_{\text{atm}}|) = 6\%$;

$2\sigma(|<m>|\ )$ used.

Nuclear Matrix Element Uncertainty

$$|<m>| \;= \zeta \; ((|<m>|_{\text{exp}})_{\text{MIN}} \pm \Delta) \;, \quad \zeta \geq 1,$$

$(|<m>|_{\text{exp}})_{\text{MIN}}$ – obtained with the **maximal physically allowed value of NME**.

A measurement of the $(\beta\beta)_{0\nu}$-decay half-life time

$$(|<m>|_{\text{exp}})_{\text{MIN}} - \Delta \leq |<m>| \; \leq \zeta((|<m>|_{\text{exp}})_{\text{MIN}} + \Delta) \;.$$

The estimated range of ζ^2:

^{48}Ca, $\zeta^2 \simeq 3.5$

^{76}Ge, $\zeta^2 \simeq 10$

^{82}Se, $\zeta^2 \simeq 10$

^{130}Te, $\zeta^2 \simeq 38.7$

S. Elliot, P. Vogel, 2002

NH vs IH (QD):

$$\zeta \; |<m>|^{\text{NH}}_{\text{max}} < |<m>|^{\text{IH(QD)}}_{\text{min}} \;, \; \zeta \geq 1 \;.$$

IH vs QD:

$$\zeta \; |<m>|^{\text{IH}}_{\text{max}} < |<m>|^{\text{QD}}_{\text{min}} \;, \; \zeta \geq 1 \;.$$

Method of Analysis

$$\Gamma_{\text{th}} = G\,|\mathcal{M}|^2\,\left(|<m>|\ (\mathbf{x})\right)^2\ ,\ \ \mathbf{x} = (\mathbf{x}_{\text{osc}}, \mathbf{x}_{\beta\beta}^{0\nu})$$

$$\mathbf{x}_{\text{osc}} = (\theta_{12}, \theta_{13}, |\Delta \mathbf{m}^2_{31}|, \Delta \mathbf{m}^2_{21})\,,$$

$$\mathbf{x}_{\beta\beta}^{0\nu} = (m_0, \text{sgn}(\Delta \mathbf{m}^2_{31}), \alpha_{21}, \alpha_{31})\,.$$

$$|<m>|\ ^{\text{obs}} \equiv \sqrt{\frac{\Gamma_{\text{obs}}}{G}}\,\frac{1}{|\mathcal{M}_0|}\,, \qquad \sigma_{\beta\beta} = \frac{1}{2}\,\frac{1}{\sqrt{\Gamma_{\text{obs}} G}}\,\frac{1}{|\mathcal{M}_0|}\,\sigma(\Gamma_{\text{obs}})\,,$$

$|\mathcal{M}_0|$ is some nominal value of the NME.

$$\chi^2(\mathbf{x}_{\beta\beta}^{0\nu}, \mathbf{F}) = \min_{\xi \in [1/\sqrt{F}, \sqrt{F}]} \frac{\left[\xi\,|<m>|\ (\mathbf{x}) - |<m>|\ ^{\text{obs}}\right]^2}{\sigma^2_{\beta\beta} + \xi^2 \sigma^2_{\text{th}}}\,.$$

$$\xi \equiv \frac{|\mathcal{M}|}{|\mathcal{M}_0|}\,, \quad \xi = [1/\sqrt{F}, \sqrt{F}], \quad F \geq 1,$$

$|\mathcal{M}|$ is the *true* value of the NME.

S. Pascoli, S.T.P., T. Schwetz, hep-ph/0505226

Majorana CPV Phases and $|<m>|$

IH spectrum: $m_{\text{min}} < 0.01$ eV, $\sin^2\theta -$ negligible

$$\sqrt{\Delta m^2_{\text{atm}}}|\cos 2\theta_\odot| \leq |<m>| \ \leq \sqrt{\Delta m^2_{\text{atm}}}.$$

"Just CP-violating" region:

$$(|<m>|_{\text{exp}})_{\text{MAX}} < \sqrt{(\Delta m^2_{\text{atm}})_{\text{MIN}}} \ ,$$

$$(|<m>|_{\text{exp}})_{\text{MIN}} > \sqrt{(\Delta m^2_{\text{atm}})_{\text{MAX}}}\ (\cos 2\theta_\odot)_{\text{MAX}} \ ,$$

$$|<m>| \ = \zeta \ ((|<m>|_{\text{exp}})_{\text{MIN}} \pm \Delta) \ , \quad \zeta \geq 1$$

Necessary condition for establishing CP-violation:

$$1 \leq \zeta < \frac{\sqrt{(\Delta m^2_{\text{atm}})_{\text{MIN}}}}{\sqrt{(\Delta m^2_{\text{atm}})_{\text{MAX}}}\ (\cos 2\theta_\odot)_{\text{MAX}} + 2\Delta} \ \simeq \frac{1}{(\cos 2\theta_\odot)_{\text{MAX}}}$$

QD spectrum, $m_{1,2,3} \simeq m_0 \gtrsim 0.20$ eV - similar condition: $\Delta m^2_{\text{atm}} \to m^2_0$.

CPV can be established provided

– $|<m>|$ measured with $\Delta \lesssim 15\%$;

– $\Delta m^2_{\rm atm}$ (IH) or m_0 (QD) measured with $\delta \lesssim 10\%$;

– $\xi \lesssim 1.5$;

– α_{21} (QD): in the interval $\sim [\frac{\pi}{4} - \frac{3\pi}{4}]$, or $\sim [\frac{5\pi}{4} - \frac{3\pi}{2}]$;

– $\tan^2\theta_\odot \gtrsim 0.40$.

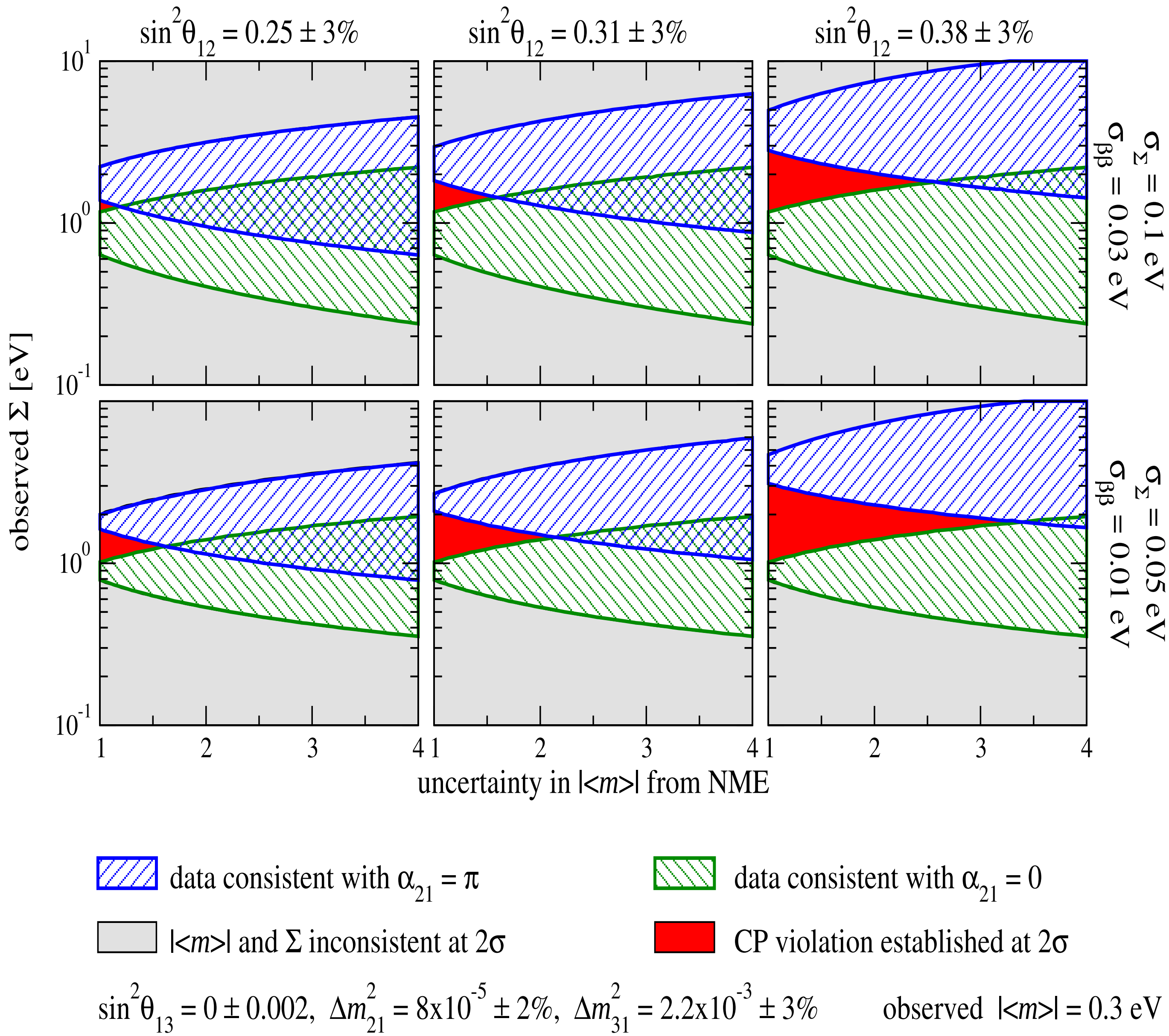
S. Pascoli, S.T.P., W. Rodejohann, 2002

S. Pascoli, S.T.P., L. Wolfenstein, 2002

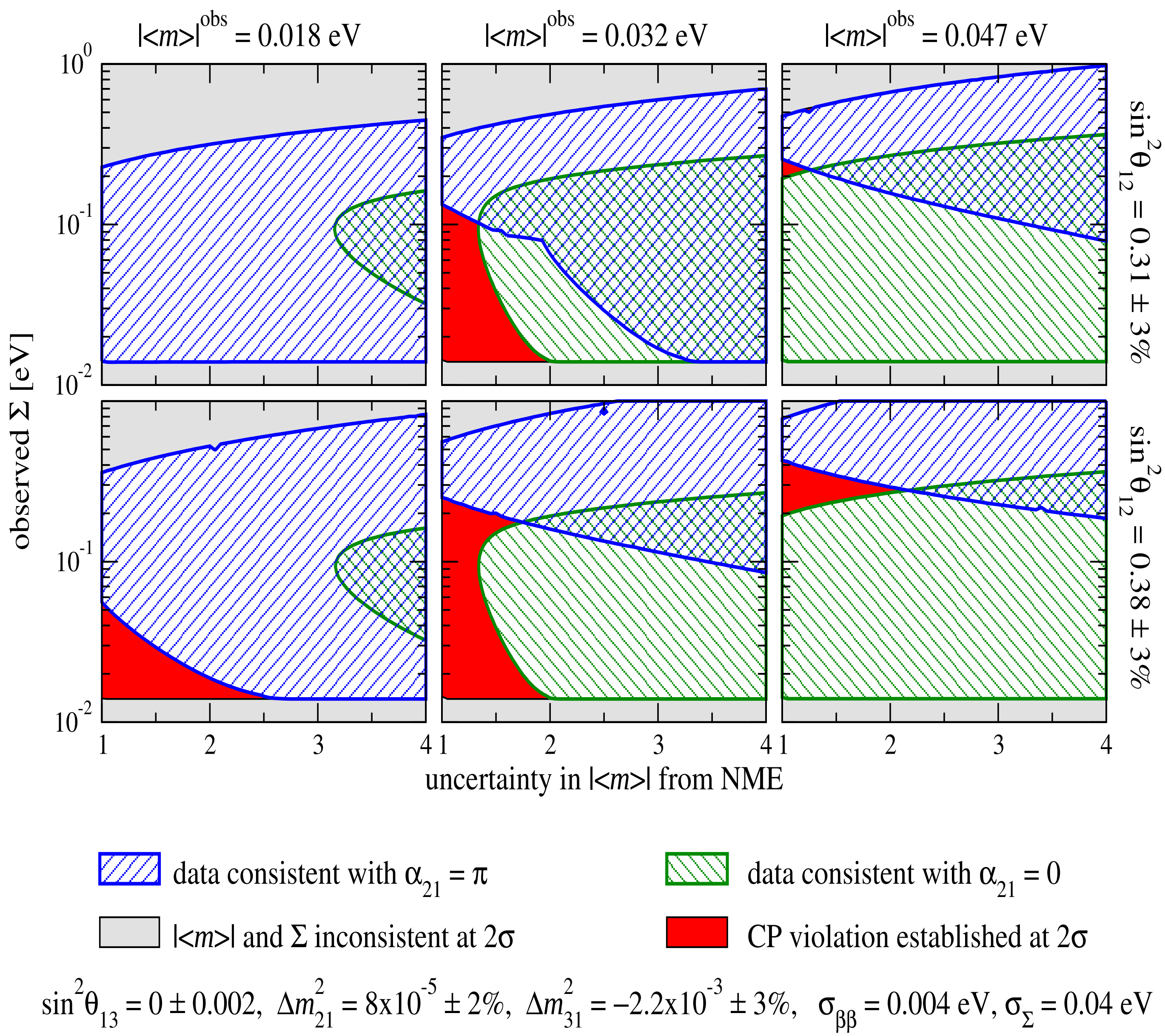
S. Pascoli, S.T.P., T. Schwetz, hep-ph/0505226

No "No-go for detecting CP-Violation via $(\beta\beta)_{0\nu}$-decay"

V. Barger *et al.*, 2002



S. Pascoli, S.T.P., T. Schwetz, hep-ph/0505226



|<m>|^obs = 0.018 eV
|<m>|^obs = 0.032 eV
|<m>|^obs = 0.047 eV
sin²θ12 = 0.31 ± 3%
sin²θ12 = 0.38 ± 3%
observed Σ [eV]
10^0
10^-1
10^-2
1 2 3 4
uncertainty in |<m>| from NME
data consistent with α21 = π
data consistent with α21 = 0
|<m>| and Σ inconsistent at 2σ
CP violation established at 2σ
sin²θ13 = 0 ± 0.002, Δm²21 = 8x10^-5 ± 2%, Δm²31 = −2.2x10^-3 ± 3%, σββ = 0.004 eV, σΣ = 0.04 eV

Absolute Neutrino Mass Measurements

The Troitzk and Mainz ^{3}H β-decay experiments

$$m_{\nu_e} < 2.3 \text{ eV} \qquad (95\% \text{ C.L.})$$

There are prospects to reach sensitivity

$$\text{KATRIN}: \qquad m_{\nu_e} \sim 0.2 \text{ eV}$$

Cosmological and astrophysical data: the WMAP result combined with data from large scale structure surveys (2dFGRS, SDSS)

$$\sum_j m_j \equiv \Sigma < (0.4 - 1.7) \text{ eV}$$

The WMAP and future PLANCK experiments can be sensitive to

$$\sum_j m_j \cong 0.4 \text{ eV}$$

Data on weak lensing of galaxies by large scale structure, combined with data from the WMAP and PLANCK experiments may allow to determine

$$\sum_j m_j : \qquad \delta \cong 0.04 \text{ eV}.$$

M_ν from the See-Saw Mechanism

P. Minkowski, 1977.
M. Gell-Mann, P. Ramond, R. Slansky, 1979;
T. Yanagida, 1979;
R. Mohapatra, G. Senjanovic, 1980.

- Explains the smallness of $\nu-$masses.

- Through **leptogenesis theory** links the $\nu-$mass generation to the generation of baryon asymmetry of the Universe Y_B.

S. Fukugita, T. Yanagida, 1986.

- In SUSY GUT's with see-saw mechanism of $\nu-$mass generation, the LFV decays

$$\mu \rightarrow e + \gamma, \quad \tau \rightarrow \mu + \gamma, \quad \tau \rightarrow e + \gamma \text{ , etc.}$$

are predicted to take place with rates within the reach of present and future experiments.

F. Borzumati, A. Masiero, 1986.

- The ν_j are **Majorana particles**; $(\beta\beta)_{0\nu}-$decay is allowed.

See-Saw: Dirac ν-mass m_D + Majorana mass M_R for N_R

The Role of the CPV Phases

$$-\mathcal{L} = \overline{N_{Ri}}\,(m_D)_{ij}\,\nu_{Lj} + \frac{1}{2}\,\overline{(N_{Ri})^c}\,(M_R)_{ij}\,N_{Rj}$$

m_D generated by the Yukawa interaction:

$$-\mathcal{L}_{\mathsf{Y}} = \overline{N_{Ri}}\,(Y_\nu)_{ij}\,L_j\,H_u,\ \ v_u = 174\ \text{GeV}\sin\beta,\ \ v_u Y_\nu = m_D - \text{complex}$$

For M_R - sufficiently large,

$$m_\nu \simeq m_D^T\,M_R^{-1}\,m_D = v_u^2\ Y_\nu^T\,M_R^{-1}\,Y_\nu = U^*_{\mathsf{PMNS}}\ m_\nu^{\mathsf{diag}}\ U^\dagger_{\mathsf{PMNS}}\ .$$

Basis: $M_R = (M_1, M_2, M_3)$; $D_N \equiv \text{diag}(M_1, M_2, M_3)$, $D_\nu \equiv \text{diag}(m_1, m_2, m_3)$.

$Y_\nu = \sqrt{D_N}\ R\ \sqrt{D_\nu}\ (U_{\mathsf{PMNS}})^\dagger / v_u$, all at M_R ; R-complex, $R^T R = \mathbf{1}$.

J.A. Casas and A. Ibarra, 2001

In GUTs, $M_R < M_X$, $M_X \sim 10^{16}$ GeV;

in GUTs, e.g., $M_R = (10^9, 10^{12}, 10^{15})$ GeV, $m_D \sim 1$ GeV.

Leptogenesis

$$Y_B = \frac{n_B - n_{\bar{B}}}{S} \sim 6 \times 10^{-10}$$

$$Y_B \cong -10^{-2} \quad \kappa\, \varepsilon$$

W. Buchmüller, M. Plümacher, 1998;

W. Buchmüller, P. Di Bari, M. Plümacher, 2004

$\kappa-$ efficiency factor; $\kappa \sim 10^{-1} - 10^{-3}$: $\varepsilon \gtrsim 10^{-7}$.

ε: $CP-$, $L-$ violating asymmetry generated in out of equilibrium $N_{Rj}-$decays in the early Universe,

$$\varepsilon_1 = \frac{\Gamma(N_1 \to \Phi^- \ell^+) - \Gamma(N_1 \to \Phi^+ \ell^-)}{\Gamma(N_1 \to \Phi^- \ell^+) + \Gamma(N_1 \to \Phi^+ \ell^-)}$$

$$\simeq \frac{1}{8\pi} \frac{1}{(Y_\nu Y_\nu^\dagger)_{11}} \sum_{j=2,3} \mathrm{Im}(Y_\nu Y_\nu^\dagger)^2_{1j} \left(f(M_j^2/M_1^2) + g(M_j^2/M_1^2) \right) \; .$$

$$f(x) = \sqrt{x} \left(1 - (1+x) \ln\left(\frac{1+x}{x}\right) \right) \; , \quad g(x) = \frac{\sqrt{x}}{1-x}$$

M.A. Luty, 1992;

L. Covi, E. Roulet, and F. Vissani, 1996

$$\frac{1}{\kappa} \simeq \frac{3.3 \times 10^{-3}\ \mathrm{eV}}{\widetilde{m}_1} + \left(\frac{\widetilde{m}_1}{0.55 \times 10^{-3}\ \mathrm{eV}} \right)^{1.16} , \quad \widetilde{m}_1 \equiv \frac{v_u^2}{M_1} \left(Y_\nu Y_\nu^\dagger \right)_{11}$$

G. F. Giudice *et al.*, 2004

LFV Charged Lepton Decays

$m_\nu \neq 0$, $U_{\rm PMNS} \neq \mathbf{1}$: L_e, L_μ, L_τ not conserved

$\mu \to e + \gamma$, $\mu \to 3e$, $\tau \to \mu + \gamma$, $\tau \to e + \gamma$, etc., allowed.

$$BR(\mu \to e + \gamma) < 1.2 \times 10^{-11} \qquad \text{(MEGA)}$$

$$BR(\tau \to \mu + \gamma) < 6.8 \times 10^{-8} \qquad \text{(BABAR)}$$

PSI: $BR(\mu \to e + \gamma) < 10^{-13} - 10^{-14}$ (MEG)

Plans: $BR(\tau \to \mu + \gamma) < 10^{-8} - 10^{-9}$ (LHC)

Standard Theory with $m_\nu \neq 0$, $U_{\rm PMNS} \neq \mathbf{1}$:

$$BR(\mu \to e + \gamma) \sim 2.5 \times 10^{-4} \, (\tfrac{m_j}{M_W})^4 \lesssim 10^{-46}$$

S.T.P, 1977;

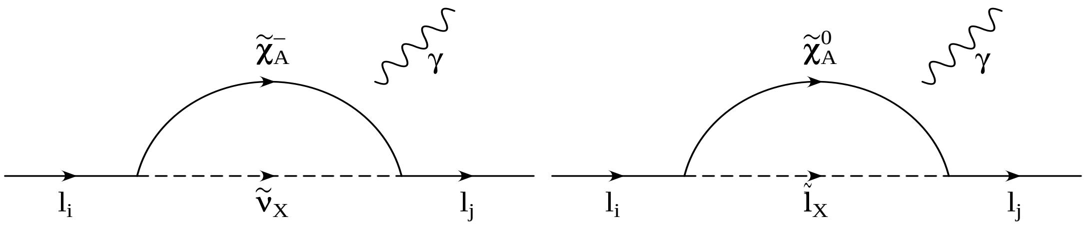
SUSY GUTs with see-saw mechanism:

• Flavour universality of SUSY breaking at $M_X \sim 10^{16}$ GeV (scalar masses m_0, trilinear couplings $A_0 \equiv a_0 m_0$, gaugino masses $m_{1/2}$)

• RG running: "large" LFV corrections to the slepton masses $\delta m^2 \sim Y_\nu^\dagger Y_\nu$

F. Borzumati, A. Masiero, 1986.

$$\left(m^2_{sL}\right)_{ji} \approx -\frac{1}{8\pi^2}(3m_0^2 + A_0^2)\left(Y_\nu^\dagger Y_\nu\right)_{ji} \log\frac{M_{\rm X}}{M_{\rm R}}, \qquad m_0, A_0 - \text{ at } M_X \,.$$



$$BR(\ell_i \to \ell_j + \gamma) \simeq \alpha^3 \left(\frac{(3 + A_0^2)\, m_0^2}{8\pi^2 m_S^4\, G_F} \right)^2 \left|(Y_\nu^\dagger L Y_\nu)_{ij}\right|^2 \tan^2\beta \sim \frac{\left|(Y_\nu^\dagger L Y_\nu)_{ij}\right|^2}{G_F^2\, m_{\rm SUSY}^4} \tan^2\beta\,,$$

m_S- sparticle mass (at M_X); $L_k = \ln M_X/M_k$, $k = 1, 2, 3$.

$$m_S^8 \simeq 0.5\ m_0^2\ m_{1/2}^2\ (m_0^2 + 0.6 m_{1/2}^2)^2\ .$$

S.T.P., S. Profumo, Y. Takanishi, C. Yaguna, 2003

For, e.g., $A_0 \sim m_S \sim m_0 \sim 10^2$ GeV, $M_X \sim 10^{16}$ GeV, $M_R \sim 10^{11}$ GeV, $\tan\beta = 10$,

$$BR(\ell_i \to \ell_j + \gamma) \sim 10^{-11} \left|(m_D^\dagger m_D)_{ij}\ \text{GeV}^{-2}\right|^2$$

"Benchmark SUSY scenario":

$m_0 = m_{1/2} = 250$ GeV, $A_0 = a_0 m_0 = -100$ GeV, $\tan\beta \sim 5 - 10$.

χ_1^0: $m_1 \sim 100$ GeV; χ_2^0, χ^+: $m_2 \sim 250$ GeV; $m_{sq} \sim (400 - 600)$ GeV (LHC).

$$BR(\ell_i \to \ell_j + \gamma) \simeq 9.1 \times 10^{-10} \left| \left(\mathbf{Y}_\nu^\dagger L \mathbf{Y}_\nu \right)_{ij} \right|^2 \tan^2\beta\,, \qquad \tan^2\beta \gtrsim 10\,.$$

- Leptogenesis: $\mathrm{Im}(Y_\nu Y_\nu^\dagger)^2$, $(Y_\nu Y_\nu^\dagger)_{11}$
- $\mu \to e + \gamma$, etc.: $Y_\nu^\dagger L Y_\nu$
- See-saw: $Y_\nu = \sqrt{D_N}\ R\ \sqrt{D_\nu}\ (U_{\mathrm{PMNS}})^\dagger$, all at M_R ; $R^T R = \mathbf{1}$.

J.A. Casas and A. Ibarra, 2001

Leptogenesis: $Y_\nu Y_\nu^\dagger = \sqrt{D_N}\ R\ D_\nu\ R^\dagger\ \sqrt{D_N}$, R should be complex.

$R = R_{12}(\omega_{12}) R_{13}(\omega_{13}) R_{23}(\omega_{23}) = R_{12}(\omega_{12}) R_{23}(\omega_{23}) R_{12}(\omega'_{12})$, ω_{ij}-complex.

$\mu \to e + \gamma$: $(Y_\nu^\dagger L Y_\nu)_{21} = (Y_\nu^\dagger)_{21} L_1 (Y_\nu)_{11} + (Y_\nu^\dagger)_{22} L_2 (Y_\nu)_{21} + (Y_\nu^\dagger)_{23} L_3 (Y_\nu)_{31}$

$(Y_\nu^\dagger)_{21} L_1 (Y_\nu)_{11} \propto \sqrt{m_j m_k}\ M_1/v_u^2$, $j,k \neq 1$ (3),   NH (IH);

$(Y_\nu^\dagger)_{22} L_2 (Y_\nu)_{21} \propto \sqrt{m_j m_k}\ M_2/v_u^2$;

$(Y_\nu^\dagger)_{23} L_3 (Y_\nu)_{31} \propto \sqrt{m_j m_k}\ M_3/v_u^2$.

Hierarchical spectrum: $M_1 \ll M_2 \ll M_3$

Low Energy Leptonic CPV and Leptogenesis

Assume:

$10^9 \lesssim M_1 \lesssim 10^{12}$ GeV, $\quad M_1 \ll M_2, M_3$

The "one-flavor" approximation used is correct only if $\mathbf{Y_{e,\mu,\tau}}$ - "small" :

Boltzmann eqn. for $n(N_1)$ and $\Delta L = \Delta(L_e + l_\mu + L_\tau)$.

$Y_l \bar{L}_l \Phi l_R$ - out of equilibrium at $T \sim M_1$.

At $M_1 \sim 10^{12}$ GeV: Y_τ - in equilibrium, $Y_{e,\mu}$ - not.

At $M_1 \sim 10^9$ GeV: Y_τ, Y_μ - in equilibrium, Y_e - not.

Thus, at $M_1 \sim 10^{12}$ GeV: L_τ, ΔL_τ - distinguishable;

L_e, L_μ, ΔL_e, ΔL_μ - individually not distinguishable;

$L_e + L_\mu$, $\Delta(L_e + L_\mu)$

A. Abada et al., 2006; E. Nardi et al., 2006

A. Abada et al., 2006;

S. Blanchet and P. Di Bari, 2006.

Individual asymmetries:

$$\varepsilon_{1\alpha} = -\frac{3M_1}{16\pi v^2} \frac{\text{Im}\left(\sum_{\beta\rho} m_\beta^{1/2} m_\rho^{3/2} U^*_{\alpha\beta} U_{\alpha\rho} R_{1\beta} R^*_{1\rho}\right)}{\sum_\beta m_\beta \left|R_{1\beta}\right|^2}$$

$Y_\alpha \sim \varepsilon_{1\alpha}\ \kappa\,(\widetilde{m_{1\alpha}}), \quad \widetilde{m_{1\alpha}} = |Y_{\nu 1\alpha}|^2 v^2/M_1$

The baryon asymmetry is

$Y_{\mathcal{B}} = -(12/37)\,((115/36) Y_2 + (37/9) Y_\tau),$

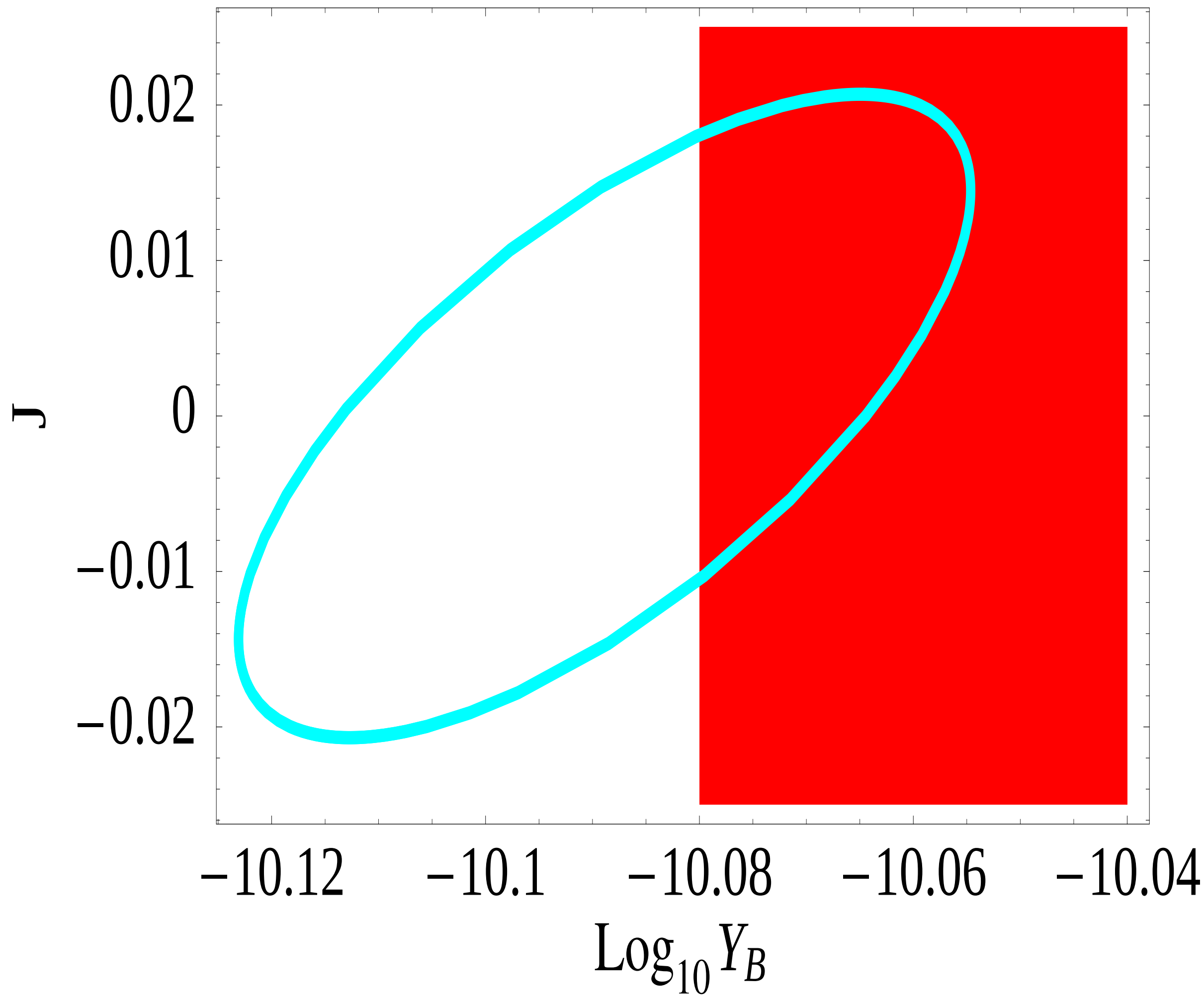
$Y_2 = Y_{e+\mu}, \quad \varepsilon_2 = \varepsilon_{1e} + \varepsilon_{1\mu}, \quad \widetilde{m_2} = \widetilde{m_{1e}} + \widetilde{m_{1\mu}}$

Real R: $\varepsilon_{1\alpha} \neq 0$, CPV from U

Example: NH spectrum, $M_1 \ll M_2 \ll M_3$,

$$\epsilon_\tau \simeq \frac{3M_1}{16\pi v^2} \frac{(\Delta m^2_\odot \Delta m^2_{31})^{1/4} R_{12} R_{13}}{\sqrt{\Delta m^2_\odot/\Delta m^2_{31} R^2_{12} + R^2_{13}}} \times \left(c_{12} c_{23} s_{23} \sin\frac{\alpha_{32}}{2} - s_{12} c^2_{23} s_{13} \sin\left(\delta - \frac{\alpha_{32}}{2}\right)\right)$$

S. Pascoli, S.T.P., A. Riotto, 2006.



$M_1 = 10^{11}$ GeV, $R_{12} = 0.95$, $R_{13} = 0.3$, $\alpha_{32} = \pi$, $s_{13} = 0.09$, $s_{23} = 1/\sqrt{2}$, $s_{12} = 1/\sqrt{3}$, $0 \leq \delta \leq 2\pi$; $J \equiv Im(U_{e1}U^*_{e2}U^*_{\mu1}U_{\mu2})$.

S. Pascoli, S.T.P., A. Riotto, 2006.

The Role of LFV Decays: $\mu \to e + \gamma$

Assume:

$M_1 \ll M_2 \ll M_3$, $M_3 \gtrsim 5 \times 10^{13}$ GeV (GUTs)
($m_\nu \cong m_D^2/M_R$; $m_D \sim 175$ GeV, $m_\nu \sim 5 \times 10^{-2}$ eV, then $M_R \sim 6 \times 10^{14}$ GeV.)

$M_{SUSY} \sim (100 - 600)$ GeV (LHC), e.g.,

$$m_0 = m_{1/2} = 250 \text{ GeV}, \quad A_0 = a_0 m_0 = -100 \text{ GeV},$$

$BR(\mu \to e + \gamma) < 1.2 \times 10^{-11}$ implies:

terms $\sim M_3$ in $|(\mathbf{Y}_\nu^\dagger L \mathbf{Y}_\nu)_{21}|$ – **suppressed**, i.e., $Y_{\nu 32} \cong 0$, or $Y_{\nu 31} \cong 0$.

One possible solution - the form of $\mathbf{R}$:

$R = R_{12}(\omega_{12}) R_{13}(\pi/2) R_{23}(\omega_{23}) = R(\omega_{12} - \omega_{23}) \equiv R(\omega)$, NH,

$R = R_{12}(\omega_{12}) R_{23}(0) R_{12}(\omega'_{12}) = R_{12}(\omega_{12} + \omega'_{12}) \equiv R_{12}(\omega)$, IH.

NH spectrum:

$$\mathbf{R} \simeq \begin{pmatrix} 0 & \sin\omega & \cos\omega \\ 0 & \cos\omega & -\sin\omega \\ -1 & 0 & 0 \end{pmatrix}, \qquad \omega = \rho + i\sigma \,. \tag{6}$$

S.T.P., W. Rodejohann, T. Shindou, Y. Takanishi, 2005

also: J. Ellis et al., 2004; A. Ibarra, G.G. Ross, 2004

The terms $\sim M_2$ in $|(\mathbf{Y}_\nu^\dagger L \mathbf{Y}_\nu)_{21}|$ – **dominant**.

Leptogenesis, NH spectrum: $M_1 \gtrsim 10^{10}$ GeV, $\omega-$complex.

$M_2 \gtrsim \times 10^{11}$ GeV: predicted $BR(\mu \to e + \gamma) \sim 10^{-12}$

$$\epsilon_1 \simeq -\frac{3}{8\pi}\left(\frac{m_3 M_1}{v_u^2}\right) \frac{\text{Im}\left[c_\omega^2 + \frac{\Delta m_\odot^2}{\Delta m_{31}^2}\; s_\omega^2\right]}{|c_\omega|^2 + \sqrt{\frac{\Delta m_\odot^2}{\Delta m_{31}^2}}\; |s_\omega|^2}$$

$$|\epsilon_1| \lesssim \frac{3}{8\pi}\left(\frac{m_3 M_1}{v_u^2}\right) \simeq 1.97 \times 10^{-7}\left(\frac{m_3}{0.05\text{ eV}}\right)\left(\frac{M_1}{10^9\text{ GeV}}\right)\left(\frac{174\text{ GeV}}{v_u}\right)^2 .$$

$$\widetilde{m}_1 \simeq m_3|c_\omega|^2 + m_2|s_\omega|^2 = \frac{1}{2}(m_3 + m_2)\cosh 2\sigma + \frac{1}{2}(m_3 - m_2)\cos 2\rho \geq m_2 \,.$$

9×10^{-3} eV $< \widetilde{m}_1 \lesssim 0.12 eV$: $1.9 \times 10^{-3} \lesssim \kappa < 3.9 \times 10^{-2}$

.

IH (QD) spectrum:

$$\mathbf{R} \simeq \begin{pmatrix} \cos\omega & \sin\omega & 0 \\ -\sin\omega & \cos\omega & 0 \\ 0 & 0 & 1 \end{pmatrix} . \quad (7)$$

The terms $\sim M_2$ in $|(\mathbf{Y}_\nu^\dagger L \mathbf{Y}_\nu)_{21}|$ – **dominant**.

Leptogenesis, IH spectrum: $M_1 \gtrsim 7 \times 10^{12}$ GeV, $\omega-$complex.

$M_2 \gtrsim 5 \times 10^{13}$ GeV: predicted $BR(\mu \rightarrow e + \gamma) \gg 10^{-11}$

$$\epsilon_1 \simeq -\frac{3}{8\pi}\left(\frac{m_2\ M_1}{v_u^2}\right)\frac{\Delta m_\odot^2}{|\Delta m_{31}^2|}\frac{\text{Im}\left[\sin^2\omega_{12}\right]}{\left(1+\frac{\Delta m_\odot^2}{2|\Delta m_{31}^2|}\right)|\sin\omega_{12}|^2+|\cos\omega_{12}|^2}$$

$$|\epsilon_1| \lesssim \frac{3}{16\pi}\left(\frac{m_2\,M_1}{v_u^2}\right)\frac{\Delta m_\odot^2}{|\Delta m_{31}^2|} \simeq 3.2\times 10^{-9}\left(\frac{m_2}{0.05\text{ eV}}\right)\left(\frac{M_1}{10^9\text{ GeV}}\right)\left(\frac{174\text{ GeV}}{v_u}\right)^2 ,$$

$$\widetilde{m}_1 \simeq m_{1,2}(|\cos\omega_{12}|^2 + |\sin\omega_{12}|^2) = m_{1,2}\cosh 2\sigma \geq m_{1,2} .$$

5×10^{-2} eV $< \widetilde{m}_1 \lesssim 0.1eV$: $2.4 \times 10^{-3} \lesssim \kappa < 5.4 \times 10^{-3}$

.

S.T.P., W. Rodejohann, T. Shindou, Y. Takanishi, 2005

Two possibilities:

- $M_{SUSY} \sim (600-2000)$ GeV,

($m_{1/2} \gg m_0$, e.g, $m_0 = 300$ GeV, $m_{1/2} = 1400$ GeV, $a_0 m_0 = 0$)

- $M_{SUSY} \sim (100-600)$ GeV, but $\mathbf{Y}_{\nu 21} = 0$, or $\mathbf{Y}_{\nu 22} = 0$

A. $\mathbf{Y}_{\nu 21} = 0$:

$$\tan\omega = e^{-i\alpha/2}\ \tan\theta_{12}.$$

B. $\mathbf{Y}_{\nu 22} \cong 0$, neglecting s_{13}:

$$\tan\omega = -e^{-i\alpha/2}\ \cot\theta_{12}.$$

Leptogenesis: $\omega-$complex; thus $\alpha \neq 0, \pi$, CP-violating values

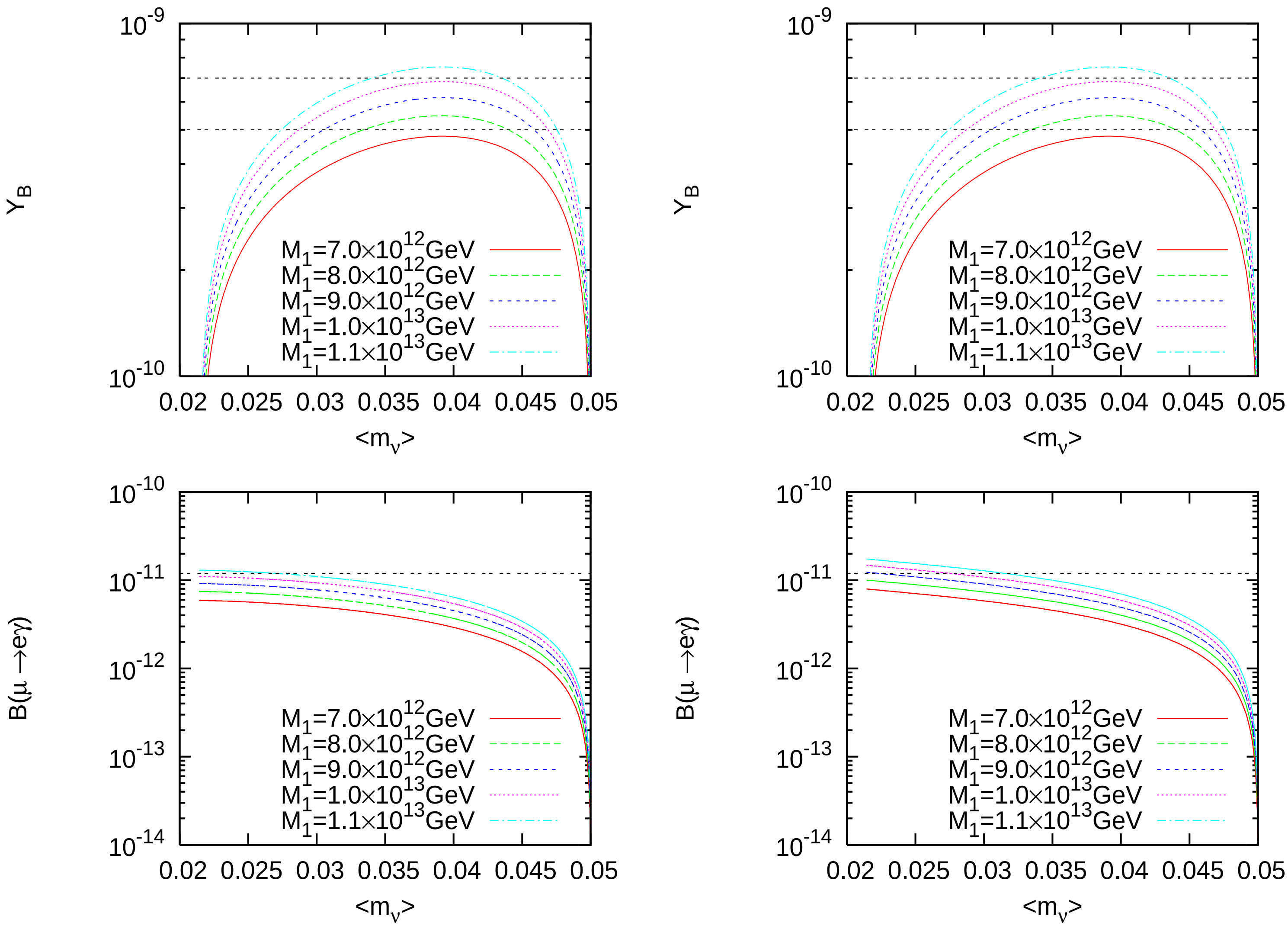
B. $\mathbf{Y}_{\nu 22} = 0$, including s_{13}:

$$\tan\omega = -\frac{c_{12} - s_{12}s_{13}e^{-i\delta}}{s_{12} + c_{12}s_{13}e^{-i\delta}}\ e^{-i\alpha/2}$$

IH spectrum:

$$|<m>|\ \cong \sqrt{\Delta m^2_{13}}\left|\cos^2\theta_{12} + e^{i\alpha}\ \sin^2\theta_{12}\right|$$

S.T. Petcov, T. Shindou, 2006



$(Y_\nu)_{21} = 0$ $(Y_\nu)_{22} \approx 0$

Figure 6: Predicted values of Y_B and B$(\mu \to e\gamma)$ for $s_{13} = 0$. The SUSY parameters are fixed as $m_0 = m_{1/2} = 450$, $A_0 = 0$, and $\tan\beta = 5$.

Conclusions

Determining the nature - Dirac or Majorana, of massive neutrinos is of fundamental importance for understanding the origin of neutrino masses.

The see-saw mechanism provides a link between ν-mass generation and BAU.

SUSY see-saw: LFV processes $\mu \to e + \gamma$, etc.

LHC: constraints on (discovery of?) SUSY.

$\mu \to e + \gamma$, leptogenesis - significant constraints on the theory.

Majorana CPV phases in $U_{\rm PMNS}$: $(\beta\beta)_{0\nu}$-decay, $Y_{\rm B}$, $\mu \to e + \gamma$.

CPV phases in $U_{\rm PMNS}$ can be the leptogenesis CPV parameters.

Low energy leptonic CPV can be directly related to the existence of BAU.

The searches for leptonic CP-violation are of fundamental importance.

Supporting Slide

Oscillation Parameters

$$\Delta m^2_\odot = 8.0 \times 10^{-5}\ \text{eV}^2\ , \quad 3\sigma(\Delta m^2_\odot) = 12\%\ ,$$

$$\sin^2\theta_\odot = 0.30\ , \quad 3\sigma(\sin^2\theta_\odot) = 24\%\ ,$$

$$|\Delta m^2_{\text{atm}}| = 2.5 \times 10^{-3}\ \text{eV}\ , \quad 3\sigma(|\Delta m^2_{\text{atm}}|) = 28\%.$$

Future:

3 kTy KamLAND: $3\sigma(\Delta m^2_\odot) = 7\%$, $3\sigma(\sin^2\theta_\odot) = 18\%$;

A. Bandyopadhyay et al., hep-ph/0410283

SK-Gd (0.1% Gd: 43×(KL $\bar{\nu}_e$ rate)), 3y: $3\sigma(\Delta m^2_\odot) \cong 4\%$

S. Choubey, S.T.P., hep-ph/0404103;
J. Beacom and M. Vagins, hep-ph/0309300

KL type reactor $\bar{\nu}_e$ detector, $L \sim 60$ km, ~ 60 GW kTy: $3\sigma(\sin^2\theta_\odot) \cong 12\%$

A. Bandyopadhyay et al., hep-ph/0410283 and hep-ph/0302243;
H. Minakata et al., hep-ph/0407326

T2K (SK): $3\sigma(|\Delta m^2_{\text{atm}}|) \cong 6\%$

sgn(Δm^2_{atm}): ν_{atm} experiments, studying the subdominant $\nu_{\mu(e)} \rightarrow \nu_{e(\mu)}$ and $\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$ oscillations; LBL $\nu-$oscillation experiments (T2K, NOνA); $\nu-$factory.

$\sin^2\theta_{13}$: reactor $\bar{\nu}_e$ experiments, $L \sim (1-2)$ km: Double CHOOZ, Daya-Bay, KASKA - factor (5 - 10).

Majorana Neutrinos

Can be defined in QFT using fields or states.

Fields: $\chi_k(x)$ - 4 component (spin 1/2), complex, m_k

Majorana condition:

$$C\ (\bar{\chi}_k(x))^{\mathsf{T}} = \xi_k \chi_k(x), \quad |\xi_k|^2 = 1$$

– Invariant under proper Lorentz transformations.
– Reduces by 2 the number of components in $\chi_k(x)$.

Implications:

$$U(1): \ \chi_k(x) \rightarrow e^{i\alpha}\chi_k(x) - \ \text{impossible}$$

– $\chi_k(x)$ cannot absorb phases.
– $Q_{U(1)} = 0: \ Q_{\text{el}} = 0, \ L_l = 0, \ L = 0, ...$
– $\chi_k(x)$: 2 spin states of a spin 1/2 absolutely neutral particle
– $\chi_k \equiv \bar{\chi}_k$

Propagators: $\Psi(x)$−Dirac, $\chi(x)$−Majorana

$$< 0|T(\Psi_\alpha(x)\bar{\Psi}_\beta(y))|0 >= S^F_{\alpha\beta}(x-y) \ ,$$

$$< 0|T(\Psi_\alpha(x)\Psi_\beta(y))|0 >= 0 \ , \quad < 0|T(\bar{\Psi}_\alpha(x)\bar{\Psi}_\beta(y))|0 >= 0 \ .$$

$$< 0|T(\chi_\alpha(x)\bar{\chi}_\beta(y))|0 >= S^F_{\alpha\beta}(x-y) \ ,$$

$$< 0|T(\chi_\alpha(x)\chi_\beta(y))|0 >= -\xi^* S^F_{\alpha\kappa}(x-y)C_{\kappa\beta} \ ,$$

$$< 0|T(\bar{\chi}_\alpha(x)\bar{\chi}_\beta(y))|0 >= \xi \ C^{-1}_{\alpha\kappa}S^F_{\kappa\beta}(x-y)$$

$$U_{CP} \ \chi(x) \ U^{-1}_{CP} = \eta_{CP} \ \gamma_0 \ \chi(x'), \quad \eta_{CP} = \pm i \ .$$

Currents of Majorana Fields