THEORY OF NEUTRINO MASSES

Ferruccio Feruglio

Neutrino Oscillation Workshop
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Plan

1. a lesson from the recent past
2. two extrapolations from lesson 1.
3. speculations

1. solution to sol and atm neutrino anomalies was the simplest

ν propagation studied with 4 independent sources
- sun
- cosmic rays
- reactors
- accelerators

spanning > 12 order of magnitudes in L/E

νs propagate as massive neutral fermions with specific mixing angles between mass and interaction eigenstates:

ν oscillations

+ possibly, a number of (still undetected) subleading effects

non-oscillation ``solutions''

ν decay	$P_{ff} = c + c' e^{-\frac{mL}{\tau E}} + \ldots$	wrong E dependence
ν decoherence	$P_{ff} = 1 - \frac{1}{2}\sin^2 2\vartheta (1 - e^{-\frac{\gamma L}{E}} \cos\frac{\Delta m^2 L}{2E})$	wrong E dependence
spin flavour precession (for solar ν)	$\mu_{ij} \approx 10^{-11}\mu_B \quad B \approx 80\mathrm{KGauss}$	rejected by KamLAND no such large B in Earth
Lorentz invariance violation	$P_{ff} = 1 - \sin^2 2\vartheta \sin^2(\delta c\, LE/2)$	wrong E dependence
non-standard ν interactions	$\delta L = \varepsilon\, G_F \psi\psi\nu\nu$ E-independent P_{ff}	sol: clash between solar and KamLAND data atm: wrong E dependence
mass varying neutrinos	$\delta m_\nu = \frac{\lambda\lambda'}{m^2} N_e$	sol: clash between solar and KamLAND data
ν oscillations with a non unitary mixing matrix U [1]	non-canonical ν kinetic terms in flavour basis from dim=6 operator	ν oscillations, W,Z decays universality tests, LFV $UU^+=1$ at the percent level

[1] Antusch, Biggio, Fernandez-Martinez, Gavela, Lopez-Pavon 0607020

all these effects can play, at most, a subleading role results can be encoded in a Lorentz-invariant Lagrangian

$$L = L_{SM} + \delta L(m_\nu) + \delta L$$

1st evidence of physics beyond the SM after more than 30 years!

additional operators giving negligibly small contributions to ν propagation in present experiments

L_{SM} invariant under

global, non-anomalous

$$\frac{B}{3} - L_e, \quad \frac{B}{3} - L_\mu, \quad \frac{B}{3} - L_\tau$$

broken individually by **δL(mν)**
possible exception: (B - L)

either dim(δL)≥6 such as e.g.

$$\frac{c_1}{\Lambda^2}\psi\psi l l + \frac{c_2}{\Lambda^2}(lH)^+ \gamma^\mu \partial_\mu (lH) + \ldots$$

or new particles in δL such as

$$\lambda \varphi \nu \nu + \ldots$$

new (pseudo)scalar

low-energy parameters in $\delta L(m_\nu)$

ν masses

[3 light active ν]

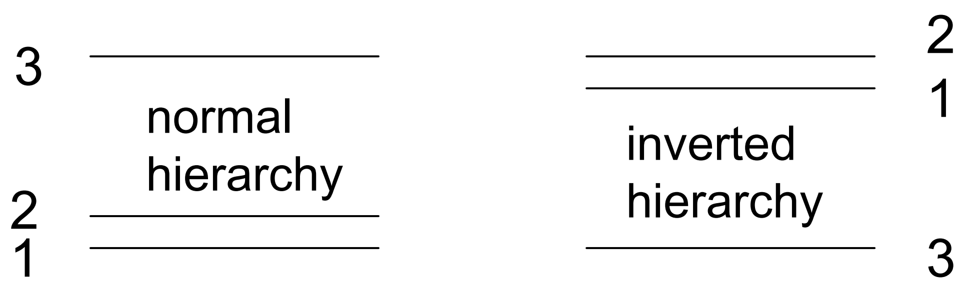
$$m_1, m_2, m_3$$

order $m_1 < m_2$

$$\Delta m^2_{21} < \left|\Delta m^2_{32}\right|, \left|\Delta m^2_{31}\right| \qquad [\Delta m^2_{ij} \equiv m_i^2 - m_j^2]$$

i.e. 1 and 2 are, by definition, the closest levels

two, still open, possibilities:



Mixing matrix (analogous to V_{CKM})

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{-i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{-i\delta} & c_{13}s_{23} \\ -c_{12}s_{13}c_{23} + s_{12}s_{23}e^{-i\delta} & -s_{12}s_{13}c_{23} - c_{12}s_{23}e^{-i\delta} & c_{13}c_{23} \end{pmatrix} \times \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i\beta} \end{pmatrix}}$$

$$c_{12} \equiv \cos\vartheta_{12}, \ldots$$

- only if ν are Majorana
- drops in oscillations

from data

[2σ errors (95% C.L.)]

$$\Delta m^2_{sol} \equiv \Delta m^2_{21} = 7.92\,(1 \pm 0.09) \times 10^{-5}\ \text{eV}^2$$

$$\Delta m^2_{atm} \equiv \left|\Delta m^2_{32}\right| = 2.6\,(1^{+0.14}_{-0.15}) \times 10^{-3}\ \text{eV}^2$$

[Marrone, ICHEP 2006 Moscow,
Fogli, Lisi, Marrone, Palazzo, 0506083]

$$\sin^2 \vartheta_{12} = 0.314\,(1^{+0.18}_{-0.15})$$

$$\sin^2 \vartheta_{23} = 0.45\,(1^{+0.35}_{-0.20})$$

$$\sin^2 \vartheta_{13} = 0.8^{+2.3}_{-0.8} \times 10^{-2}$$

two lepton mixing angles are large

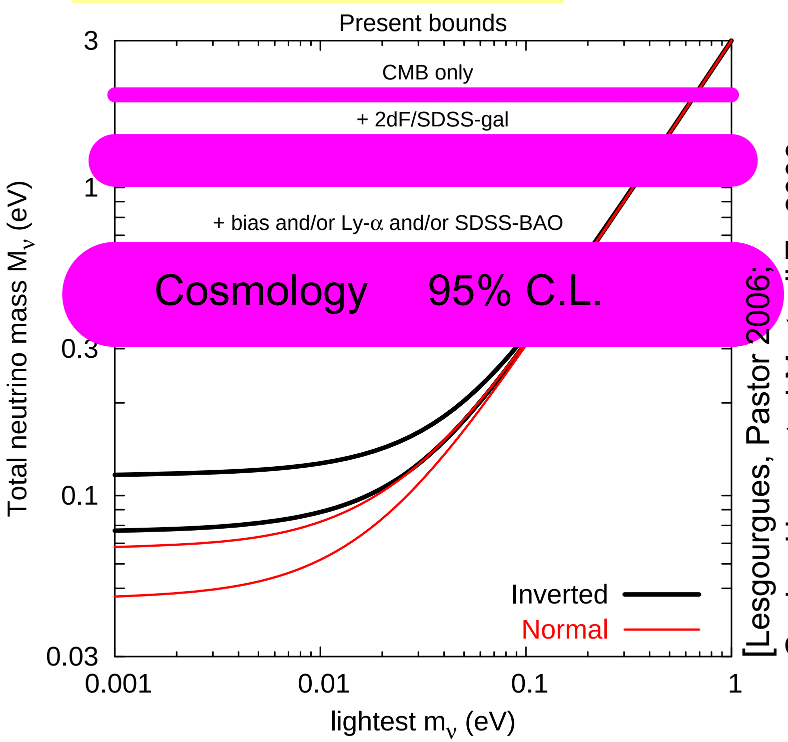
$$\text{sign}\,[\Delta m^2_{32}] \quad \text{unknown}$$

$$m_i < 2\ \text{eV} \quad 95\%\ \text{C.L.}$$ [Tritium β-decay]

$$V_{us} \approx \lambda \quad V_{cb} \approx \lambda^2 \quad V_{ub} \approx \lambda^3$$

$$\lambda \approx 0.22$$

$$\delta, \alpha, \beta \quad \text{unknown}$$



[Lesgourgues, Pastor 2006; Goobar, Hannestad, Mortsell, Tu 2006 Seljak, Slosar, Mc Donald 2006; Fukugita, Ichikawa, Kawasaki, Elgaroy, Lahav 2006; Cirelli, Strumia 2006]

some parameters measured already quite precisely

$$\vartheta_{12} = (34.1\ ^{+1.7}_{-1.6}\ )^0 \quad [1\sigma]$$

$$\vartheta_{12} + \underbrace{\vartheta_C}_{\text{Cabibbo angle}} = (47.0\ ^{+1.7}_{-1.6}\ )^0$$

quark-lepton complementarity?

[Raidal 0404046
Minakata, Smirnov 0405088]

next CMB satellite + weak grav. lensing + improved galaxy survey

$$\sum m_i < (0.02 \div 0.08)\ \text{eV}\ (1\sigma)$$

[2015?]

part 2.

two extrapolations from lesson 1

1st extrapolation: only ν_e ν_μ and ν_τ take part in ν oscillations

- (almost) all experiments explained by 3 ν_a

  hint for a 3rd independent Δm^2 from an accelerator ν beam (LSND)

$$P(\bar{\nu}_\mu \to \bar{\nu}_e) = (2.6 \pm 0.8)\, 10^{-3} \qquad \Delta m^2 \geq 0.1\,\text{eV}^2$$

only 3 active neutrinos

$$N = 2.984 \pm 0.009$$

$(m_t, m_H) = (174.3, 115)$ GeV

(invisible Z width)

no room for LSND with 3 ν_a

CPT violation, i.e. different Δm^2 in ν and anti-ν sectors, disfavoured by now

[Pakvasa&Valle 0301061
Gonzalez-Garcia, Maltoni, Schwetz 0306226]

if confirmed $3\,\nu_a + [\text{at least}]\, 1\,\nu_s$ → sterile

- inclusion of ν_s **worsens** the global fits

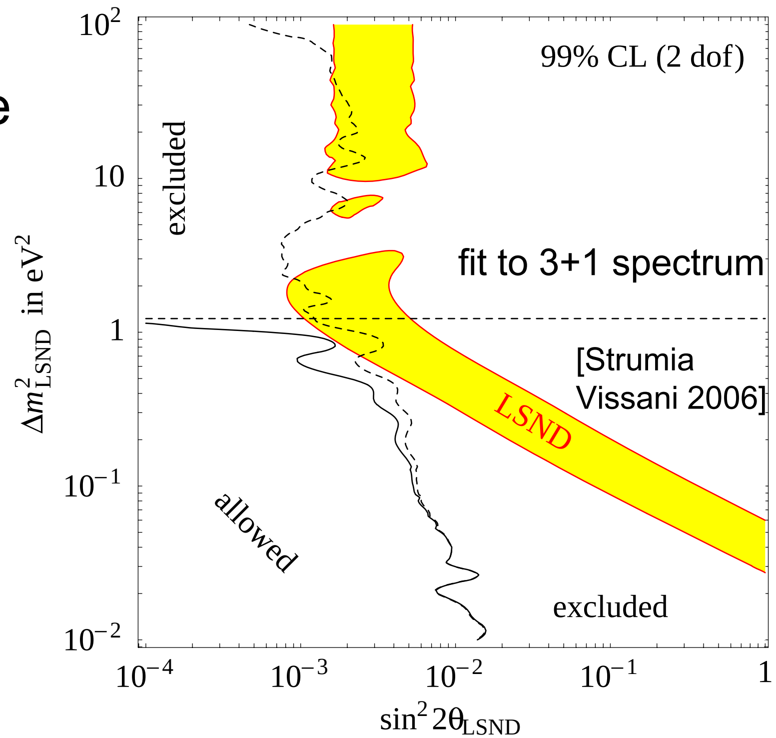
- WMAP + LSS

$$m_s < (0.4 \div 0.7)\,\text{eV} \qquad (99.9\%\,\text{CL}) \qquad \text{for } 3\nu_a + 1\nu_s$$

[Dodelson, Melchiorri, Slosar 0511500
Seljak,Slosar,McDonald 0604335; Cirelli, Strumia 0607335]

- LSND soon checked by MiniBooNE

  data analysis under way



2^{nd} extrapolation: B - L is violated

a theorist viewpoint:
- all other global symmetries of the SM are violated; B-L is violated in many GUTs
- B-L violation is welcome in baryogenesis
- global quantum numbers are expected to be violated by quantum gravity effects at $\Lambda \cong M_{Planck}$
- **simplest explanation of $m_\nu << m_f$ (f=e,u,d) is in term of B-L violation**

Weinberg's list $$L = L_{SM} + \frac{c_5}{\Lambda} L_5 + \frac{c_6}{\Lambda^2} L_6 + \ldots$$

[80 independent dim=6 operators]

Λ= scale of new physics

a unique operator of dim=5 (up to flavour combinations)

$$\frac{\mathcal{L}_5}{\Lambda} = \frac{(Hl)(Hl)}{\Lambda} = \frac{1}{2}\frac{v^2}{\Lambda}\nu\nu + \ldots$$

$$\Delta(B-L) = 2$$

$$m_\nu = y\frac{v^2}{\Lambda} \longleftrightarrow m_f = \frac{y_f}{\sqrt{2}} v$$

❖ smallness of m_ν due to $\frac{v}{\Lambda} << 1$

❖ $m_3 \approx \sqrt{|\Delta m^2_{32}|} \approx 0.05\,\mathrm{eV} \rightarrow \quad \Lambda \approx 10^{15}\ \mathrm{GeV}$ not that far from GUT scale

❖ L_5 is the leading operator in Weinberg expansion: 1^{st} effect of New Physics

❖ L_5 perfectly matches $\delta L(m_\nu)$

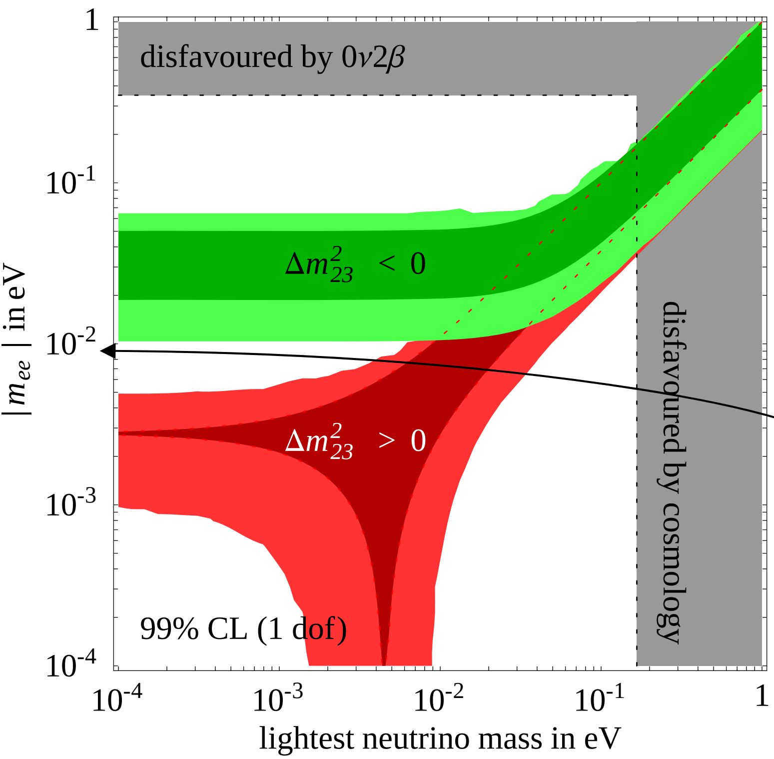
experimental constraints

- oscillations are insensitive to L violation
- L violation can be tested in $0\nu\beta\beta$ decay

$$\text{HM } (^{76}Ge) \quad T_{1/2} > 1.9\times10^{25}\ yr \quad |m_{ee}| < 0.35\ \text{eV}$$
$$\text{IGEX } (^{76}Ge) \quad T_{1/2} > 1.6\times10^{25}\ yr \quad |m_{ee}| < (0.33\div1.35)\ \text{eV}$$
$$\text{Cuoricino } (^{130}Te) \quad T_{1/2} > 1.8\times10^{24}\ yr \quad |m_{ee}| < (0.2\div1.1)\ \text{eV}$$

[90%CL]

uncertainty from nuclear matrix elements



- expected range of $|m_{ee}|$ can be predicted from $(\Delta m^2_{ij}, \vartheta_{ij})$

$$|m_{ee}| = \left|\cos^2\vartheta_{13}(\cos^2\vartheta_{12}\, m_1 + \sin^2\vartheta_{12} e^{2i\alpha}\, m_2) + \sin^2\vartheta_{13} e^{2i\beta}\, m_3\right|$$

Future expected sensitivity on $|m_{ee}|$ **:** 10 meV

CUORE	^{130}Te	(30-50) meV
Majorana	^{76}Ge	(20-70) meV
GERDA	^{76} Ge	(90-290) meV (phase II)
		10 meV (phase III ?)

[F, Strumia, Vissani 2003
Petcov&Pascoli 0310003 Bilenky 0403245; Bahcall, Murayama, Pena-Garay 0403167]

How to explain $\frac{y_{\nu_e}}{y_e} < 10^{-6}$ if B-L is conserved ?

[smallest ratio is 1/100 for charged fermions in same gen.]

Interesting attempts in models with extra dimensions

[Dienes, Dudas, Gherghetta, Arkani-Hamed, Dimopoulos, Dvali, March-Russell, Barbieri, Creminelli, Strumia]

large ED: standard Yukawa couplings to a singlet fermion ν_s who lives in the bulk

$$\mathcal{L}_{Yuk} = \frac{y_\nu \mathrm{v}}{\sqrt{2}} \left(\frac{M_D}{M_P} \right) \nu_a(x) \nu_s^{(0)}(x)$$

$$\nu_s(x, y) = \frac{\nu_s^{(0)}(x)}{\sqrt{V_\delta}} + \ldots$$

no experimental hints from oscillations

$\nu_s^{(n)}$ effects subdominant, if present

dimension 5, L-violating operators not sufficiently suppressed by $M_D \approx 1\,\mathrm{TeV}$

alternative models: warped compactifications, L gauged in the bulk,…

not fully realistic in their minimal realization [Grossman&Neubert'99 Gherghetta 0312392]

part 3.

speculations

PREMISE

theory of neutrino masses

it does not exist! Neither for neutrinos nor for charged fermions. We lack a unifying principle.

like weak interactions before the electroweak theory

$SU(2)_L \otimes U(1)_Y$
gauge invariance → all fermion-gauge boson interactions in terms of 2 parameters: g and g'

? → Yukawa interactions between fermions and spin 0 particles: many free parameters (up to 22 in the SM!)

only few ideas and prejudices about neutrino masses and mixing angles

caveat: several prejudices turned out to be wrong in the past!
- $m_\nu \cong 10$ eV because is the cosmologically relevant range
- solution to solar is MSW SA
- atm problem will go away because it implies a large angle

Model building in two pages

hierarchies in fermion spectrum

quarks

$$\frac{m_u}{m_t} << \frac{m_c}{m_t} << 1 \qquad \frac{m_d}{m_b} << \frac{m_s}{m_b} << 1 \qquad |V_{ub}| << |V_{cb}| << |V_{us}| \equiv \lambda < 1$$

leptons

$$\frac{m_e}{m_\tau} << \frac{m_\mu}{m_\tau} << 1$$

$$\frac{\Delta m^2_{sol}}{\Delta m^2_{atm}} = (0.025 \div 0.049) \approx \lambda^2 << 1 \quad (2\sigma)$$

$$|U_{e3}| < 0.18 \leq \lambda \quad (2\sigma)$$

large parameters can be seen as small deviations from special numbers

$$\left(\frac{\pi}{4} - \vartheta_{23}\right) = 0.06^{+0.10}_{-0.12} \quad \text{rad} \quad (2\sigma)$$

$$\vartheta_{12} + \vartheta_C - \frac{\pi}{4} = 0.035^{+0.060}_{-0.056} \quad \text{rad} \quad (2\sigma)$$

call $\xi_i \equiv$ small parameters

in modern model building we have two ways of understanding $|\xi_i| << 1$

1 ξ_i **are small breaking terms of an approximate flavour symmetry**

[Froggatt&Nielsen 1978]

when $\xi_i \to 0$ the theory becomes invariant under a flavour symmetry F

very appealing approach,
unfortunately freedom is huge

example $m_e \to 0$ in QED

$U(1)_A \qquad e \to e^{i\alpha} e \qquad e^c \to e^{i\alpha} e^c$

- symmetries global or local continuous or discrete
- breaking terms from SSB, ad-hoc explicit breaking,…

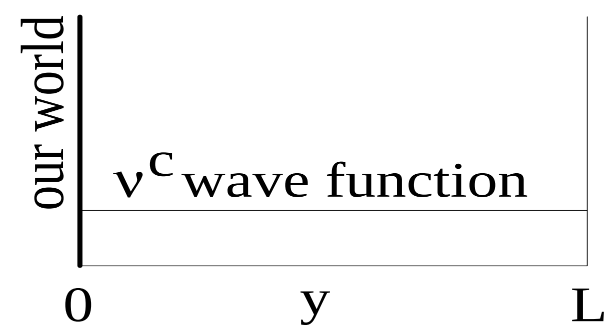
2 ξ_i **are small due to geometry**

$\Lambda_F \equiv$ scale of flavour physics [unknown at present]

$E \approx \Lambda_F$ a four-dimensional description of particle interactions might break down

example: 1 extra dimension 0≤y≤L

$$\frac{y_\nu}{y_f} \approx \frac{1}{\sqrt{L\Lambda}} << 1 \quad \text{if} \quad L >> \frac{1}{\Lambda}$$



many other possibilities in field and string theories

embedding in GUTs is fruitful

- ν as a window on GUT physics $\quad m_3 \approx (v^2/\Lambda) \approx 0.05\,\mathrm{eV} \rightarrow \quad \Lambda \approx 10^{15}\,\mathrm{GeV}$

- ν^c present in many GUTs as SO(10), E_6,…

- L_5 can be obtained from the **see-saw mechanism**

$$L = -\frac{1}{2}\nu^c M \nu^c - \nu^c m_D \nu + h.c.$$

$$m_\nu = m_D^T M^{-1} m_D$$

- link to baryogenesis through **leptogenesis** [Fukugita, Yanagida ’86]
  CP violating, out-of-equilibrium decay of lightest ν^c
  net $\Delta L \neq 0$ (converted into $\Delta B \neq 0$ by sphalerons)
  $m_i < (0.12 \div 0.15)\,\mathrm{eV}$ in simplest models [Buchmuller, Di Bari, Plumacher 0401240 Hambye, Lin, Notari, Papucci,Strumia 0312203]

- a small mixing in m_D can be enhanced into a large mixing in m_ν by the see-saw [Smirnov 1993; Altarelli,F, Masina 2000]

  a small $\Delta m^2_{sol} / \Delta m^2_{atm}$ can be produced by the see-saw [King 1998]

  in SU(5) even without see-saw a large θ_{23} mixing can be produced from large mixing between l_2 and l_3 in $\bar{5}_2 \equiv (l_2, d_2^c)$ and $\bar{5}_3 \equiv (l_3, d_3^c)$ [Babu&Barr 1996]

  [large mixing between d_2^c and d_3^c : unobservable in 1st approximation]

- link to LFV in specific models

many possibilities…

1+2: no compelling model from data at the moment

Here: any general feature of direct experimental interest, independent on the details of model building?

now: solar & KamLAND data quite precise (Δm^2_{sol}, θ_{12})
Δm^2_{atm} soon improved by LBL

close future (<10 yr from now): precision/sensitivity on θ_{23} and θ_{13} down to

$$\lambda^2 \approx 0.04 \div 0.05 \text{ rad} \, (2.1^0 \div 2.9^0)$$

significant level of precision for model building

most of existing models predict

$$\vartheta_{13} > \lambda^2 \qquad \left|\frac{\pi}{4} - \vartheta_{23}\right| > \lambda^2$$

model	θ_{23}	θ_{13}	comments
\`NATURAL' TEXTURES [1] providing 2 relations	O(1)	>0.03 (90% C.L.)	for all cases but case \`\`D'': θ_{13}<0.02
3 ZERO TEXTURES [2] for m_ν + large θ_{23} from U_e	O(1)	>0.025	
ANARCHY [3]	O(1)	O(1)	structure-less neutrino mass matrix
FLAVOUR DEMOCRACY [4]	35.3^0 (off by 2σ)	$(0.03 \div 0.1)$	
INVERTED HIERARCHY U_ν bimaximal, θ_{12} corrected by U_e	O(1)	>0.1	θ_{13} much smaller if U_e does not contribute to θ_{12}
NORMAL HIERARCHY see-saw dominance of light ν_R lopsided m_ν and m_e	O(1)	$\underbrace{(0.03 \div 0.2)}_{\text{from } U_\nu} \oplus \underbrace{(0.02 \div 0.1)}_{\text{from } U_e}$	
SU(5)xU(1) [5] [abelian flavour symmetries]	O(1)	O(0.1)	U(1) SB parameter optimized to fit the data; unknown O(1) coefficients generated at random

[1] Barbieri, Hambye, Romanino 0302118
[2] Watanabe, Yoshioka 0601152
[3] Hall, Murayama, Weiner 9911341
[4] Fritzsch, Xing PLB 372 (1996)
[5] Altarelli, F, Masina 0210342

θ_{23} maximal only by a fine-tuning

an example: inverted hierarchy

enforced by a flavour symmetry acting as $L_e - L_\mu - L_\tau$ on lepton doublets

at the leading order: $|m_1| = |m_2| = \sqrt{|a|^2 + |b|^2}$ $\quad m_3 = 0$

$\vartheta_{13} = 0$

$\tan\vartheta_{23} = -\frac{b}{a}$

large θ_{23} expected, maximal only by a fine-tuning

$\vartheta_{12} = 45^o$ $\quad \vartheta_{12}^{exp} = 34.1^{+1.7\,o}_{-1.6}$ it's off by 6σ

SB terms are too small to correct θ_{12}

$$1 - \tan^2\vartheta_{12} \approx O\left(\frac{\Delta m^2_{sol}}{\Delta m^2_{atm}}\right)$$

$$0.36 \div 0.70\,(3\sigma) >> 0.015 \div 0.07\,(3\sigma)$$

-either we accept a tuning 1/10

[Altarelli&Franceschini 0512202]

- or we fix θ_{12} by a contribution from the charged lepton sector

if $\vartheta_{23} \approx 45^o$, to 1st order in $|u| \equiv \sin\vartheta_{12}^e >> |\sin\vartheta_{13}^e| \approx 0$

$$1 - \tan^2\vartheta_{12} = 2\sqrt{2}\,\mathrm{Re}(u)$$

$$|U_{e3}| = \frac{1}{\sqrt{2}}|u|$$

$$\delta_{CP} = \arg(u)$$

[Frampton, Petcov, Rodejohann 0401206
Altarelli, F, Masina 0402155
Romanino 0402508]

if $u \approx \vartheta_C \approx 0.22$

$$1 - \tan^2\vartheta_{12} \approx 2\sqrt{2}\,\vartheta_C \approx 0.6$$

[right amount]

$\theta_{13} > 0.1$ expected

a special class of models

$\left|\frac{\pi}{4}-\vartheta_{23}\right|<\lambda^2$ and/or $\vartheta_{13}<\lambda^2$ would signal some special mechanism at work

example: tri-bimaximal mixing

$$\sin^2\vartheta_{13}=0.9^{+2.3}_{-0.9}\times 10^{-2}$$

$$\vartheta_{23}=(41.6^{+10.4}_{-5.7})^0 \quad [2\sigma]$$

$$\vartheta_{12}=(34.1\ ^{+1.7}_{-1.6}\ )^0 \quad [1\sigma]$$

$$\sin^2\vartheta_{13}=0$$

$$\sin^2\vartheta_{23}=\frac{1}{2}$$

$$\sin^2\vartheta_{12}=\frac{1}{3} \leftrightarrow \quad \vartheta_{12}=35.3^0$$

not a bad 1st order approximation!

[Harrison, Perkins and Scott≡HPS]

mixing matrix

$$U_{PMNS}=\begin{pmatrix}\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0\\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}}\\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}}\end{pmatrix}$$

assume future data will confirm $\theta_{23}=45^0$ to $O(\lambda^2)$ precision, etc…

$\vartheta_{23}=45^0$ in some symmetry limit? [as $m_e=0$ when $U(1)_A$ exact in QED]

No, if the symmetry is realistic [≡no huge breaking terms]

θ_{23} always undetermined in the symmetry limit
$\theta_{23}=45^0$ entirely determined by breaking effects

easiest possibility: flavour symmetry F spontaneously broken along different subgroups in e and ν sectors

$$\langle\varphi_\nu\rangle\neq\langle\varphi_e\rangle$$

vacuum alignment problem

minimal example (not unique, many produced in the last year!)

[other discrete groups: Hagedorn, Lindner, Plentinger, Mohapatra 2006]

flavour symmetry $A_4 \times \ldots$

group of even permutations of four objects

controls charged lepton mass hierarchies

[Ma, Rajasekaran 2001; Babu, Ma, Valle 2003 Ma 0409075; Altarelli & F 0504165 & 0512103 He, Keum, Volkas 0601001and many more…]

	l	e^c	μ^c	τ^c	φ_e	φ_ν	ξ_ν
A_4	3	1	1”	1’	3	3	1

minimization of V can produce the special alignment

$$\langle \varphi_e \rangle \propto (1,0,0) \qquad \langle \varphi_\nu \rangle \propto (1,1,1) \qquad \langle \xi_\nu \rangle \neq 0$$

- m_e is diagonal

can be made small, below λ^2

$$m_\nu = \begin{pmatrix} a+\frac{2}{3}b & -\frac{b}{3} & -\frac{b}{3} \\ -\frac{b}{3} & \frac{2}{3}b & a-\frac{b}{3} \\ -\frac{b}{3} & a-\frac{b}{3} & \frac{2}{3}b \end{pmatrix} \frac{v_u^2}{\Lambda} + O\left(\frac{VEV}{\Lambda}\right)$$

$$U_{PMNS} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} + O\left(\frac{VEV}{\Lambda}\right)$$

ν spectrum is of normal type, between hierarchical and degenerate

$m_1 \approx m_2 \approx |a| \quad m_3 \approx 3|a| \quad$ [units $\frac{v_u^2}{\Lambda}$]

$b \approx -2a$ to reproduce $\frac{\Delta m^2_{sol}}{\Delta m^2_{atm}} \approx \frac{1}{35}$

predictions:

$$|m_3|^2 = |m_{ee}|^2 + (10/9)\Delta m^2_{atm}\left(1 - \Delta m^2_{sol} / \Delta m^2_{atm}\right)$$

$$m_1 > 0.017 \text{ eV} \qquad \sum_i m_i > 0.09 \text{ eV}$$

if naively extended to the quark sector V_{CKM} too close to 1, unrealistic

SUMMARY

- Experimental side: $(\Delta m^2_{21}, \vartheta_{12})$ entered a precision era
  $(|\Delta m^2_{32}|, \vartheta_{23})$ reasonably well-known
  ϑ_{13}, absolute spectrum, δ,... still missing!

- several key points still unknown: - how many light neutrinos?
  - is L violated or not?

- theory of neutrino (and fermion) masses lacks a unifying principle …

- light neutrino masses are naturally explained by L violation at a large scale, possibly close to GUT scale

  several ``common'' mechanisms that accommodate small quark mixing angles and large lepton mixing angles in GUTs are available

- aimed for sensitivities might provide a significant progress in theory

  $$\vartheta_{13} \approx \delta\vartheta_{23} \approx \lambda^2 \approx 0.04 \div 0.05$$

most of existing models predict

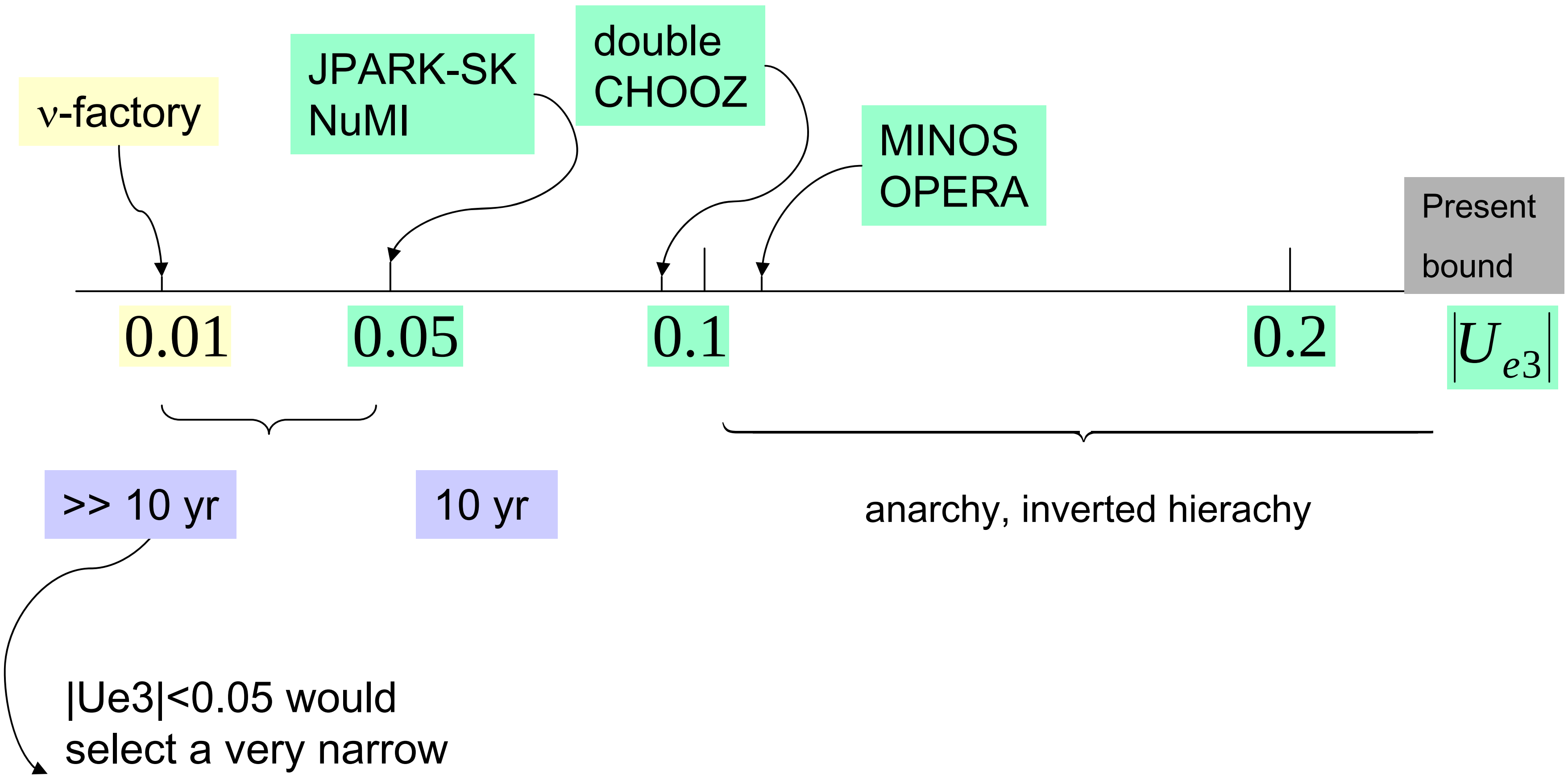
$$\vartheta_{13} > \lambda^2 \qquad \left|\frac{\pi}{4} - \vartheta_{23}\right| > \lambda^2$$

$$\vartheta_{13} < \lambda^2 \qquad \left|\frac{\pi}{4} - \vartheta_{23}\right| < \lambda^2$$

only in ``special models''
e.g. SB flavour symmetry
with a natural vacuum alignment

OTHER SLIDES

- Most of plausible range for Ue3 explored in 10 yr from now



|Ue3|<0.05 would select a very narrow (not empty) subset of existing models

similar conclusion by:

Barbieri, Hambye, Romanino 0302118
Ibarra, Ross 0307051
Chen, Mahanthappa 0305088
Lebed, Martin 0312219
Joshipura @ NOON 2004

too many models. Here: try to classify models by their predictions

Present and (near) future sensitivities

	current precision	future < 10 yr
Δm^2_{12}	$(8.0 \pm 0.3)\times 10^{-5}\ \mathrm{eV}^2\ \ [\approx 4\%]$	few percent [KamLAND]
$\lvert\Delta m^2_{23}\rvert$	$(2.5 \pm 0.3)\times 10^{-3}\ \mathrm{eV}^2\ \ [\approx 12\%]$	$0.15\times 10^{-3}\ \mathrm{eV}^2$ LBL conventional beams $0.05\times 10^{-3}\ \mathrm{eV}^2\ [\approx 2\%]$ superbeams
ϑ_{12}	$\tan^2\vartheta_{12} = 0.45^{+0.09}_{-0.08}$ $\vartheta_{12} = 33^0 \pm 2^0$	$\delta\tan^2\vartheta_{12} \approx 2\delta\sin^2\vartheta_{12}$ ν_e scattering rate of pp neutrinos to 1% down by about a factor 2: challenging
ϑ_{13}	$< 0.23\,(13^0)$ 90% C.L.	0.10 rad LBL, ChoozII 0.05 rad superbeams
ϑ_{23}	$\sin^2\vartheta_{23} = 0.52^{+0.07}_{-0.08}$ $\vartheta_{12} = 46^{0}{}^{+4^0}_{-5^0}$	$\delta\sin^2\vartheta_{23} \approx \delta\vartheta_{23}$ down by about a factor 2 superbeams
sign Δm^2_{23}	---	> 10 yr
δ	---	> 10 yr

normal models: some examples

-- degenerate spectrum

❖ anarchy [Hall, Murayama, Weiner 2000 De Gouvea, Murayama 0301050]

$$m_\nu = m \begin{pmatrix} O(1) & O(1) & O(1) \\ O(1) & O(1) & O(1) \\ O(1) & O(1) & O(1) \end{pmatrix}$$

$$\frac{\Delta m^2_{sol}}{\Delta m^2_{atm}} << 1$$

can be produced in part by the see-saw

$$\vartheta_{13} << \vartheta_{23}$$

accidental

$$\vartheta_{23} \approx \frac{\pi}{4}$$

fortuitous

❖ flavour democracy [Fritzsch, Xing]

$$\underset{f \neq \nu}{m_f} \propto \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \ldots$$

$$m_\nu \propto \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \ldots$$

$$\sin^2 2\vartheta_{23} = \frac{8}{9} \approx 0.89 \;\; \text{(out by about } 2\,\sigma \text{ now)}$$

$$\vartheta_{13} \approx \sqrt{2m_e / 3m_\mu} \approx (0.03 \div 0.1)$$

[estimates by allowing 3σ exp. and (factors ½ and 2) th. uncertainties]

- substantial contribution to ϑ_{12} from charged leptons needed

$$U_{PMNS} = U_e^+ U_\nu$$

standard parametrization $U_e = U_{23}^e \cdot U_{13}^e \cdot U_{12}^e$

by expanding to 1st order in $|u| \equiv \sin\vartheta_{12}^e,\ |v| \equiv \sin\vartheta_{13}^e << 1$ $\quad \vartheta_{23} \approx 45^o$

$$1-\tan^2\vartheta_{12} = 2\sqrt{2}\,\mathrm{Re}(u+v)$$

$$|U_{e3}| = \frac{1}{\sqrt{2}}|u-v|$$

[Frampton, Petcov, Rodejohann 0401206
Altarelli, F, Masina 0402155
Romanino 0402508]

$$\tan^2\vartheta_{23} = 1 + O(u^2, v^2, uv)$$

$$\delta_{CP} = \arg(u-v)$$

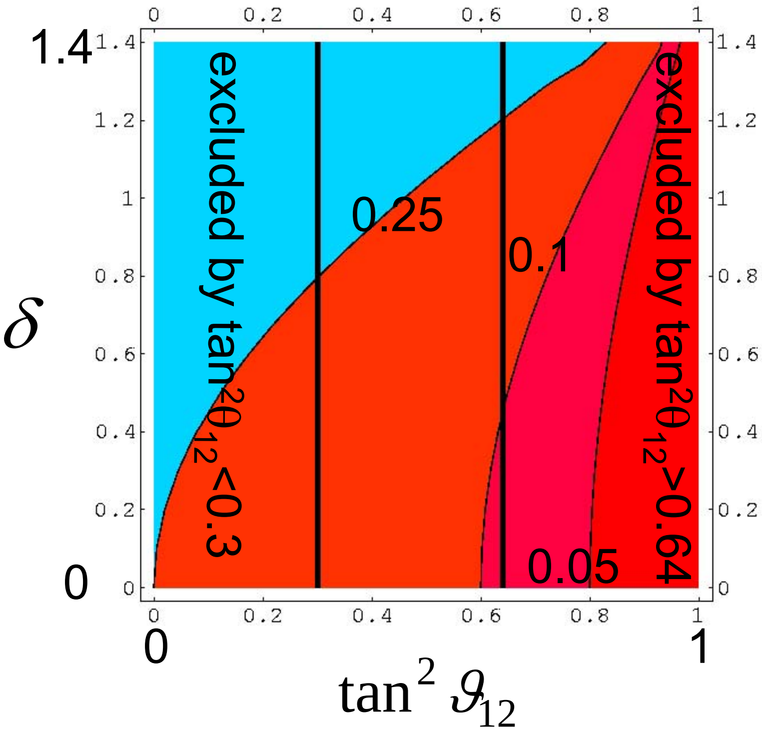
if, by analogy with the quark sector:

$$|v| << |u| \approx \vartheta_C \approx 0.22$$

- $1-\tan^2\vartheta_{12} \approx 2\sqrt{2}\,\vartheta_C \approx 0.6$

[right amount] [Raidal 0404046
Minakata, Smirnov 0405088]

- $|U_{e3}| = \dfrac{(1-\tan^2\vartheta_{12})}{4\cos\delta_{CP}}$



$\theta_{13} > 0.1$ expected

Normal Hierarchy

- Several viable mechanisms for ϑ_{23} large
  - ϑ_{23}^{e} and ϑ_{23}^{ν} small but $\vartheta_{23} \equiv \vartheta_{23}^{\nu} - \vartheta_{23}^{e} \approx O(1)$
  - see-saw dominance of light ν^{c} equally coupled to ν_{μ} and ν_{τ} [King]
  - lopsided structure of m_e or/and m_D^{ν} : $\bar{R}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & a & b \end{pmatrix} L$ [Albright, Barr Altarelli, F]

large θ_{23} expected, maximal only by a fine-tuning

$$U_{e3} \approx \sin\vartheta_{13}^{\nu} - \sin\vartheta_{23} \cdot \sin\vartheta_{12}^{e}$$

[1st order in $\sin\vartheta_{12}^{e} >> \sin\vartheta_{13}^{e}$]

ν-dominated

$$\approx \sin\vartheta_{12}^{\nu}\sqrt{\frac{\Delta m_{sol}^{2}}{\Delta m_{atm}^{2}}}$$

$$\approx (0.03 \div 0.3)$$

e-dominated

$$\approx -\sin\vartheta_{23}\sqrt{\frac{m_e}{m_\mu}}$$

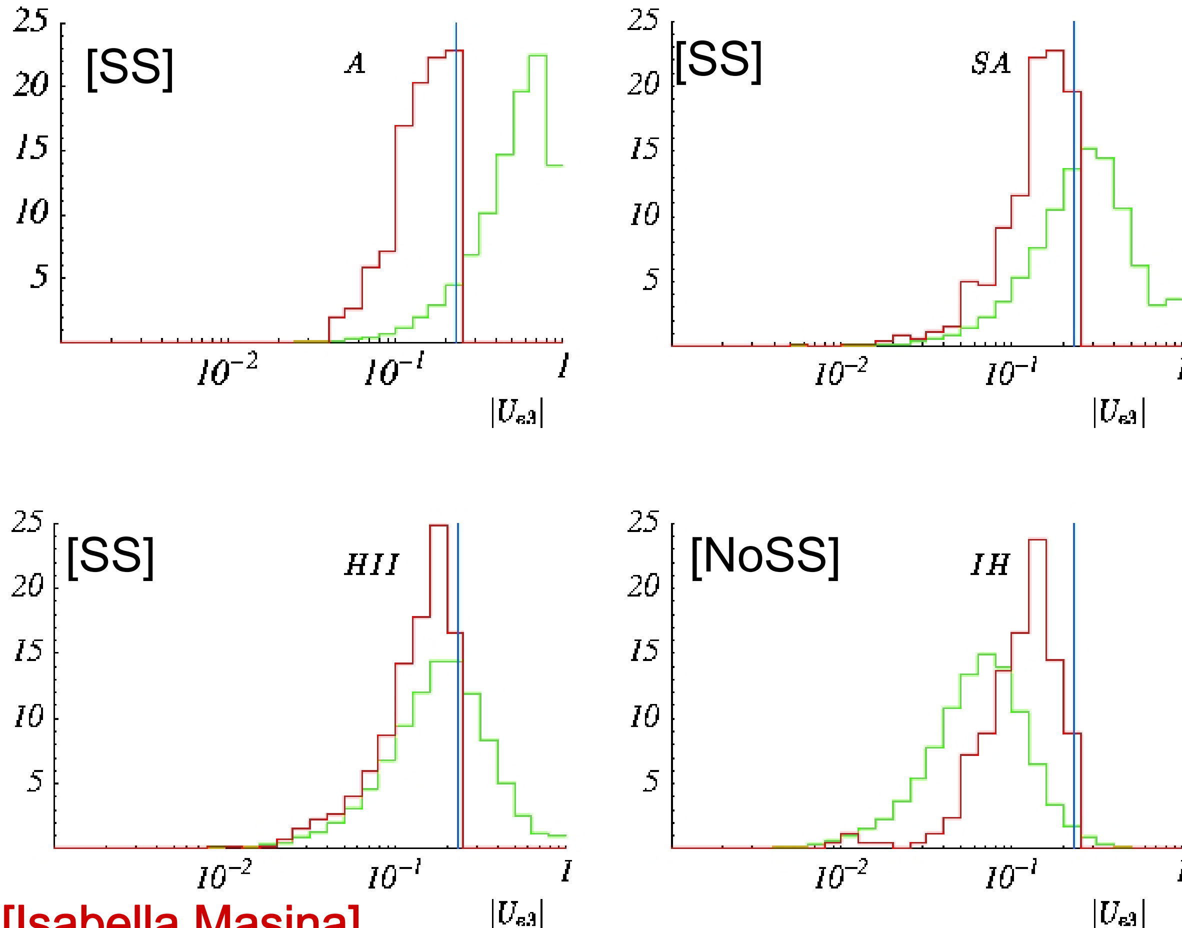
$$\approx (0.02 \div 0.1)$$

if we make a similar estimate in the quark sector

$V_{ub} \approx \lambda^{3} \ (\lambda \approx 0.22)$

θ_{13} not tiny, barring cancellations

U_{e3} in models with U(1) flavour symmetry



[Isabella Masina]

$$\begin{cases} 0.018 < r \equiv \left|\dfrac{\Delta m^2_{12}}{\Delta m^2_{23}}\right| < 0.053 \\ |U_{e3}| < 0.23 \\ 0.30 < \tan^2 \vartheta_{12} < 0.64 \\ 0.45 < \tan^2 \vartheta_{23} < 2.57 \end{cases}$$

ε optimised case by case to fit

$$m_\nu = m \begin{pmatrix} \varepsilon^2 & \varepsilon & \varepsilon \\ \varepsilon & 1 & 1 \\ \varepsilon & 1 & 1 \end{pmatrix}$$

$\varepsilon = 1$ anarchy=A

$\varepsilon < 1$ semianarchy=SA

$\varepsilon < 1$ normal hierarchy=H

$\det 23 \approx \varepsilon$

inverse hierarchy=IH

$\varepsilon < 1$

$$m_\nu = m \begin{pmatrix} \varepsilon^2 & 1 & 1 \\ 1 & \varepsilon^2 & \varepsilon^2 \\ 1 & \varepsilon^2 & \varepsilon^2 \end{pmatrix}$$

matrix elements up to unknown O(1) coeff.

explanations of LSND signal

model	comments	MiniBooNE
$3\nu_a + 1\nu_S$ unstable $\vartheta_{\mu S} \neq 0 \quad \nu_S \to \overline{\nu_e} + \varphi$ [1]	reactor bounds evaded by U_{eS}=0	expected signal
$3\nu_a + 1\nu_S$ and mass-varying parameters air $\neq$ earth [2]	pure 3 mass varying neutrinos do not work	no signal
$3\nu_a + 1\nu_S$ and CPT violation [3]		?
anomalous μ decay $\mu^+ \to e^+ \overline{\nu_e \nu_\mu}$ [4]	Karmen: BR<0.009 90% C.L. ρ=0.7485 versus ρ=0.7508(10)	no signal

[1] Palomares Ruiz, Pascoli, Schwetz 0505216
[2] Barger, Marfatia, Whisnant 0509163
[3] Barger, Marfatia, Whisnant 0308299
[4] Babu, Pakvasa 0204236