Resolving the Octant θ_{23} Degeneracy by Neutrino Oscillation Experiments

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Based on the works

K. Hiraide, H. Minakata, T. Nakaya, H. Sugiyama, W.J.C. Teves, R. Zukanovich Funchal, HN, hep-ph/0601258 [PRD73, 093008 (2006)]

T. Kajita, H. Minakata, S. Nakayama, HN, to appear

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Outline

- Introduction
- Resolving of the θ_{23} Octant Degeneracy by (1) accelerator (superbeam) and reactor
- Resolving of the θ_{23} Octant Degeneracy by (2) accelerator (superbeam) with 2 identical detectors
- Summary

Introduction

θ_{23} octant degeneracy is a part of the so called 8 fold parameter degeneracy we encounter when we try to determine neutrino mixing paramters by oscillation experiments

(1) θ_{23} octant degeneracy (Fogli and Lisi, 1996)

(2) Intrinsic θ_{13}-δ degeneracy (Burguet-Castell et al, 2001)

(3) sign of Δm^2_{13} degeneracy (Minakata, HN, 2001)

→ 8 fold degeneracy (Barger et al, 2002)

Currently, we don't know if θ_{23} is maximal

From SK atmospheric neutrino data

$\sin^2 2\theta_{23} > 0.92$ @ 90% CL

$0.34 < \sin^2\theta_{23} < 0.64$

From Theory Consideration: μ-τ symmetry

Typically predict
$\theta_{23} = \pi/4$ and $\theta_{13} = 0$
in the symmetry limit

What is the θ_{23} Octant degeneracy?

Fogli and Lisi, PRD 54, 3667 (1996)

Let us first consider the disapperance mode $\nu_\mu \to \nu_\mu$

One Δm^2 dominance approximation and vanishing θ_{13}

$$P(\nu_\mu \to \nu_\mu) \approx 1 - \sin^2 2\theta_{23} \sin^2\left(\frac{\Delta m^2}{4E} L\right)$$

Disappearance mode can determine $\sin^2 2\theta_{23}$

$$\sin^2\theta_{23} = \frac{1}{2}\left[1 \pm \sqrt{1 - \sin^2 2\theta_{23}}\right]$$

For example, $\sin^2 2\theta_{23} = 0.96 \to \sin^2\theta_{23} = 0.4$ or 0.6

To be more precise...

$$P(\nu_\mu \rightarrow \nu_\mu) \equiv 1 - \sin^2 2\theta_{eff} \sin^2\left(\frac{\Delta m^2_{eff}}{4E} L\right)$$

$$\sin^2 2\theta_{eff} \approx 4|U_{\mu 3}|^2 (1-|U_{\mu 3}|^2)$$

T2K I can measure this quantity with ≈ 1 % accuracy

For a given value of $\sin^2 2\theta_{eff}$

$$\sin^2\theta_{23} \approx \frac{1}{2}\left[1 \pm (1+\sin^2\theta_{13}) \sqrt{1 - \sin^2 2\theta_{eff}} \right]$$

$$\sin^2 2\theta_{eff} = 0.96 \rightarrow \sin^2\theta_{23} \approx (0.4 \text{ or } 0.6) + \underline{0.1 \sin^2\theta_{13}}$$

What will happen if we add $\nu_\mu \to \nu_e$?

One Δm^2 dominance approximation

$$P(\nu_\mu \to \nu_e) \approx \sin^2\theta_{23} \sin^2 2\theta_{13} \sin^2\left(\frac{\Delta m^2}{4E} L\right)$$

Appearance mode can determine $\sin^2\theta_{23}\sin^2 2\theta_{13}$

(Not θ_{23} and θ_{13} separately !)

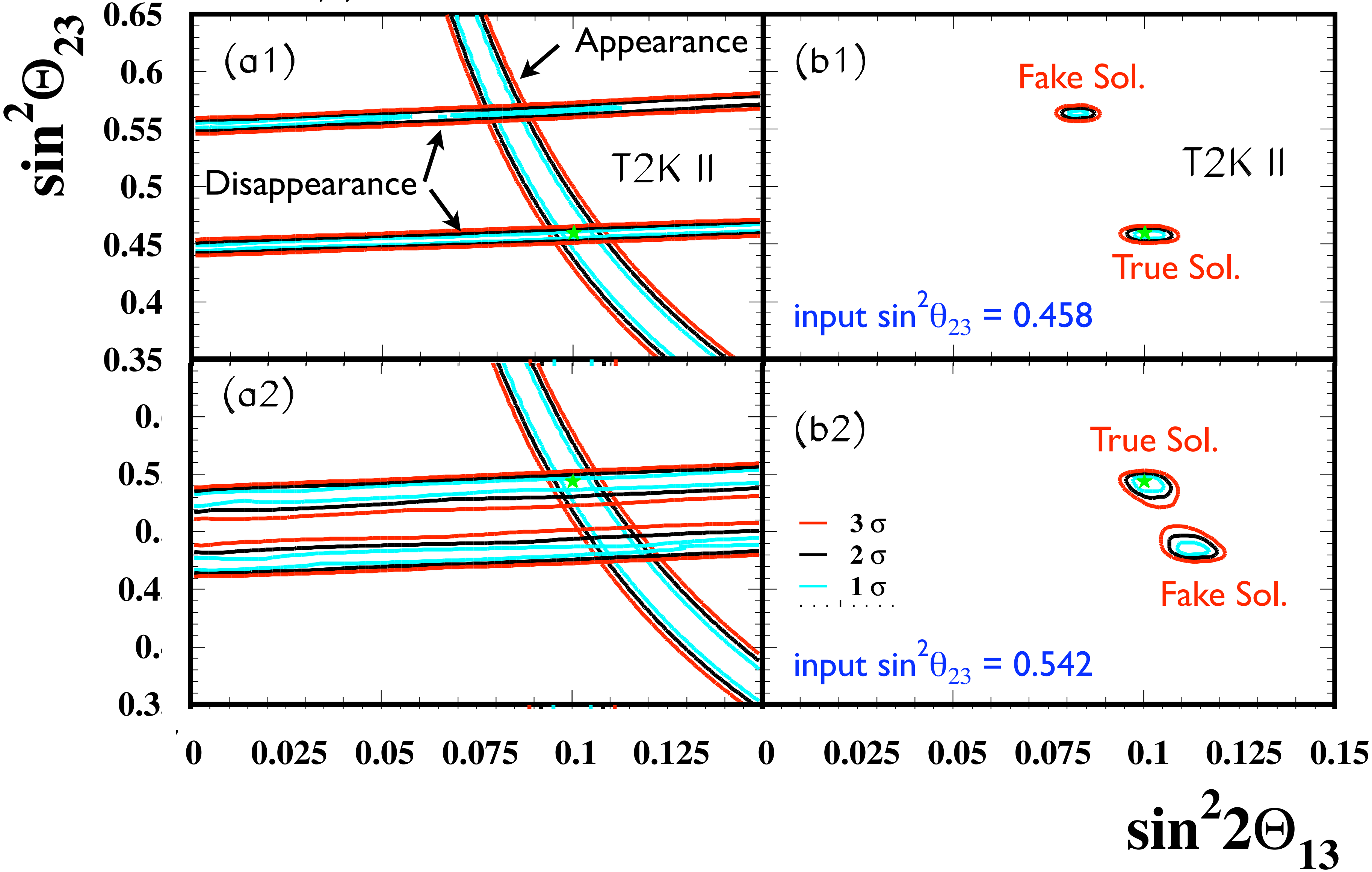
Unless we know θ_{13} determined by some OTHER experiment, we can not distinguish 2 value of $\sin^2\theta_{23}$ obtained by the disappearance mode!

Two solutions of $(\theta_{23}, \theta_{13})$

$$\sin^2\theta_{23}^{fake}\sin^2 2\theta_{13}^{fake} \approx \sin^2\theta_{23}^{true}\sin^2 2\theta_{13}^{true}$$

Some Examples of Degenerate Solutions

In appearance mode: $\Delta m^2_{31} = 2.5 \times 10^{-3} eV^2$ is fixed



Assume T2K Phase II: 4MW (2yr ν + 6 yr $\bar{\nu}$), HK@Kamioka

How Can We Resolve This Degeneracy?

First Possible Strategy

Combine Reactor Data

Minakata et al, PRD 68, 033017 (2003)

See also Fogli and Lisi, PRD 54, 3667 (1996)

$$P(\nu_e \to \nu_e) \approx 1 - \sin^2 2\theta_{13} \sin^2\left(\frac{\Delta m^2_{13}}{4E} L\right) + O(\sin^2 2\theta_{13} \Delta m^2_{12})$$

Reactor with $L \approx$ 1-1.5 km can provide a clean measurement of θ_{13} free from degeneracy

Since 2 degenerate solutions correspond to different values of θ_{13}, it will be possible to eliminate one of the solutions

Experimental Setup and Assumptions

(1) T2K Phase II: 4MW 2.5 deg. OA beam (2yr ν + 6 yr $\bar{\nu}$)
HK 0.54 Mt, appearance and disappearance modes

(2) High Statistics/Sensitivity Reactor Experiment
with an exposure of ~10 GW·kt·yr , L = 1.5 km
beyond Double CHOOZ

Inputs

$\Delta m^2_{23} = \pm 2.5\times10^{-3}$ eV^2

$\Delta m^2_{12} = 8\times10^{-5}$ eV^2

$\sin^2\theta_{12} = 0.31$

$\delta = 0$

χ^2 definitions

(1) $\nu_\mu \to \nu_e$ appearance mode

$$\chi^2_{\rm app} \equiv \frac{(N^{\rm obs}_{\rm sig} + N^{\rm obs}_{\rm BG} - N^{\rm theo}_{\rm sig} - N^{\rm theo}_{\rm BG})^2}{N^{\rm obs}_{\rm sig} + N^{\rm obs}_{\rm BG} + (\sigma_{\rm sig} N^{\rm obs}_{\rm sig})^2 + (\sigma_{\rm BG} N^{\rm obs}_{\rm BG})^2},$$

(2) $\nu_\mu \to \nu_\mu$ disappearance mode

$$\chi^2_{\rm dis} \equiv \min_{\alpha_{\rm sig}, \alpha_{\rm BG}} \sum_i \frac{[N^{\rm obs}_i + N^{\rm obs}_{i,\rm BG} - (1+\alpha_{\rm sig}) N^{\rm theo}_i - (1+\alpha_{\rm BG}) N^{\rm theo}_{i,\rm BG}]^2}{N^{\rm obs}_i + N^{\rm obs}_{i,\rm BG}} + \left(\frac{\alpha_{\rm sig}}{\sigma_{\rm sig}}\right)^2 + \left(\frac{\alpha_{\rm BG}}{\sigma_{\rm BG}}\right)^2,$$

$\sigma_{sig}=\sigma_{BG}= 2\%$: sytematic erros

(3) $\bar{\nu}_e \to \bar{\nu}_e$ reactor disappearance mode

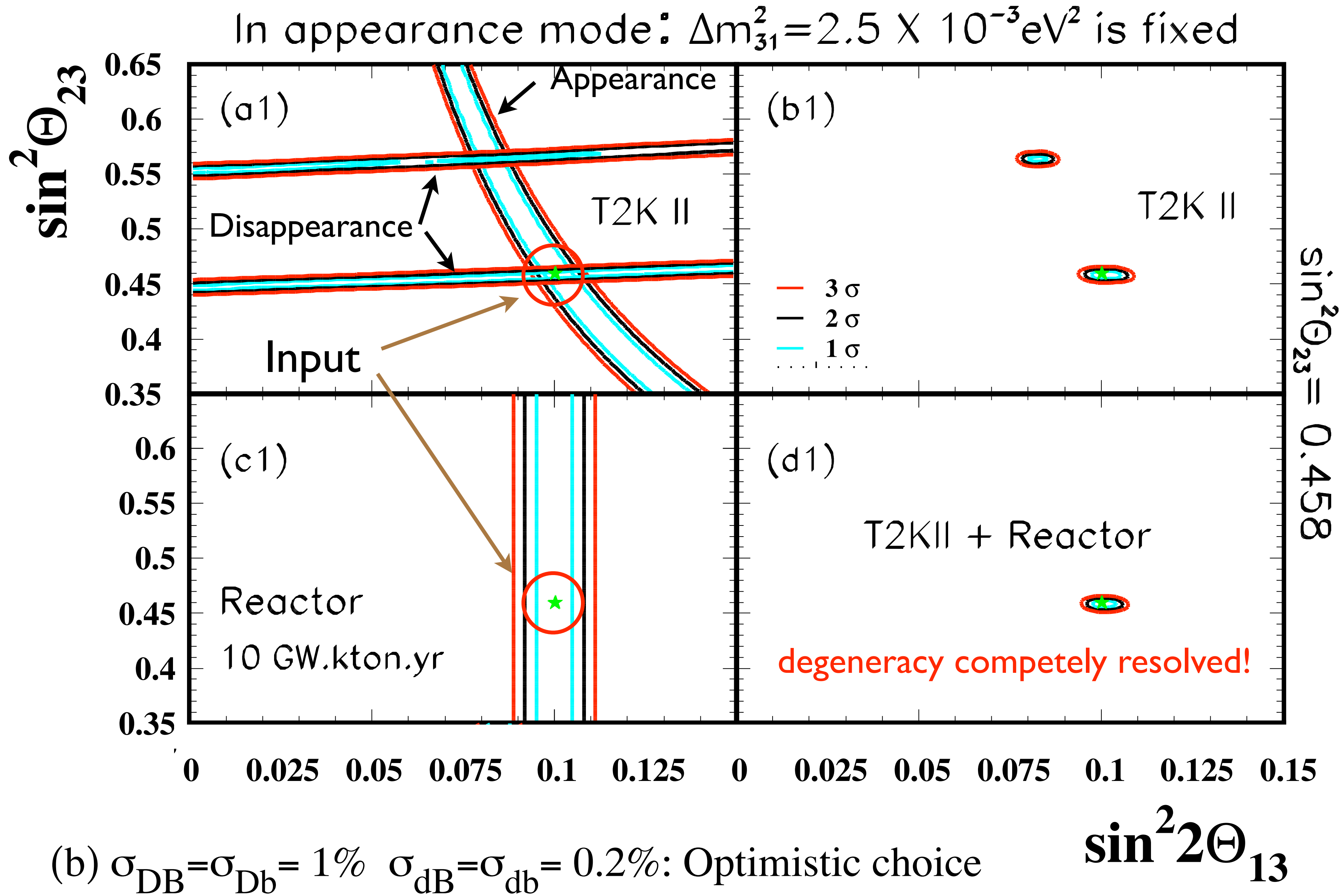
$$\chi^2_{\rm reac} \equiv \min_{\alpha\text{'s}} \sum_{a=f,n} \left[\sum_{i=1}^{17} \left\{ \frac{\left(N^{\rm theo}_{ai} - (1+\alpha_i+\alpha_a+\alpha) N^{\rm obs}_{ai}\right)^2}{N^{\rm obs}_{ai} + \sigma^2_{\rm db} (N^{\rm obs}_{ai})^2} + \frac{\alpha_i^2}{\sigma^2_{\rm Db}} \right\} + \frac{\alpha_a^2}{\sigma^2_{\rm dB}} \right] + \frac{\alpha^2}{\sigma^2_{\rm DB}},$$

(a) $\sigma_{DB}=\sigma_{Db}= 2\%$ $\sigma_{dB}=\sigma_{db}= 0.5\%$: Conservative choice of sys. error

(b) $\sigma_{DB}=\sigma_{Db}= 1\%$ $\sigma_{dB}=\sigma_{db}= 0.2\%$: Optimistic choice of sys. error

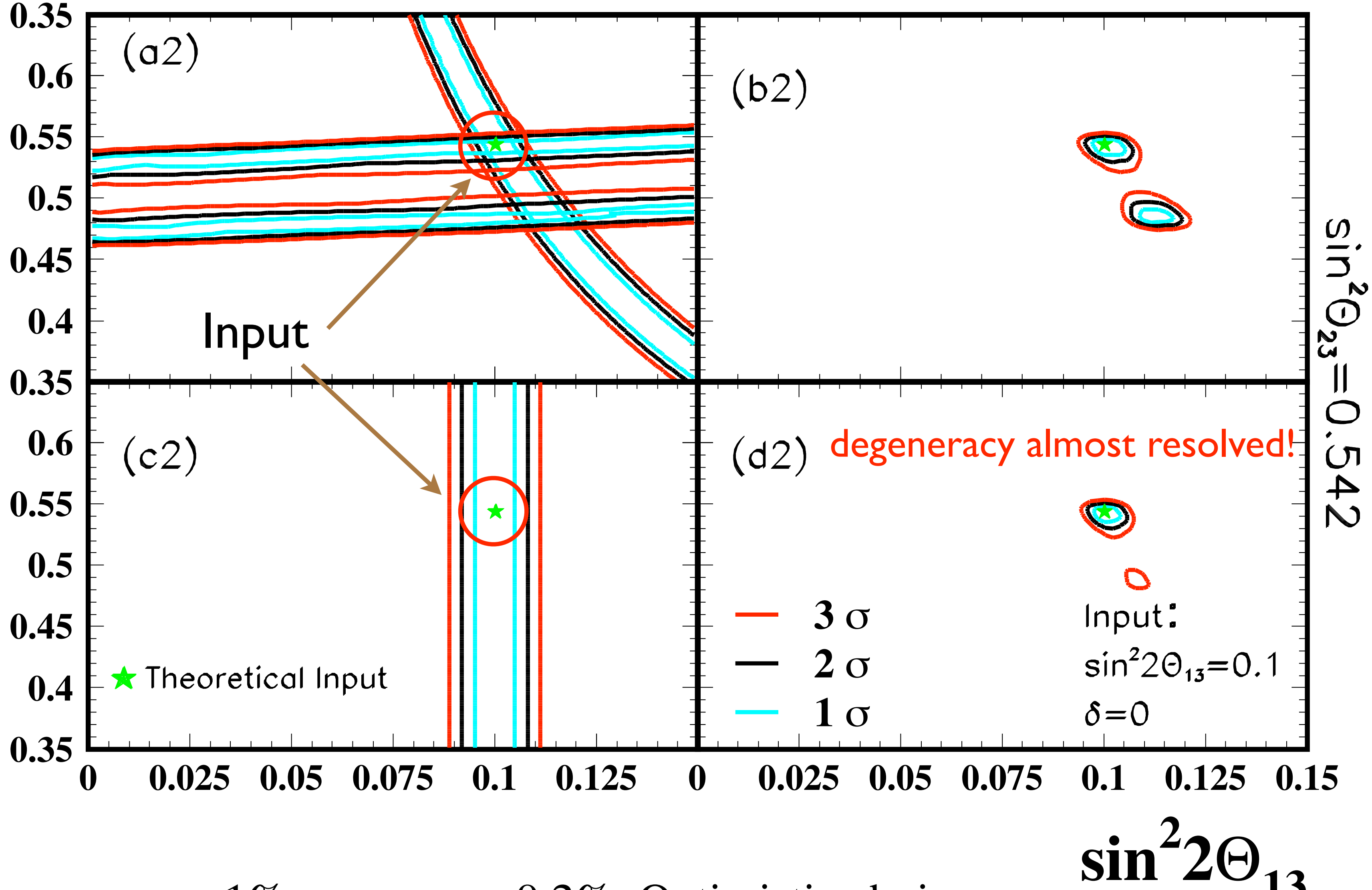
D(B): Correlated between Detectors(Bins)   d(b): Uncorrelated between Detectors(Bins)

Impact of adding Reactor data



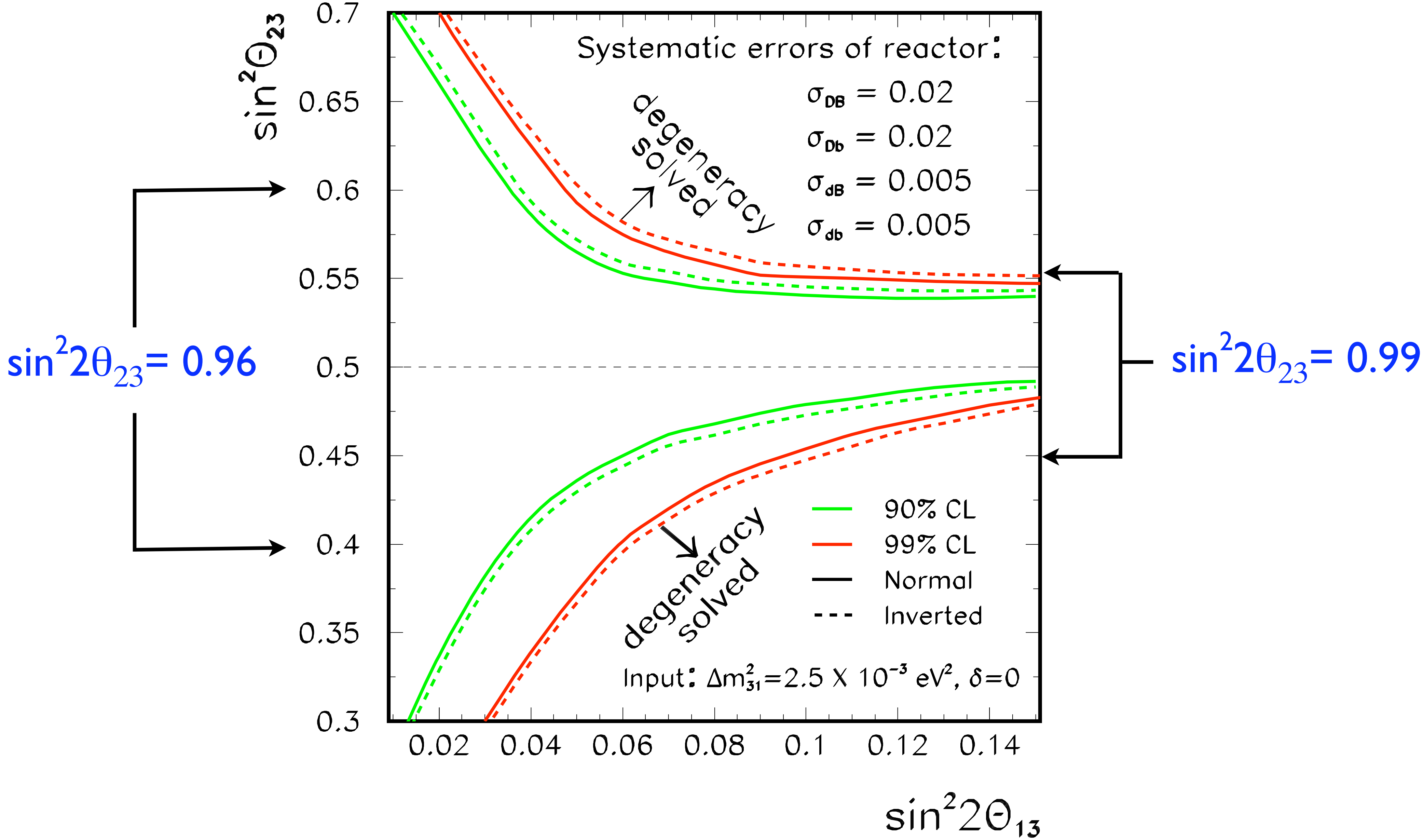
(b) $\sigma_{DB}=\sigma_{Db}= 1\%$ $\sigma_{dB}=\sigma_{db}= 0.2\%$: Optimistic choice

Impact of adding Reactor data

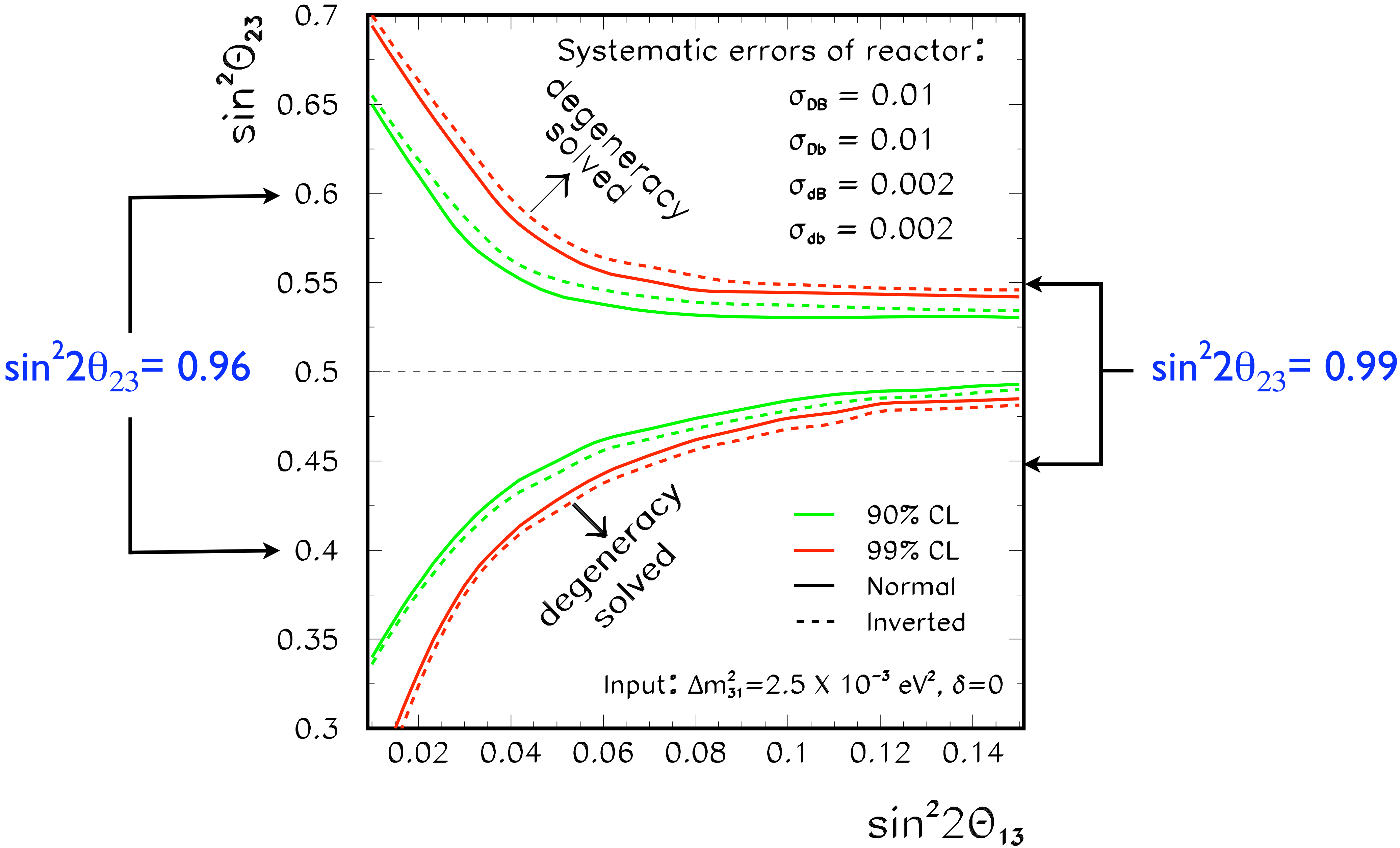


(b) $\sigma_{DB}=\sigma_{Db}= 1\%$ $\sigma_{dB}=\sigma_{db}= 0.2\%$: Optimistic choice

Expected Sensitivity: Regions of paramters where the hierarchy can be determined



Expected Sensitivity: Regions of paramters where the hierarchy can be determined



Second Possible Strategy

T. Kajita, H. Minakata, S. Nakayama and HN, to appear

Superbeam with 2 detector system

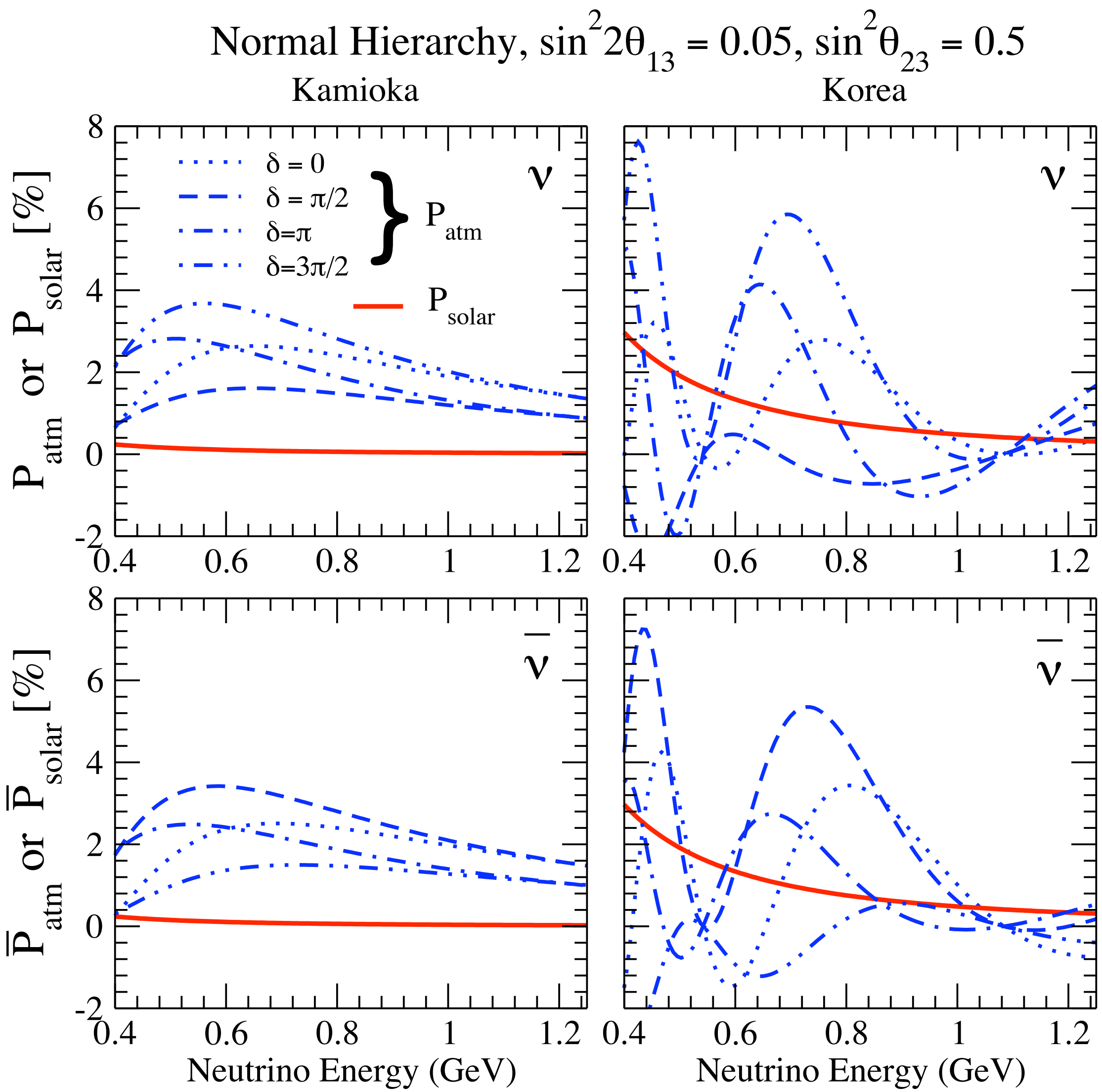
M. Ishitsuka, T. Kajita, H. Minakata, HN, hep-ph/0504026

$$
\begin{aligned}
P[\nu_\mu(\bar{\nu}_\mu) \rightarrow \nu_\mathrm{e}(\bar{\nu}_e)] =\ & c_{23}^2 \sin^2 2\theta_{12} \left(\frac{\Delta m_{21}^2 L}{4E}\right)^2 \\
+\ & \sin^2 2\theta_{13} s_{23}^2 \left[\sin^2\left(\frac{\Delta m_{31}^2 L}{4E}\right) - \frac{1}{2} s_{12}^2 \left(\frac{\Delta m_{21}^2 L}{2E}\right) \sin\left(\frac{\Delta m_{31}^2 L}{2E}\right) \right. \\
& \left. \pm \left(\frac{4Ea}{\Delta m_{31}^2}\right) \sin^2\left(\frac{\Delta m_{31}^2 L}{4E}\right) \mp \frac{aL}{2} \sin\left(\frac{\Delta m_{31}^2 L}{2E}\right)\right] \\
+\ & 2J_r \left(\frac{\Delta m_{21}^2 L}{2E}\right) \left[\cos\delta \sin\left(\frac{\Delta m_{31}^2 L}{2E}\right) \mp 2 \sin\delta \sin^2\left(\frac{\Delta m_{31}^2 L}{4E}\right)\right],
\end{aligned}
$$

← Solar Term = P_{solar}

$\equiv P_{solar} + P_{atm}$

Magnitude of the Solar term: $P_{solar} \equiv \cos^2\theta_{23}\sin^2 2\theta_{12}\sin^2\left(\frac{\Delta m^2_{12}}{4E}L\right)$



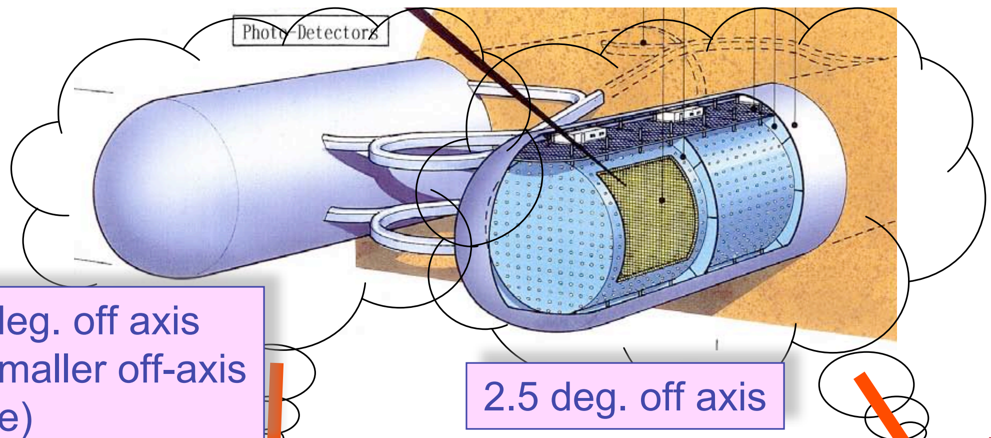
$\Delta m^2_{12} = 8.0 \times 10^{-5}$ (eV²)
$\Delta m^2_{23} = 2.5 \times 10^{-3}$ (eV²)
$\sin^2\theta_{12} = 0.31$

smaller the value of θ_{13}
larger the relative magnetide of P_{solar}

Effect of solar term is very small at Kamioka (L=295km) but sizable at Korea (L=1050km)

Experimental Setup and Assumptions

T2KK (Tokai to Kamioka-Korea)



Total cost must be similar to the baseline design.

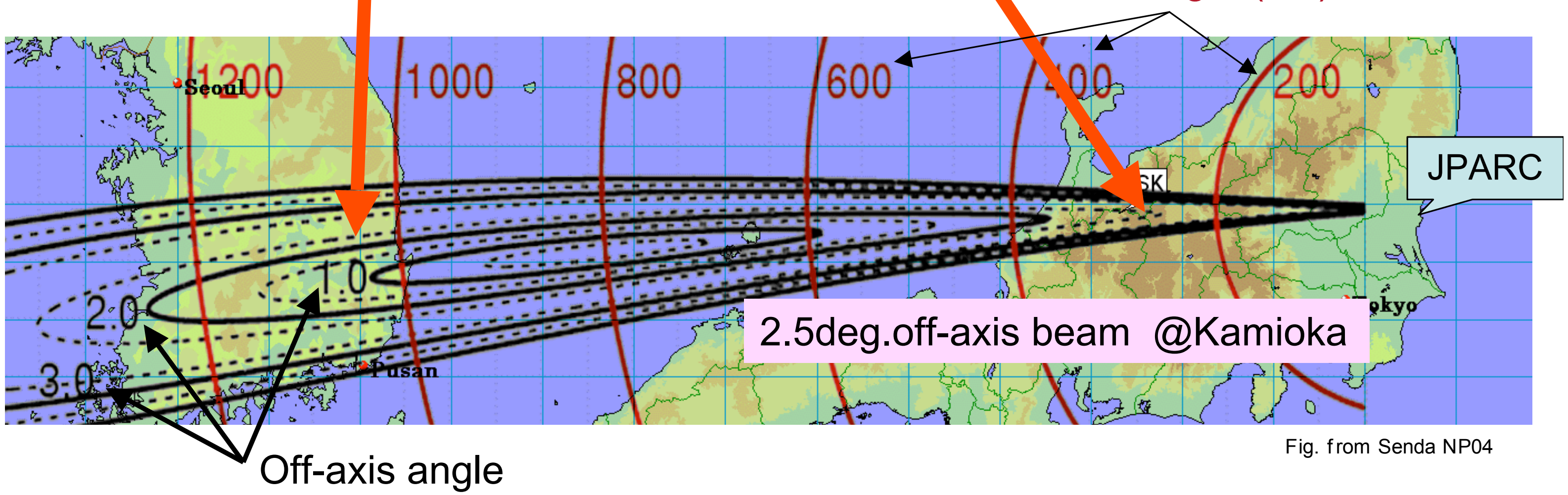


Fig. from Senda NP04

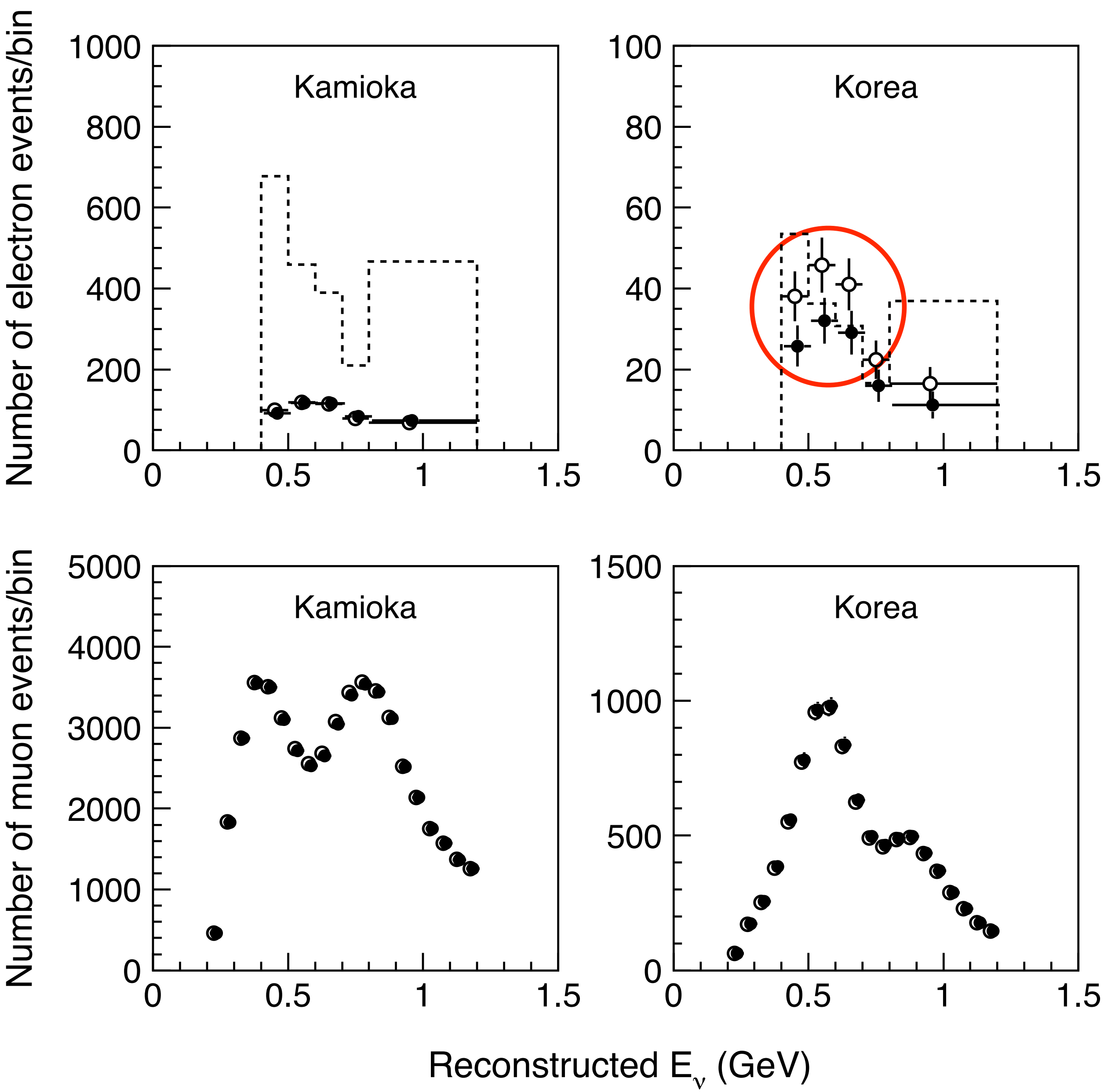
4MW 2.5 deg. OA beam from JPARC (4yr ν + 4 yr $\bar{\nu}$)

0.27 Mt detector @Kamioka (L=295 km)
0.27 Mt detector @Korea (L=1050km)
Appearance and disappearance modes

2 Identitical detectors at the same off axis angle

$\Delta m^2_{23} = \pm 2.5\times10^{-3}\ eV^2$ $\quad \Delta m^2_{12} = 8\times10^{-5}\ eV^2$ $\quad \sin^2\theta_{12} = 0.31$

Impact of the Solar Term



4MW 2.5 deg. OA beam
(4yr ν + 4 yr $\bar{\nu}$)

0.27 Mt@Kamioka (L=295km)

0.27 Mt@Korea (L=1050km)

Δm^2_{12} = 8.0×10^{-5} (eV^2)
Δm^2_{23} = 2.5×10^{-3} (eV^2)
$\sin^2\theta_{12}$ = 0.31
$\sin^2 2\theta_{23}$ = 0.96
δ = $3/4\ \pi$
normal mass hierarchy

○ $\sin^2\theta_{23} = 0.4$, $\sin^2 2\theta_{13} = 0.01$
● $\sin^2\theta_{23} = 0.6$, $\sin^2 2\theta_{13} = 0.0067$

χ^2 definition

$\begin{pmatrix} \nu \\ \bar{\nu} \end{pmatrix} \times \begin{pmatrix} \text{kamioka} \\ \text{korea} \end{pmatrix}$ = 4 combinations

$\nu_e(\bar{\nu}_e)$ event (5 bins) $\nu_\mu(\bar{\nu}_\mu)$ event (20 bins)

$$\chi^2 = \sum_{k=1}^{4} \left(\sum_{i=1}^{5} \frac{\left(N(e)_i^{\rm obs} - N(e)_i^{\rm exp}\right)^2}{\sigma_i^2} + \sum_{i=1}^{20} \frac{\left(N(\mu)_i^{\rm obs} - N(\mu)_i^{\rm exp}\right)^2}{\sigma_i^2} \right) + \sum_{j=1}^{7} \left(\frac{\epsilon_j}{\tilde{\sigma}_j} \right)^2$$

$$N(e)_i^{\rm exp} = N_i^{\rm BG} \cdot (1 + \sum_{j=1,2,7} f(e)_j^i \cdot \epsilon_j) + N_i^{\rm signal} \cdot (1 + \sum_{j=3,7} f(e)_j^i \cdot \epsilon_j) \ ,$$

$$N(\mu)_i^{\rm exp} = N_i^{\rm non-QE} \cdot (1 + \sum_{j=4,6,7} f(\mu)_j^i \cdot \epsilon_j) + N_i^{\rm QE} \cdot (1 + \sum_{j=4,5,7} f(\mu)_j^i \cdot \epsilon_j) \ .$$

f^i_j: fractional change in the predicted event rate in the i-th bin due to the variation of the parameter ϵ_j

ϵ_j : systematic error parameters varied freely to minimize χ^2

Systematic Errors

5 % BG (Overall)

5 % BG (Energy Dep.)

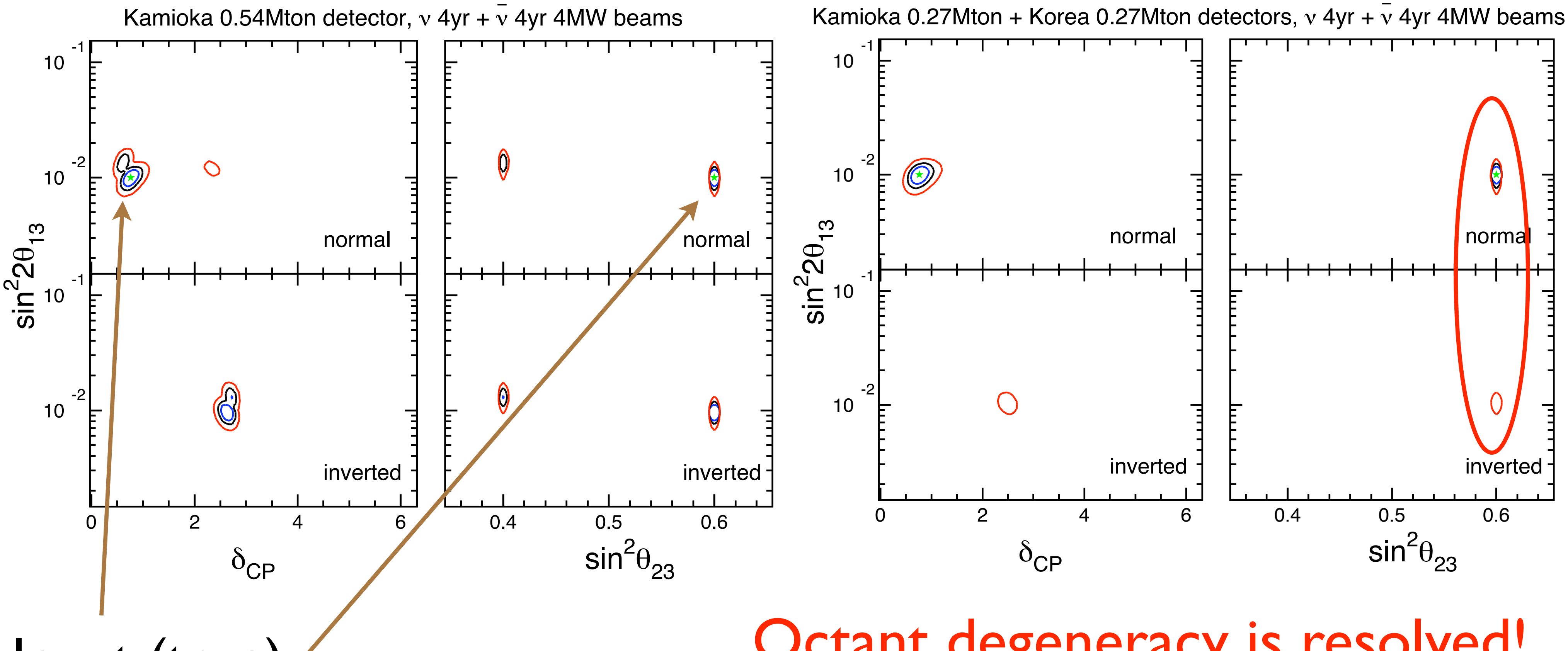
5 % Signal Efficiency

20 % Separation of QE/nQE

Example of the case where the octant degeneracy is resolved

HK(0.54Mt) only at Kamioka

2HKs(0.27x2Mt) at Kamioka and Korea



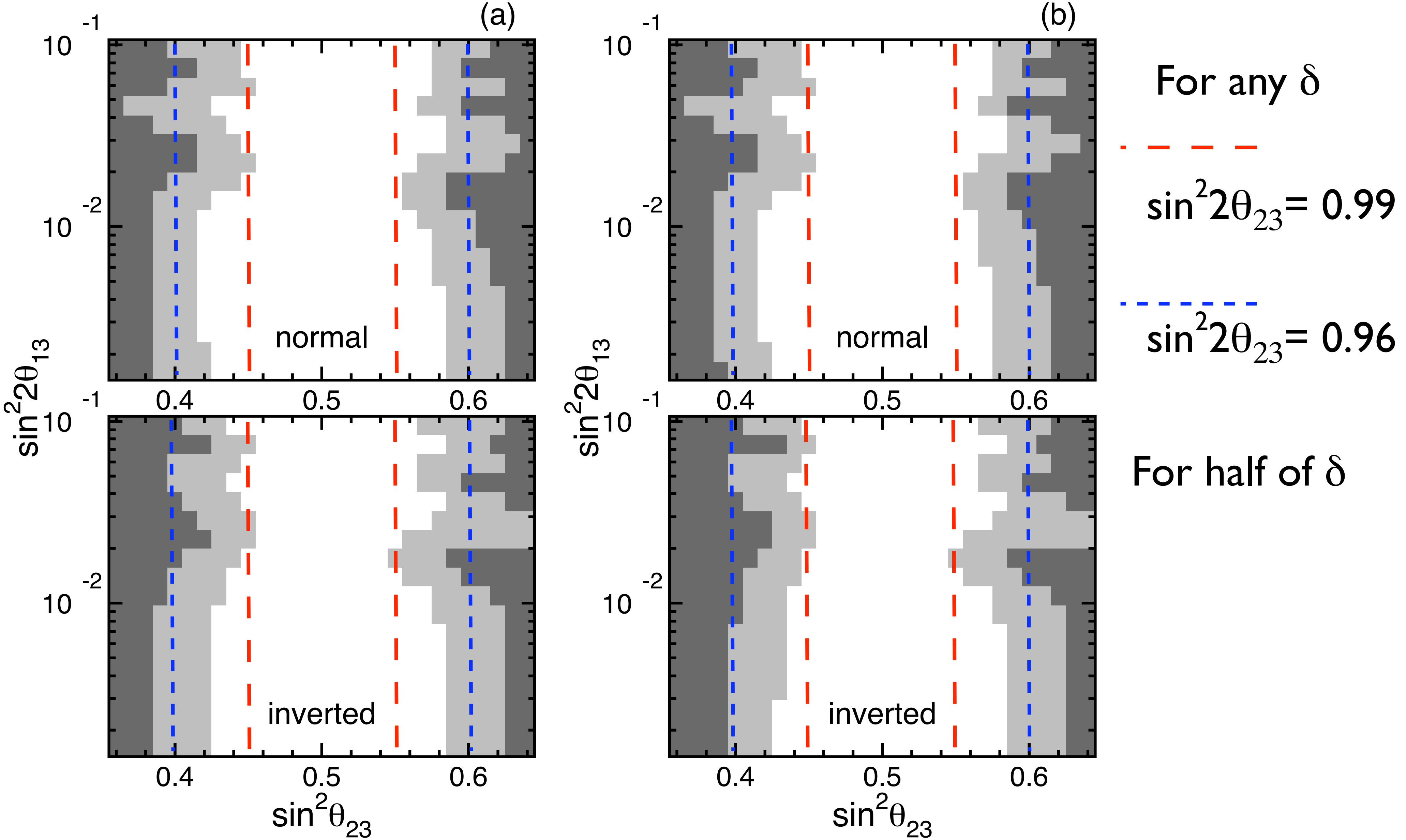
Input (true)

Octant degeneracy is resolved!

$\sin^2\theta_{23}$= 0.60 (true)

(Despite that sign Δm^2 degeneracy is not completely resolved → decoupling of degeneracy)

Parameter regions where the octant degeneracy can be resolved



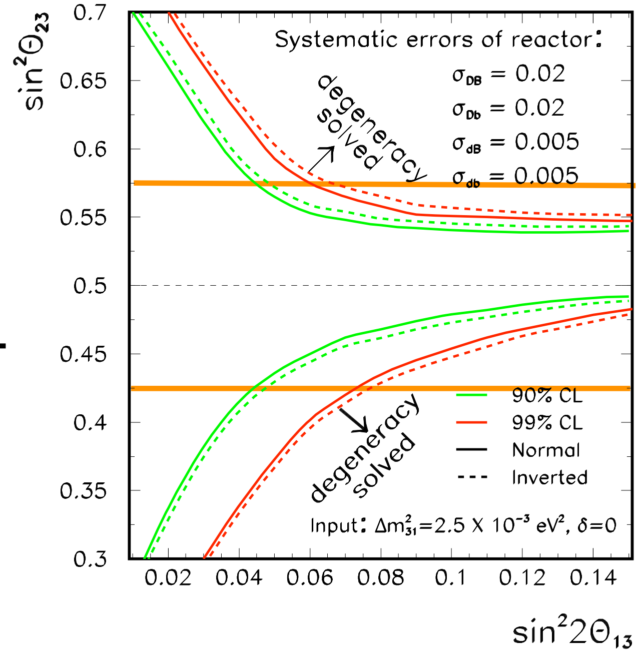
Dependence on the mass hierarchy is weak

Comparing two methods...

T2K II + Reactor

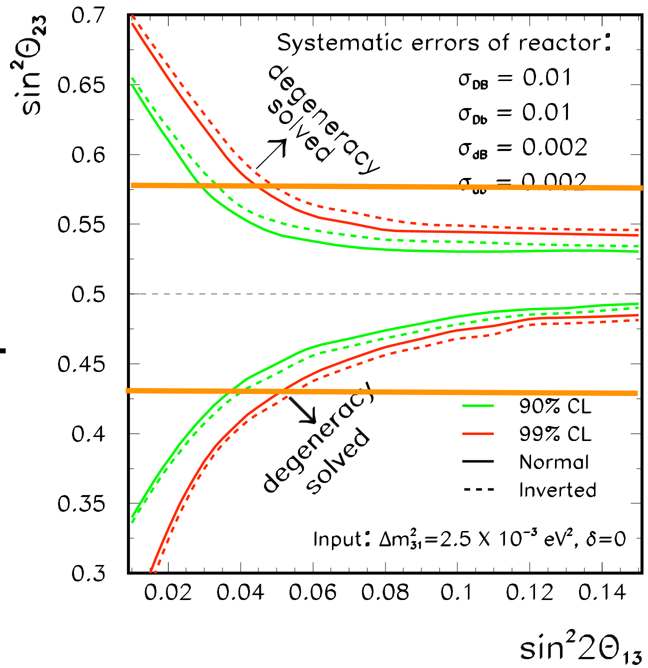
T2KK

~95% CL

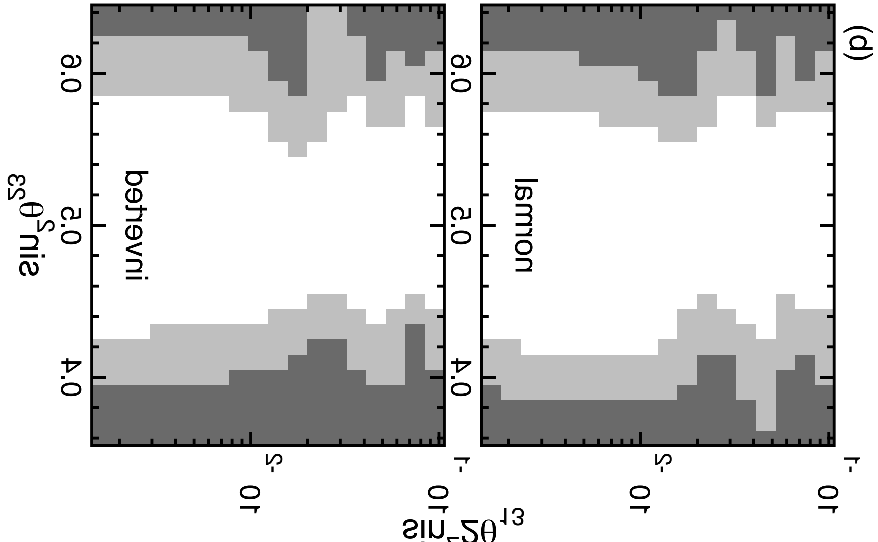


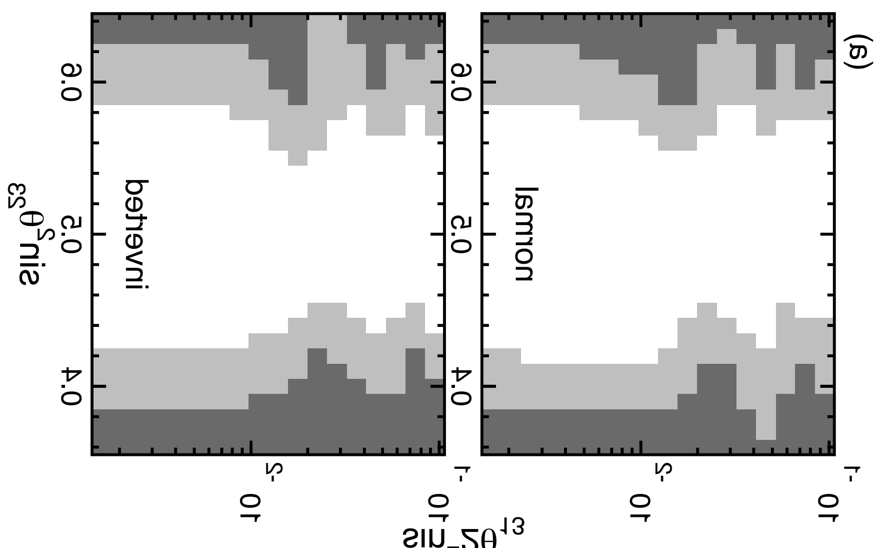
T2KK

~95% CL



T2KK





$\sin^2 2\theta_{13} \gtrsim 0.04\text{-}0.06$: T2KII+Reactor is better

$\sin^2 2\theta_{13} \lesssim 0.04\text{-}0.06$: T2KK is better

Summary

- Octant degeneracy exist if θ_{23} is different from $\pi/4$
- Method1: Superbeam (T2K II) + Reactor can resolve the octant degeneracy for $\sin^2 2\theta_{23} \approx 0.96$ (0.99) if $\sin^2 2\theta_{13} \approx 0.05$ (0.1) but not work for small θ_{13}
- Method II: Superbeam with 2 Far Detector System (T2KK) can Resolve the octant degeneracy for $\sin^2 2\theta_{23} < 0.97$ even for very small θ_{13}
- Both Methods are Complementary: For larger (smaller) θ_{13}, $\sin^2 2\theta_{13} \gtrsim (\lesssim)\ 0.05$, Method I (II) would be better