Many-Body Theory of the Electroweak Nuclear Response

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work done in collaboration with

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Outline

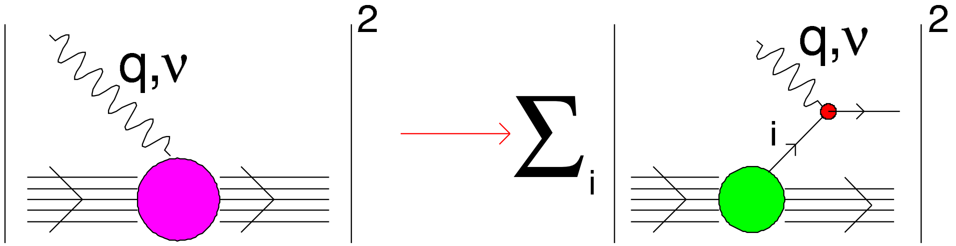
- Motivation
- Many body theory of electroweak interactions with nuclei
- Results for (e, e') and comparison to data
- Results for (ν, ℓ)
- Conclusions and prospects

Motivation

- Neutrino experiments use nuclei as detectors
- Quantitative understanding of the weak nuclear response at $E_\nu \sim 0.5 - 3$ GeV required for data analysis
- Need to develop a theoretical framework
  - able to explain electron scattering data
  - applicable to a wide range of kinematical conditions and targets
  - easily implementable in Monte Carlo simulations

Many-body theory of $e + A \to e' + X$

In the Impulse Approximation scheme (IA) the scattering process off a nucleus reduces to the incoherent sum of the elementary processes involving individual nucleons:



Cross section can be written:

$$\frac{d\sigma_A}{d\Omega_{e'} dE_{e'}} = \int d^4p \, P(p) \left(\frac{d\sigma_N}{d\Omega_{e'} dE_{e'}} \right)$$

Ingredients of IA calculations

- $e + N \to e' + X$ elementary cross-section

$$\frac{d\sigma_{eN}}{d\Omega_{e'} dE_{e'}} = \frac{\alpha^2}{Q^4} \frac{E'_e}{E_e} \frac{m}{E_{\mathbf{p}}} L_{\mu\nu} w_N^{\mu\nu}$$

$$w_N^{\mu\nu} = w_1^N \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) + \frac{w_2^N}{m^2} \left(p^\mu - \frac{(pq)}{q^2} q^\mu \right) \left(p^\nu - \frac{(pq)}{q^2} q^\nu \right)$$

w_1^N, w_2^N *from data*

- Target spectral function $P(\mathbf{p}, E)$: probability of removing a nucleon of momentum $\mathbf{p}$ from the target, leaving the residual spectator system with excitation energy E

$P(\mathbf{p}, E)$ obtained from non-relativistic nuclear many-body theory, using as *only* input the *bare* NN interaction (fitted to the properties of the NN system).

- no theoretical bias on the determination of the dynamics
- no adjustable parameters involved
- exact or highly accurate calculations feasible for A=2,3,4 and ∞
- spectral functions for Carbon, Oxygen, Iron and Gold obtained using Local Density Approximation (LDA)

Local Density Approximation (LDA) $P(\mathrm{p}, E)$ for oxygen

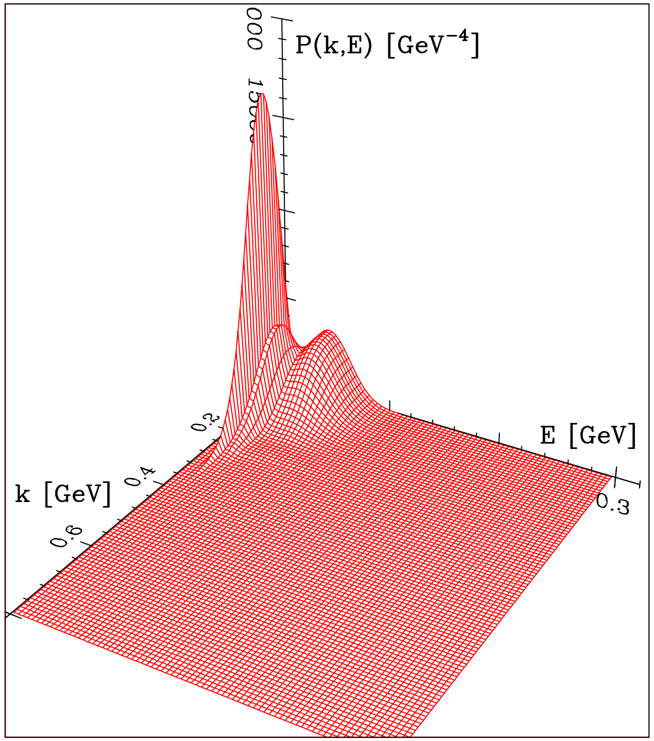
$$P(\mathbf{p}, E) = P_{MF}(\mathbf{p}, E) + P_{corr}(\mathbf{p}, E)$$

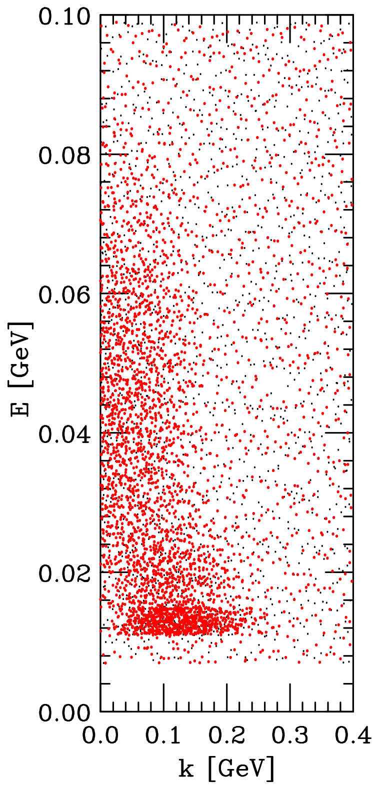
$P_{MF}(\mathbf{p}, E) \longrightarrow$ from $(e, e'p)$ data

$P_{corr}(\mathbf{p}, E) \longrightarrow$ from uniform nuclear matter calculation at different densities

$$P_{MF}(\mathbf{p}, E) = \sum_n Z_n |\phi_n(\mathbf{p})|^2 \; F_n(E - E_n)$$

$$P_{corr}(\mathbf{p}, E) = \int d^3r \; \rho_A(r) \; P_{corr}^{NM}(\mathbf{p}, E; \rho = \rho_A(r))$$





- the shell model contribution $P_{MF}(\mathbf{p}, E)$ accounts for $\sim 80\%$ of the strenght;
- the remaining $\sim 20\%$, accounted for by $P_{corr}(\mathbf{p}, E)$, is located at high momentum *and* large removal energy ($\mathbf{p} \gg p_F, E \gg e_F$)

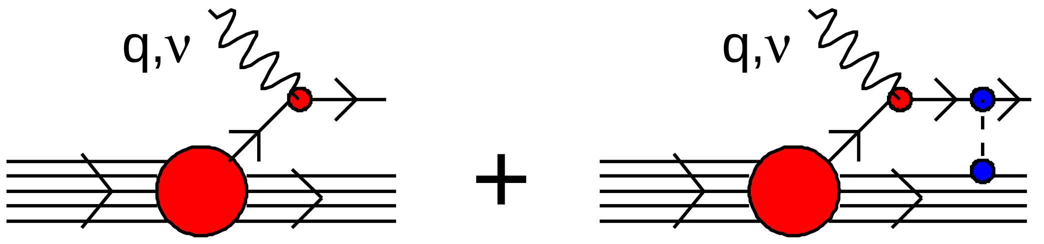
$e + N \to e' + X$

$$\frac{d\sigma_{eN}}{d\Omega_{e'} dE_{e'}} = \frac{\alpha^2}{Q^4} \frac{E'_e}{E_e} \frac{m}{E_{\mathbf{p}}} L_{\mu\nu} w_N^{\mu\nu}$$

$$\begin{aligned} w_i^{\mu\nu} &= \sum_x \langle \mathbf{p}, \mathrm{N} | j_i^\mu | x, \mathbf{p} + \mathbf{q} \rangle \langle \mathbf{p} + \mathbf{q}, x | j_i^\nu | \mathrm{N}, \mathbf{p} \rangle \\ &\times \quad \delta(\widetilde{\nu} + \sqrt{\mathbf{p}^2 + m^2} - E_x) \ . \end{aligned}$$

Binding of the struck nucleon is taken into account by replacing:

$$\begin{aligned} q &\equiv (\nu, \mathbf{q}) \to \widetilde{q} \equiv (\widetilde{\nu}, \mathbf{q}) \\ \widetilde{\nu} &= \nu - E + m - \sqrt{\mathbf{p}^2 + m^2} = \nu - \delta\nu \end{aligned}$$



- Going beyond the IA: FSI (Final State Interactions)

  A) energy shift $\rightarrow$ mean field of the spectators

  B) redistributions of the strenght $\rightarrow$ coupling of $1p1h$ final state to $np-nh$

High energy approximation:

i) the struck nucleon moves along a straight trajectory with constant velocity;

ii) the fast struck nucleon "sees" the spectator system as a collection of fixed scattering centers.

FSI described by:

$$\bar{U}^{FSI}_{\mathbf{p}+\mathbf{q}}(t) = \langle 0 | \frac{1}{A} \sum_{i=1}^{A} \mathrm{e}^{i \sum_{j \neq i} \int_0^t dt' \Gamma_{\mathbf{p}+\mathbf{q}}(|\mathbf{r}_{ij} + \mathbf{v}t'|)} \, |0\rangle \ ,$$

Γ_{if} is the Fourier Transform of NN scattering amplitude.

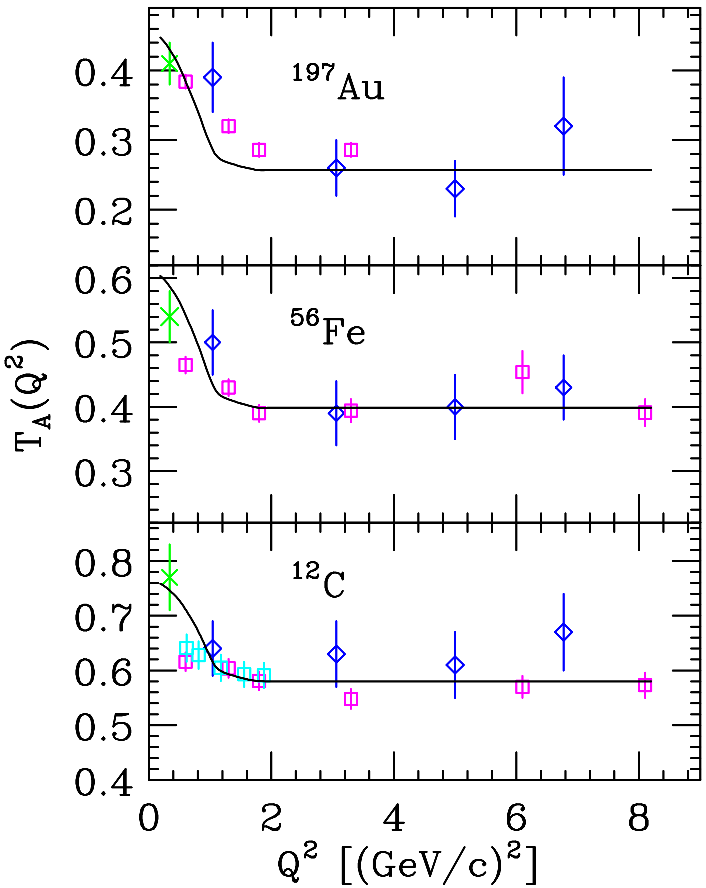
A) $\rightarrow$ Re Γ_{ij} B) $\rightarrow$ Im Γ_{ij}

At high energy B) dominates.

$$\frac{d\sigma}{d\Omega_{e'} d\nu} = \int d\nu' \ f_{\mathbf{q}}(\nu - \nu') \ \left(\frac{d\sigma}{d\Omega_{e'} d\nu'} \right)_{IA}$$

$$f_{\mathbf{q}}(\nu) = \delta(\nu)\sqrt{T_A} + \int \frac{dt}{2\pi} \, \mathrm{e}^{i\nu t} \left[U^{FSI}_{\mathbf{q}}(t) - \sqrt{T_A} \right]$$

$$T_A = \lim_{t \to \infty} \langle 0 | |U^{FSI}_{\mathbf{q}}(R;t)|^2 | 0 \rangle$$



Calculated transparency compared to MIT-Bates, SLAC and JLAB data (D. Rohe et al., (JLAB E97-006) - nucl-ex/0506007)

Going beyond IA (continued): statistical FSI $\rightarrow$ Pauli blocking

A rather crude prescription: modify the spectral function

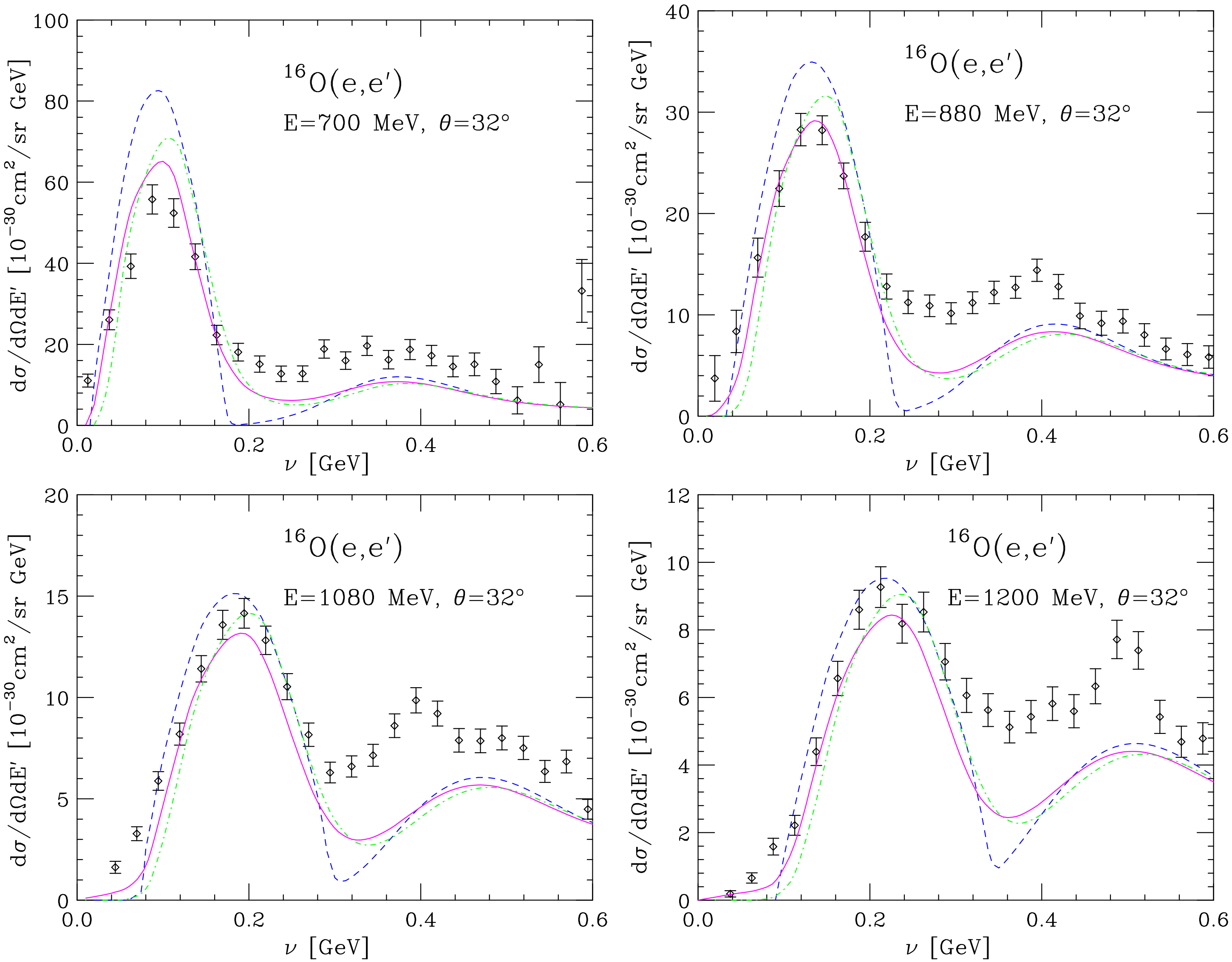
$$P(\mathbf{p},E) \rightarrow \; P(\mathbf{p},E)\,\theta(|\mathbf{p}+\mathbf{q}| - \overline{p}_F)$$

Average nuclear Fermi momentum $\overline{p}_F$ defined as:

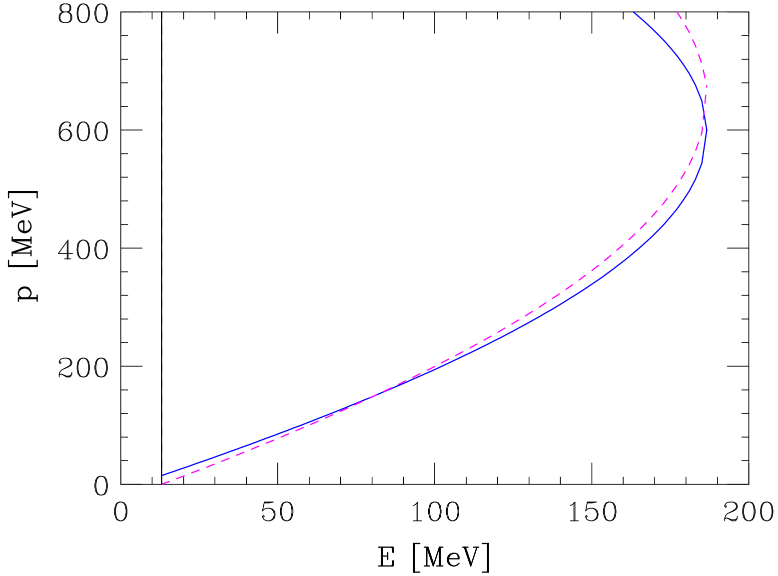
$$\overline{p}_F = \int d^3r \; \rho_A(\mathbf{r}) p_F(\mathbf{r}) \quad , \quad p_F(\mathbf{r}) = \frac{3}{2}\pi^2 \rho_A(\mathbf{r})$$

Inclusion of Pauli blocking is hardly visible in the lepton energy distribution at a fixed angle, but becomes dominant in the Q^2 distribution in the low Q^2 region.

Comparison to Frascati ^{16}O (e, e') data

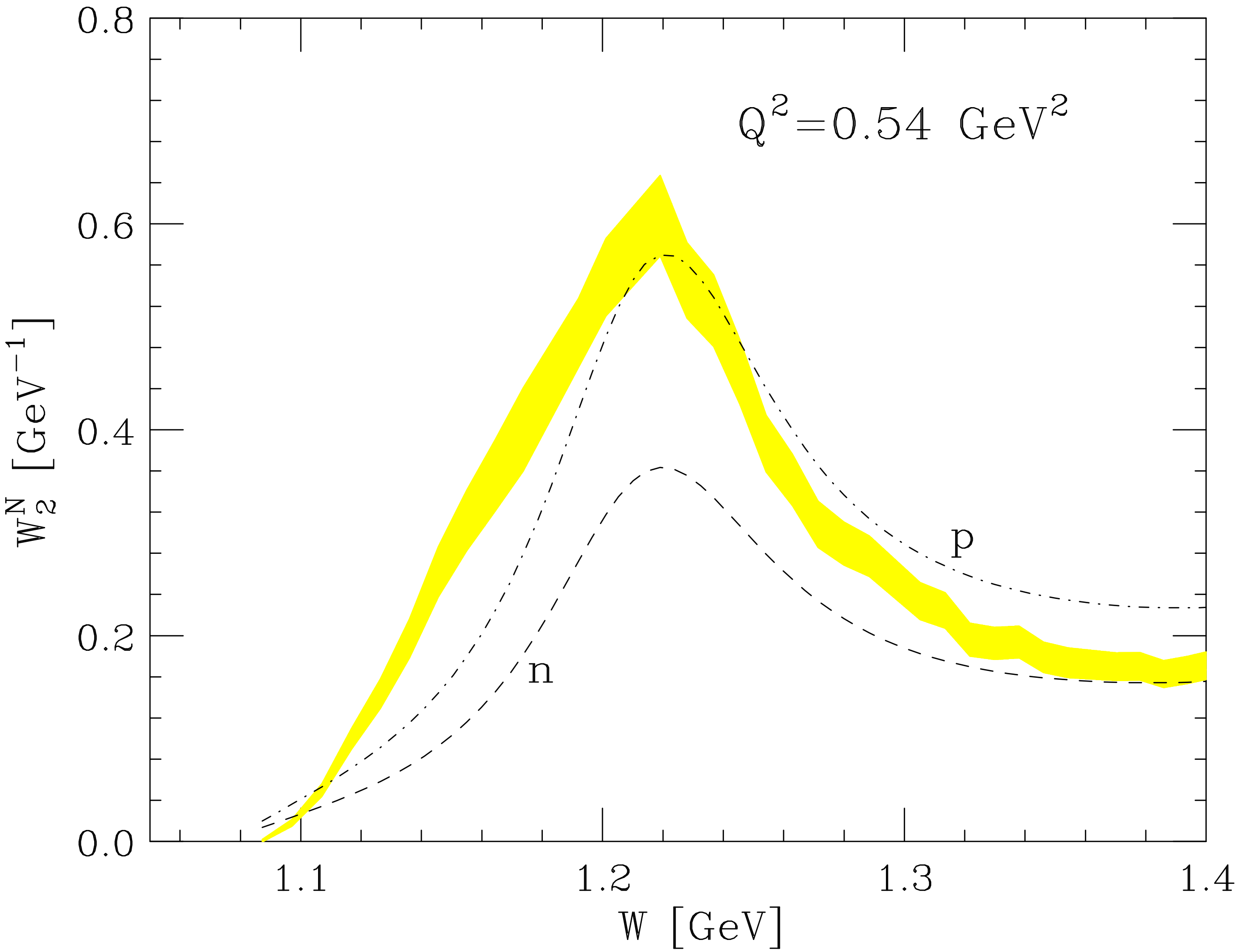


- OK in the region of quasi elastic peak
- data significantly underestimated above π production threshold
- deficiencies of the SF unlikely

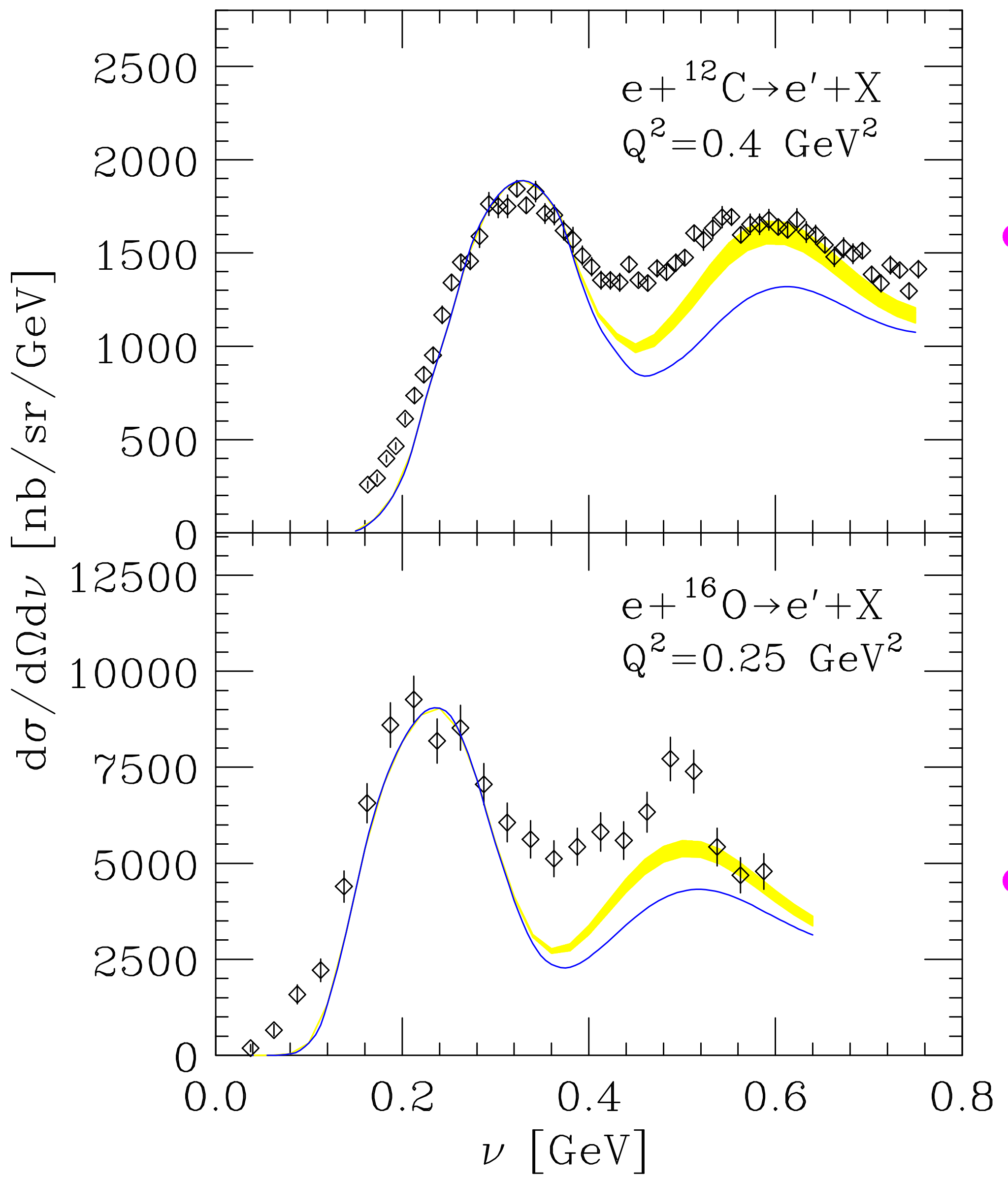


- low Q^2 nucleon structure functions in the Δ production region poorly known. For example, the Bodeck & Ritchie fit does only inlcudes data @ $Q^2 > 2$ GeV^2.

W_2^n from data @ $E_e = 2.445$ GeV and $\theta_e = 20°$



Nuclear cross sections



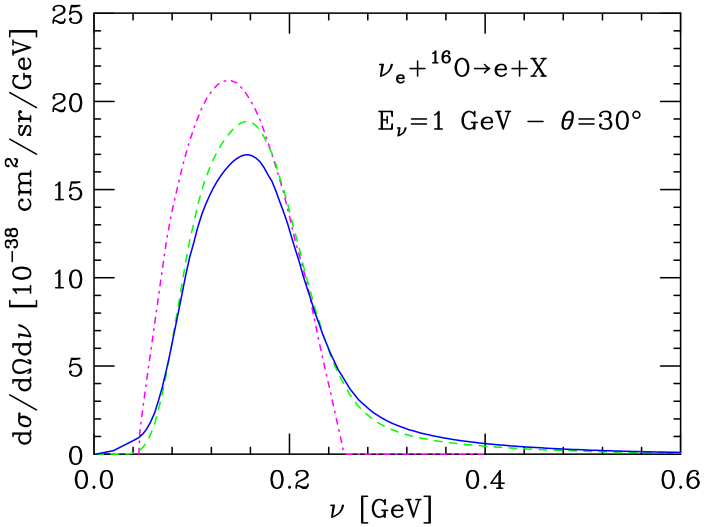
- SLAC data

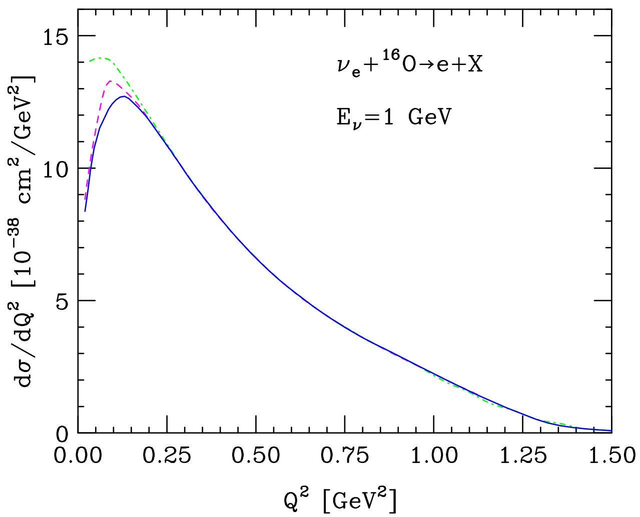
  (Sealock *et al* (1989))

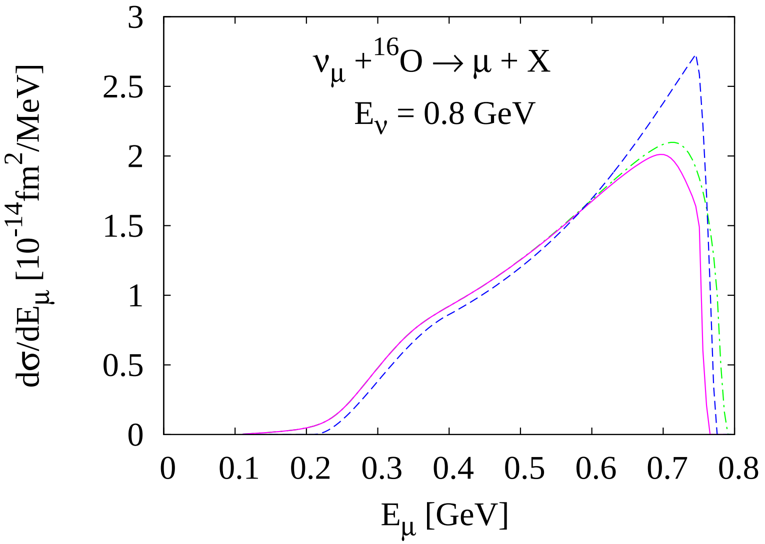
- LNF data

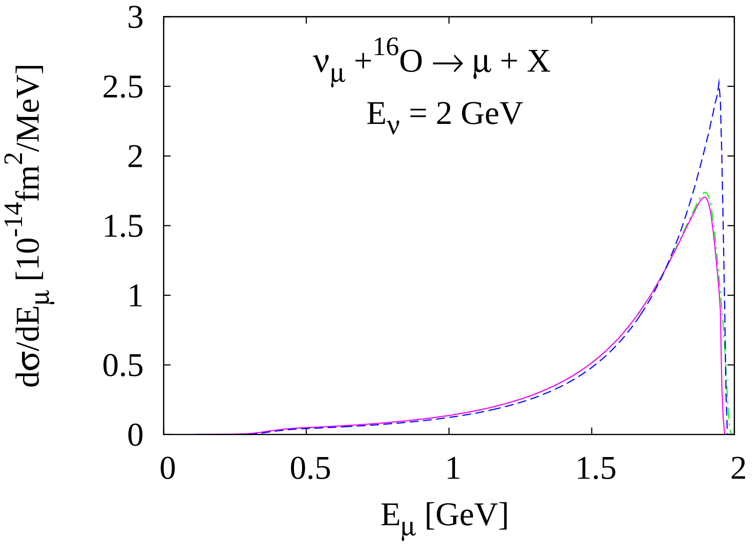
  (Anghinolfi *et* al (1996))

Results for $^{16}O\ (\nu_e, e)$ scattering

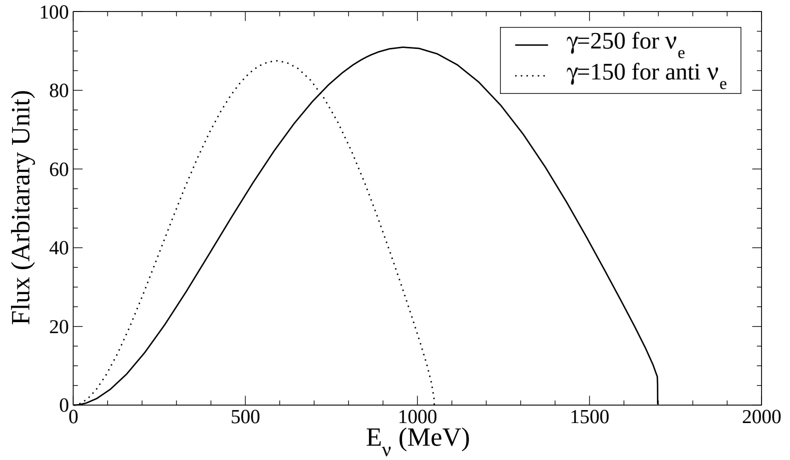


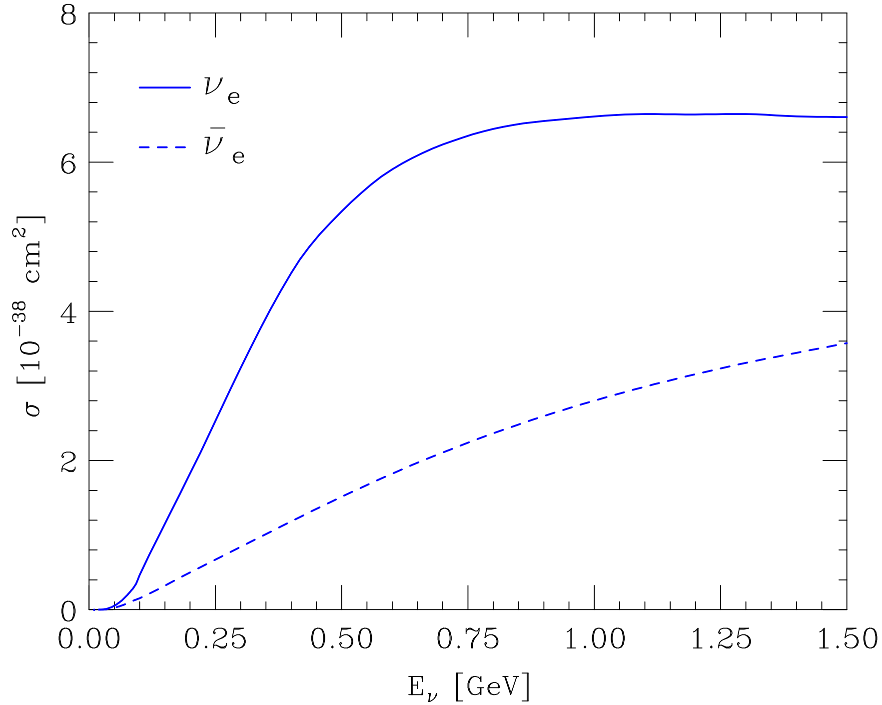






Total CC cross section for β-beams of ν_e and $\bar{\nu}_e$ (QE only)





Conclusions and prospects

- Nuclear many-body theory provides quantitative parameter-free predictions of the inclusive cross sections for a broad range of targets and kinematical conditions
- Data in the region of quasi elastic peak reproduced with accuracy better than 10%
- Problems in the region of quasi-free Δ production, likely to be ascribed to the uncertainty associated with the nucleon structure functions. New data from JLab will certainly help to fix this problem.
- Pauli blocking important at $Q^2 < 0.2$ GeV^2
- Extension to semi-inclusive processes and implementation in Monte Carlo simulations is under way