CRYSTALLINE COLOR SUPERCONDUCTIVITY

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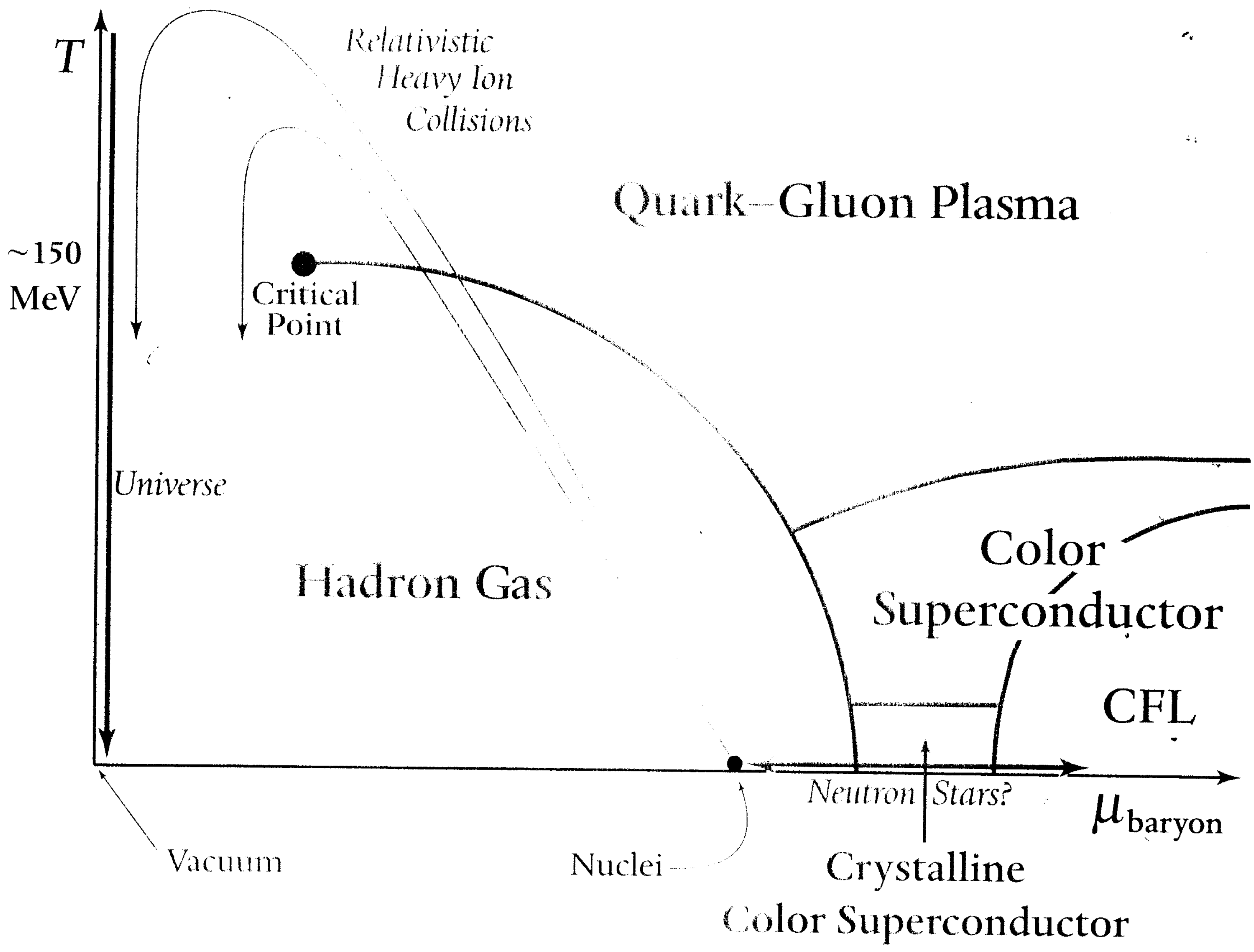
QCD@WORK, Martina Franca, 17/6/01

EXPLORING *the* PHASES *of* QCD



BUT FIRST ....

A LENGTHY INTRODUCTION

1. Why color superconductivity?
2. The "ideal" color-flavor locked phase. ($M_s = M_u = M_d$)
3. How the ideal CFL phase responds to the stresses of a less symmetric reality.
4. One answer to (3): crystalline color superconductivity.
5. → Pulsar glitches.

WHY COLOR SUPERCONDUCTIVITY?

- Large μ → quarks filling Fermi sea up to a large Fermi energy. (E_F)
- Asymptotic freedom → <u>weak</u> interactions between quarks at Fermi surface.
- <u>BUT</u> <u>any</u> attractive interaction, no matter how weak, → COOPER PAIRS ; $\langle qq \rangle$
- One gluon exchange (& in<u>s</u>tanton interaction) attractive in color $\bar{3}$.
  (no need to resort to phonons; ∴ superconductivity more robust in QCD than in metals. Higher T_c/E_F.)
- $\langle qq \rangle$, ie Cooper pairs of quarks,
  ⇒ - electric + color currents supercondu[ct]
  - mass for photon + (some) gluons(?)
  - Meissner effects. (Magnetic + color magnetic fields excluded.)

GAPS & T_c

Much recent work (dozens of papers) suggests:

$$\Delta \lesssim 100 \text{ MeV}$$
$$T_c \lesssim 50 \text{ MeV}$$

Alford, KR, Wilczek, Rapp, Schäfer, Shuryak, Velkovs[ky], Berges, Carter, Diakonov, Evans, Hsu, Schwetz...

I will not review this work.

Two methods ~ agree:

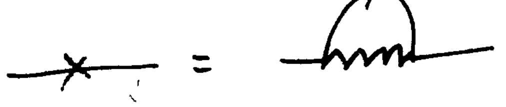
i) models normalized to $\mu = 0$ physics

ii) weak-coupling QCD calculations valid for $\mu \to \infty$; $g \to 0$

Quantitatively valid for $\mu \gg 10^9$ MeV. KR + Shuster

$$\frac{\Delta}{\mu} \sim \underbrace{256\pi^4 \left(\frac{N_f}{2}\right)^{-5/2}}_{\text{Schaefer + Wilczek; Pisarski + Rischke; Hong, Miransky, Shovkovy, Wijewardhana; Evans, Hsu, Schwetz; Brown, Liu, Ren; Beane, Bedaque, Savage; Rajagopal, Shuster} \ldots} \underbrace{\frac{1}{g^5} \exp\left(-\frac{3\pi^2}{\sqrt{2}\, g}\right)}_{\text{Son}}$$

– $\exp(-1/g)$ comes from divergence in small angle scattering via exchange of unscreened magnetic gluons.



$$\to 1 = g^2 \underbrace{\ln \frac{\Delta}{\mu}}_{\text{BCS}} \underbrace{\ln \frac{\Delta}{\mu}}_{\text{collinear divergence}}$$

THE "IDEAL" COLOR-FLAVOR LOCKED PHASE

Cold dense quark matter with $m_s = m_u = m_d$.

Attraction between quarks which are antisymmetric in color makes Fermi surface unstable to Cooper pairing.

Favored channel (all 9 quarks pair) has $\langle ud \rangle = \langle us \rangle = \langle ds \rangle$

Δ of order 100 MeV

T_c of order 50 MeV

Color-flavor locking $\to$ broken chiral symmetry

All 9 quarks gapped.

Light degrees of freedom:

- superfluid mode (massless)
- 8 light pseudo-NGB from χSB.
- massless $\tilde{Q}$-photon; lin. comb. of $\gamma + g$.

(8 gluons + 1 photon $\to$ 8 massive vectors + 1 $\tilde{Q}$-photon.)

At $T=0$: CFL is a TRANSPARENT INSULATOR

$N_f = 3$: COLOR-FLAVOR LOCKING

color

$$\langle q^\alpha_{La} q^\beta_{Lb} \rangle = -\langle q^\alpha_{Ra} q^\beta_{rb} \rangle$$

flavor

$$= \Delta\, \epsilon^{\alpha\beta A}\, \epsilon_{abA}$$

not quite correct.
(symmetries correct)

Locks $SU(3)_L$ to $SU(3)_{color}$

Locks $SU(3)_R$ to $SU(3)_{color}$

Result:

$$SU(3)_{color} \times \underbrace{SU(3)_L \times SU(3)_R}_{\text{contains } U(1)_{EM}} \times U(1)_B$$

$$\longrightarrow \underbrace{SU(3)_{color+L+R}}_{\text{contains unbroken gauged } U(1)_{\tilde{Q}}}$$

gauge symmetries: $SU(3)_{color} \times U(1)_{EM} \to U(1)_{\tilde{Q}}$

Classify excitations using $\tilde{Q}$ charge
(unbroken, but modified, "photon")
and "isospin".

$\vec{B}$ Evolution

Alford Berges KR

in a color superconductor

ordinary photon — ordinary gluon

$$A_\mu^{\tilde{Q}} = \cos\alpha_0 \, A_\mu + \sin\alpha_0 \, G_\mu^8$$

$$A_\mu^X = -\sin\alpha_0 \, A_\mu + \cos\alpha_0 \, G_\mu^8$$

$A_\mu^{\tilde{Q}}$: massless. Satisfies Maxwell

A_μ^X : massive. Like Z-boson.

α_0 : like Weinberg angle

$$\sin\alpha_0 \sim e/g \sim \frac{1}{20} - \frac{1}{40}$$

$\Rightarrow$ Propagating photon in c.s. is "mostly photon"

$$\sin\alpha_0 = \frac{\eta e}{\sqrt{\eta^2 e^2 + g^2}}$$

$\bar{w}$ $\eta = 1/\sqrt{3}$ CFL

$-2/\sqrt{3}$ 2SC

$\tilde{Q}$ IN THE CFL PHASE

$$\langle q_a^\alpha q_b^\beta \rangle \sim \Delta_1 \delta_a^\alpha \delta_b^\beta + \Delta_2 \delta_b^\alpha \delta_a^\beta$$

$$\tilde{Q} = Q_{EM} + \frac{1}{\sqrt{3}} T_8$$

Q_{EM}: $\begin{cases} \frac{2}{3} \text{ for } u \\ -\frac{1}{3} \text{ for } d \\ -\frac{1}{3} \text{ for } s \end{cases}$ $\qquad \frac{1}{\sqrt{3}} T_8$: $\begin{cases} -\frac{2}{3} \text{ for } b \\ \frac{1}{3} \text{ for } r \\ \frac{1}{3} \text{ for [illegible]} \end{cases}$

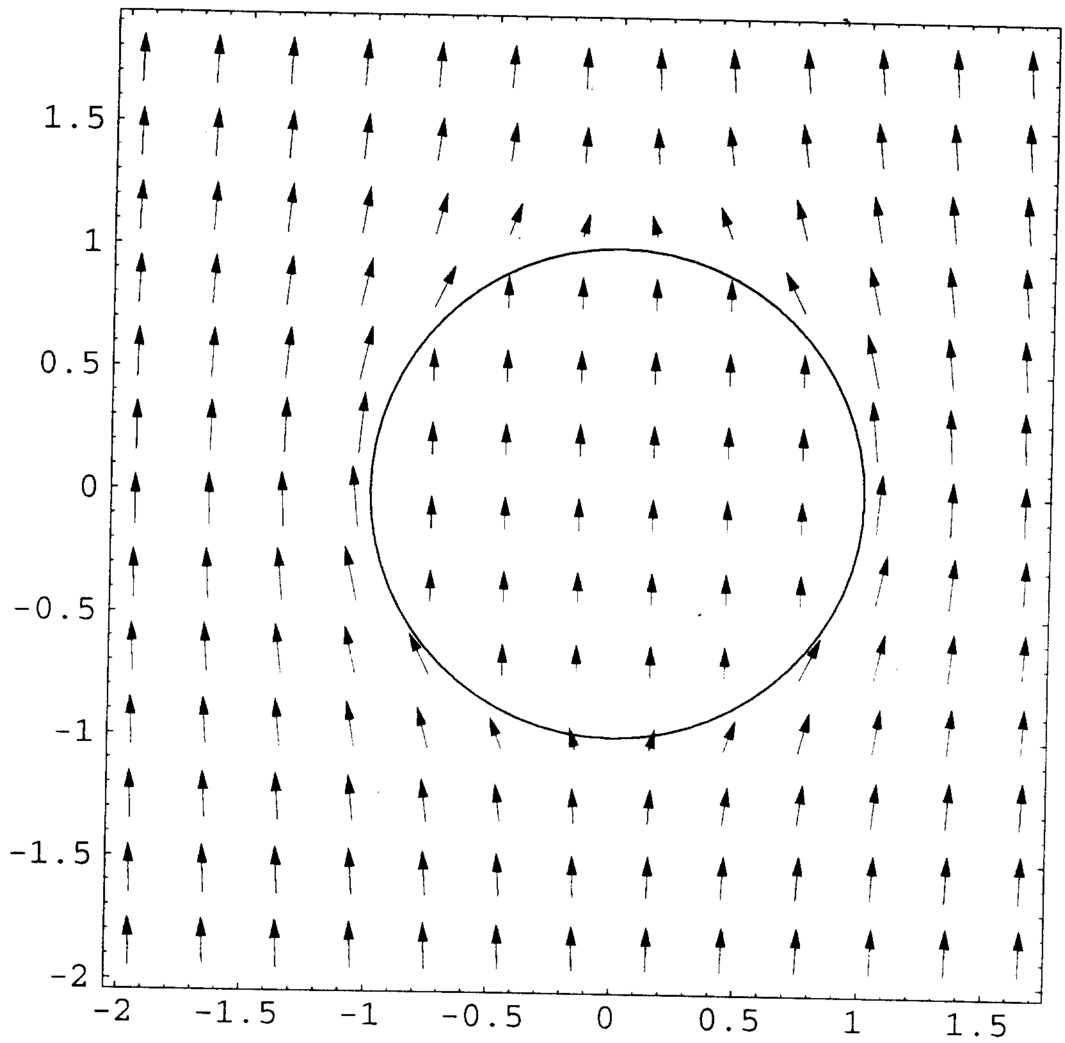
$\tilde{Q}$ charges of quarks:

u	+1
u	+1
u	0
d	0
d	0
d	−1
s	0
s	0
s	−1

Similarly, $\tilde{Q}$ charges of gluons all integer-valued. Also for $\tilde{Q}$ charges of Goldstone bosons.

Magnetic field solution (sharp boundary)

Stitching together the inside and outside solutions, we find the solution. For $\cos\alpha_0 = 0.5$



In the real world α_0 is small, so the field is mostly converted into $\tilde{Q}$ flux by the supercurrents and monopoles, and penetrates the interior. Only a weak field is excluded.

HOW DOES THE IDEAL CFL PHASE RESPOND TO THE "STRESS" OF $m_s \neq m_{u,d}$?

Ideal CFL: $m_s = m_u = m_d$

$N_s = N_u = N_d$

$\langle us \rangle = \langle ds \rangle = \langle ud \rangle$

Reality: $m_s > m_{u,d}$

If there were no pairing, this (and the requirement of electric neutrality)
$\Rightarrow$ $N_s \neq N_u \neq N_d$ $N_e \neq 0$

BUT In CFL phase, $N_s \neq N_u \neq N_d$ requires breaking pairs, and $\therefore$ costs.
CFL pairing enforces $N_s = N_d = N_u$; $N_e = 0$ in reality as in ideality....

as long as $\Omega_{CFL} < \Omega_{unpaired}$

$\Rightarrow$ as long as $\dfrac{m_s^2}{4\mu} < \Delta$ (for $m_s, \Delta \ll \mu$)

KR, Wilczek; Alford, KR, Reddy, Wilczek

WHEREVER THE CFL PHASE IS PRESENT

i) It has $N_u = N_d = N_s$ and is electrically neutral in absence of electrons.

ii) It keeps electrons out, even if you try to push them in via contact with (nuclear) matter with $\mu_e \neq 0$.
→ does so via charged boundary layers; electric field at interface
→ Figure. (Ask Sanjay to give talk)

iii) It is a Transparent Insulator

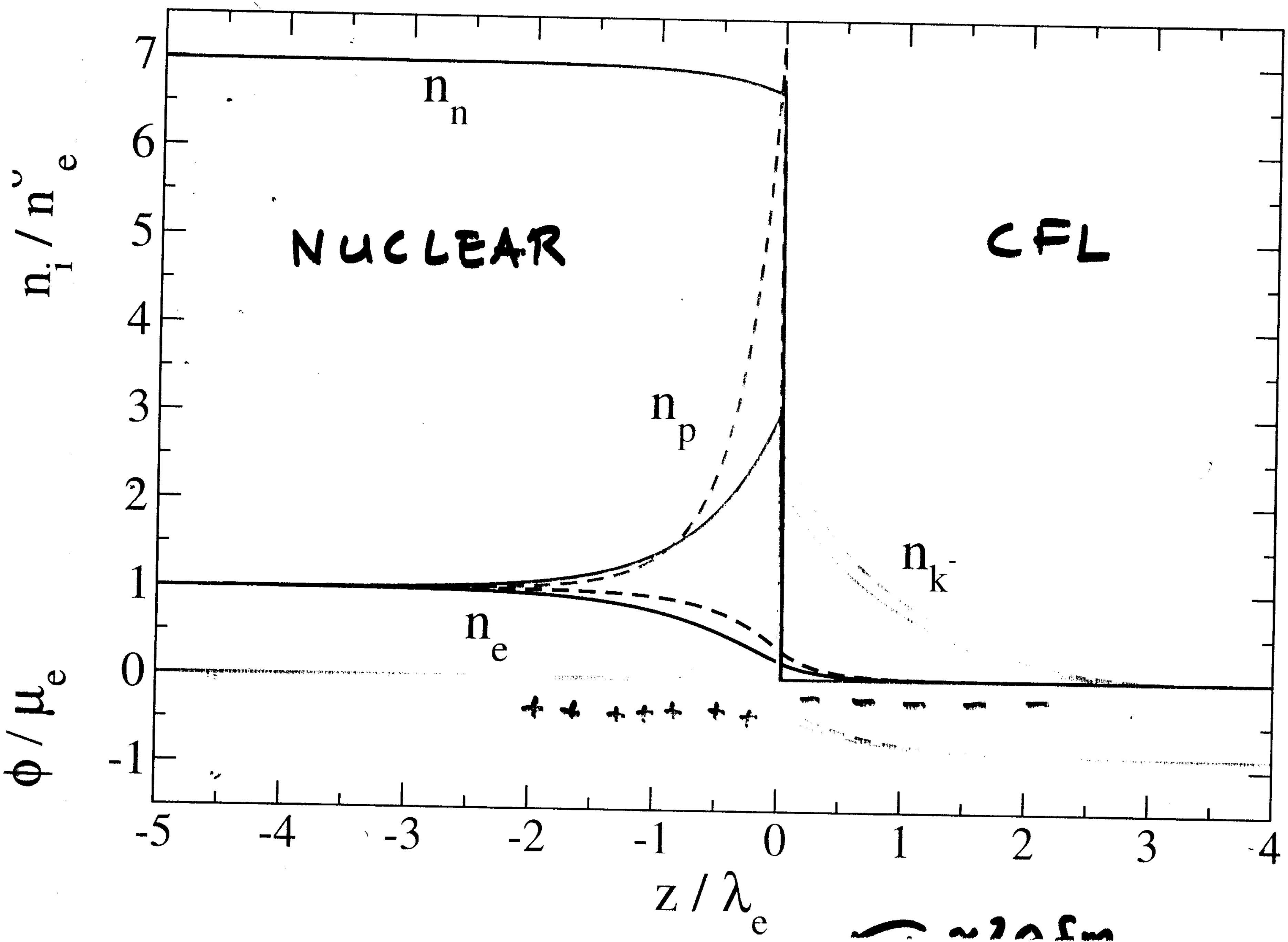
iv) Specific heat (small) and transport dominated by CFL excitations:
- superfluid mode
- mesons from χSB
- $\tilde{Q}$-photon

WHEREVER CFL BREAKS...

Get less symmetrically paired QM.
- may have, eg, <ud> BCS condensate

Possibility of CRYSTALLINE COLOR SUPERCONDUCTIVITY

MINIMAL CFL - NUCLEAR INTERFACE



CRYSTALLINE COLOR SUPERCONDUCTIVITY

Pairing between quarks with <u>different</u> Fermi momenta.

- Expected when CFL phase "breaks".
- We shall study in context of $\langle ud \rangle$ pairing (called "2SC Phase") which we stress by applying

$$\mu_u = \bar{\mu} \underline{\underline{- \delta\mu}} \qquad \mu_d = \bar{\mu} \underline{\underline{+ \delta\mu}}$$

- Redoing analysis with stress imposed by M_s instead of $\delta\mu$ is work in progress. Kundu, KR
- First review BCS ~~state~~; then explain c.c.s. .....

BCS ANSATZ FOR 2SC PHASE

3 colors ; 2 massless flavors.

$$\mu_u = \mu_d = \mu \quad (\sim 400 \text{ MeV})$$

Pair quarks with:

- Momenta $\vec{p}, -\vec{p}$ with $|\vec{p}| \approx p_F = \mu$
- Color $\bar{3}$ $\Rightarrow$ Attractive interaction
- Spin 0 $\Rightarrow$ Pair quarks of same helicity. LL or RR
- Flavor antisymmetric. (Pauli)

$$|2SC\rangle = A_L^+ A_R^+ |0\rangle$$

$$A_L^+ = \prod_{\vec{p}} \prod_{\alpha,\beta} \left(\cos\theta_L(\vec{p}) + e^{i\xi_L(\vec{p})} \epsilon^{\alpha\beta 3} \sin\theta_L(\vec{p}) \cdot a^+_{Lu\alpha}(\vec{p})\, a^+_{Ld\beta}(-\vec{p}) \right)$$

A_R^+ = same, with $L \to R$

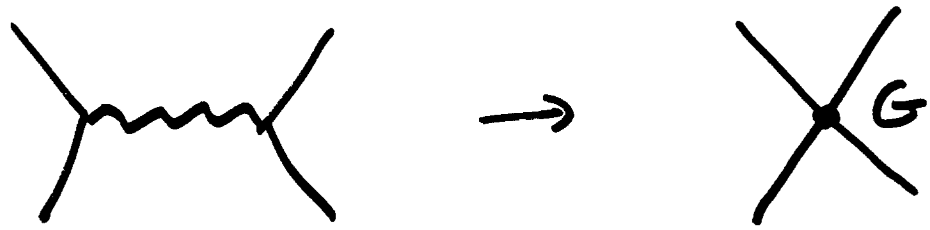
Now, evaluate $\langle 2SC | H - \mu N | 2SC \rangle$
and minimise w.r.t. $\theta_L(\vec{p}), \theta_R(\vec{p}), \xi_{L,R}(\vec{p})$.

Flavor antisymmetry comes out of ξ's.

Find $\exp i\xi(-\vec{p}) = -\exp i\xi(\vec{p})$ $\lrcorner$

CHOICE OF H ?

For ~~today~~ now, we use toy model in which QCD replaced by 4 fermion interaction with quantum numbers of gluon exchange:



Choose the 2 parameters of model (G & cutoff Λ) to give reasonable $\mu=0$ physics.

End of talk: other choices of H.

GAP EQUATION

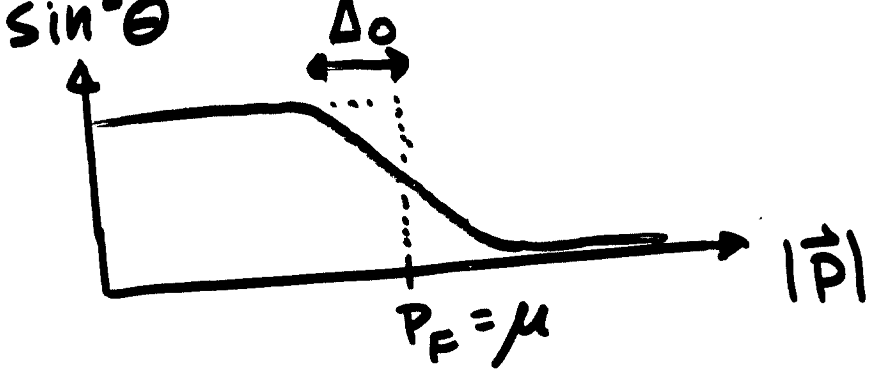
Minimize $\langle F \rangle$ and find

$$\tan 2\theta = \frac{\Delta_0}{|\vec{p}| - \mu}$$

where Δ_0 satisfies

$$1 = G \int^{\Lambda} d^3p \, \frac{1}{\sqrt{(|\vec{p}|-\mu)^2 + \Delta_0^2}}$$

NB $\Delta_0 \neq 0$ if $G > 0$ regardless of how small G.



Small G $\Rightarrow \Delta_0 \ll \mu$

Energy of elementary excitations

$$E(\vec{p}) = \sqrt{(|\vec{p}|-\mu)^2 + \Delta_0^2} \qquad \therefore \Delta_0 = \text{gap}$$

$$F_{BCS} - F_{NORMAL} = -\mu^2 \Delta_0^2 / 3\pi^2$$

Condensate:

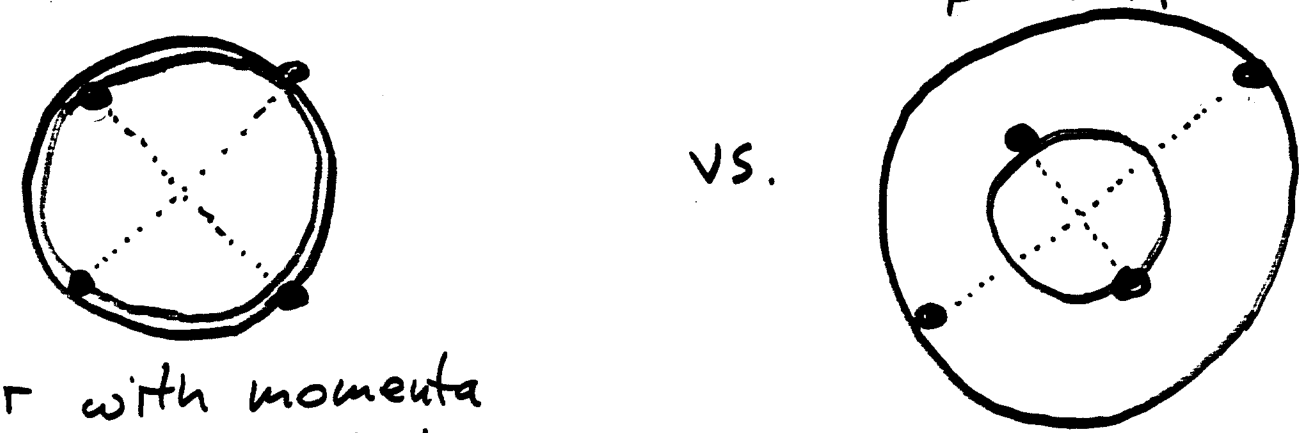
$$\Gamma \equiv \langle 2SC | \underbrace{\epsilon_{ij}}_{\text{flavor singlet}} \underbrace{\epsilon_{\alpha\beta 3}}_{\text{color } \bar{3}} \psi^T_{i\alpha}(\vec{x}) \underbrace{C\gamma_5}_{\text{Lorentz scalar}} \psi_{j\beta}(\vec{x}) | 2SC \rangle$$

$$= \Delta_0 / G$$

$\delta\mu \neq 0$: BAD NEWS FOR BCS

$$\mu_u = \mu - \delta\mu \quad ; \quad \mu_d = \mu + \delta\mu$$

$p_F^u = p_F^d$ $\qquad$ vs. $\qquad$ $p_F^u < p_F^d$



air with momenta $(\vec{p}, -\vec{p})$ can both be on respective F.S.

unpaired normal state : $p_F^u = \mu_u \quad p_F^d = \mu_d$

Favored BCS state (best choice within BCS ansatz) has $p_F^u = p_F^d = \mu$, and $\therefore$ pays free energy price.

$$F_{BCS} - F_{NORMAL} \simeq \frac{\mu^2}{3\pi^2}\left(2\,\delta\mu^2 - \Delta_0^2\right)$$

$\Rightarrow$ 1st order Phase Transition at

$$\delta\mu = \delta\mu_1 \simeq \Delta_0/\sqrt{2}$$

$(\simeq \rightarrow = \text{ for } \Delta_0 \ll \mu)$

LOFF ANSATZ

Larkin, Ovchinnikov; Fulde, Ferrell. 1964

Considered effect of Zeeman splitting on $\langle e_\uparrow e_\downarrow \rangle$ superconductor.

(Thinking of magnetic impurities.)

.OFF state never unambiguously seen in condensed matter. Problem:

$\vec{B} \to$ orbital effects, not just Zeeman.

:n QCD, we have JUST "Flavor Zeeman".

:DEA: Try Cooper pairs with total momentum!

$$(\vec{p} + \vec{q}, -\vec{p} + \vec{q})$$

Total momentum $= 2\vec{q}$ for all pairs.

$|\vec{q}| \leftarrow$ determined variationally

$\hat{q} \leftarrow$ " spontaneously.

GOAL: allow both u & d in pair to be on respective F.S.

$\langle LOFF | \psi(x)\psi(x) | LOFF \rangle \sim e^{i 2\vec{q}\cdot\vec{x}}$

LOFF state breaks rotational & translational symmetry.

Next....

- Write $|LOFF\rangle$ ansatz.
- Vary $\rightarrow$ gap equation.
- For what $\delta\mu$ does LOFF win over BCS, Normal?

Wherever "LOFF plane wave" beats BCS & Normal, expect LOFF crystal to do better.

Eg: $\langle \psi\psi \rangle \sim \cos(2\vec{q}\cdot\vec{x})$

OR $\sim$ cubic crystal; or....

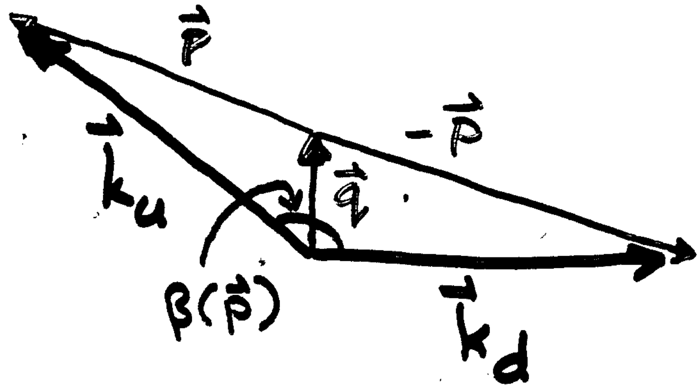
Determination of favored crystal structure $\rightarrow$ FUTURE WORK J. Bowers, KR in progress

LOFF KINEMATICS

Pair momenta:

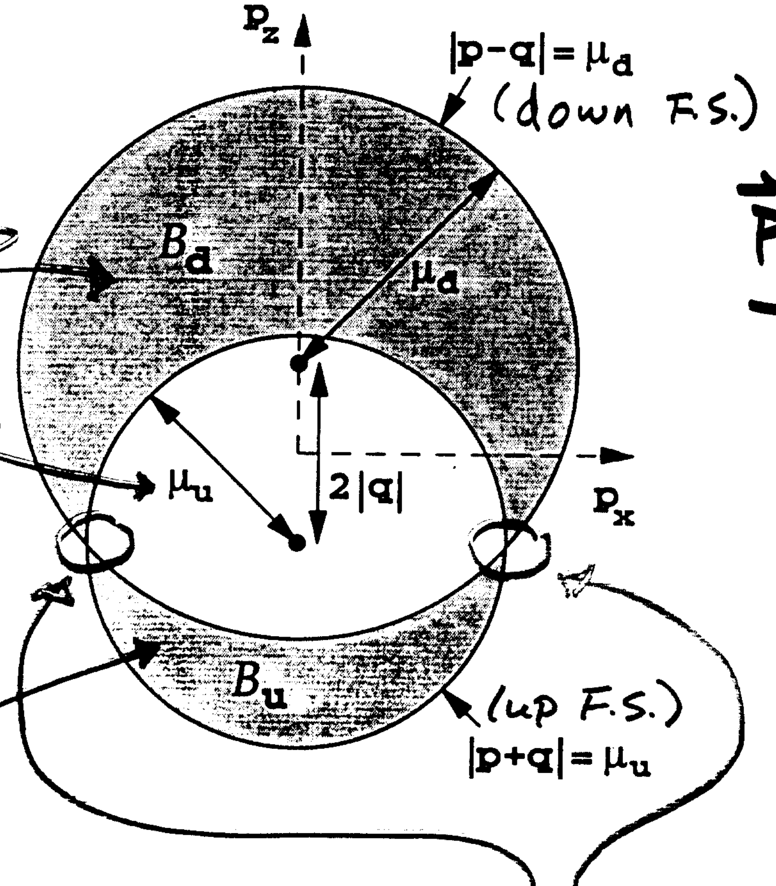
$(\vec{k}_u, \vec{k}_d)$

$= (\vec{p}+\vec{q}, -\vec{p}+\vec{q})$



Unpaired State

- -quarks only
- & u-quarks
- u-quarks only



$\vec{P}$ - space

Expect LOFF pairing here.

Back in $\vec{k}_u, \vec{k}_d$ - space, corresponds to a ring on up F.S. and a ring on down F.S.

Expect not to be able to form pairs (particle - particle; hole - hole) in B_u, B_d ....

LOFF GAP EQUATION

$$1 = G \int^{\Lambda}_{\substack{E_1(\vec{p})>0 \\ \& \, E_2(\vec{p})>0}} d^3p \, \frac{2\sin^2(\beta(\vec{p})/2)}{\sqrt{(|\vec{q}+\vec{p}| + |\vec{q}-\vec{p}| - \mu_u - \mu_d)^2 + 4\Delta^2 \sin^2(\beta/2)}}$$

Evaluate energy of elementary excitations:

$$E_1(\vec{p}) = \delta\mu + \frac{1}{2}\left(|\vec{q}+\vec{p}| - |\vec{q}-\vec{p}|\right) + \frac{1}{2}\sqrt{\quad}$$

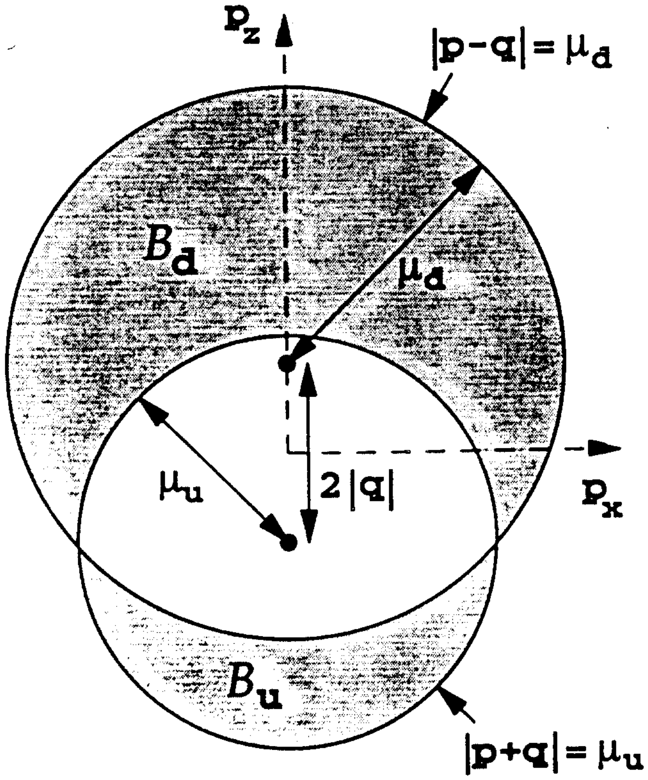
$$E_2(\vec{p}) = -\delta\mu - \frac{1}{2}\left(|\vec{q}+\vec{p}| - |\vec{q}-\vec{p}|\right) + \frac{1}{2}\sqrt{\quad}$$

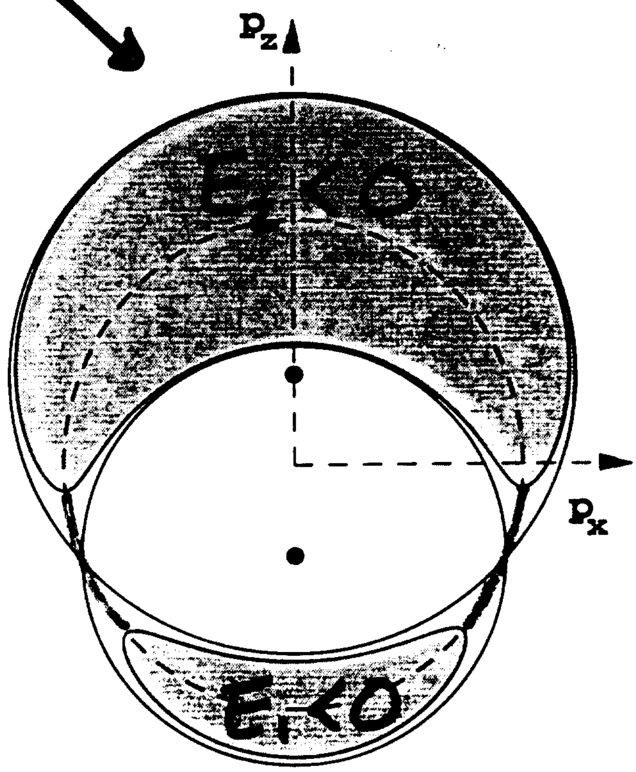
E_1 : cost of removing LOFF pair and replacing with up quark with $\vec{q}+\vec{p}$

E_2 : " " " " " " down " " $\vec{q}-\vec{p}$

FIND: There are regions in $\vec{p}$-space where $E_1 < 0$; $E_2 < 0$!!

Must go back and fix ansatz so that unpaired quarks occur in shaded region....





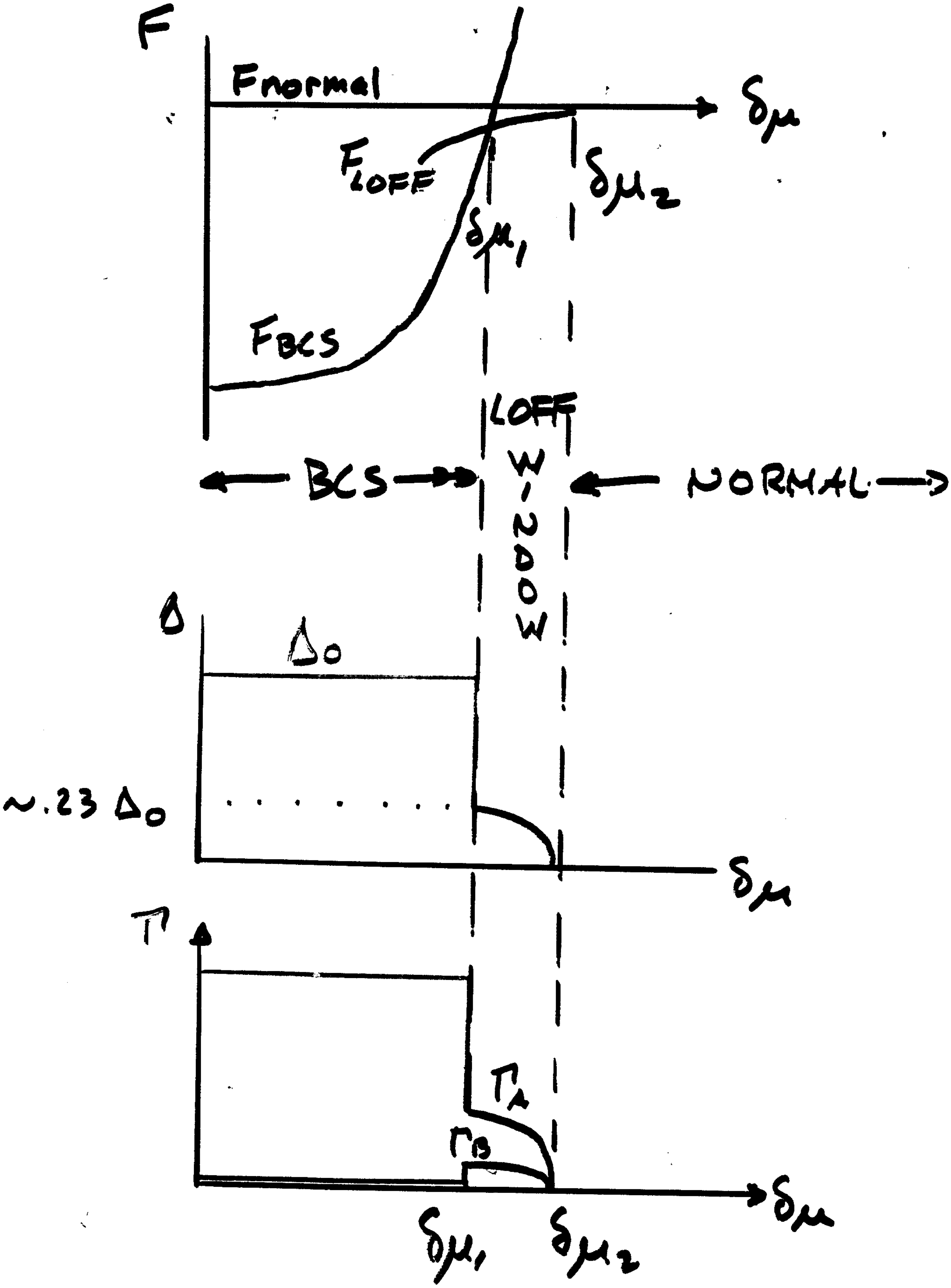
Green arc: Place where RHS of gap equation would diverge if Δ were 0.

<u>BUT</u> green arc disappears as $\Delta \to 0$

$\Rightarrow$ LOFF state not guaranteed to occur in weak-coupling limit at fixed $\delta\mu$.

(LOFF <u>is</u> guaranteed for $\Delta_0 \to 0$; $\frac{\delta\mu}{\Delta_0}$ = fixed)

LOFF vs. BCS vs. NORMAL



THE LOFF WINDOW

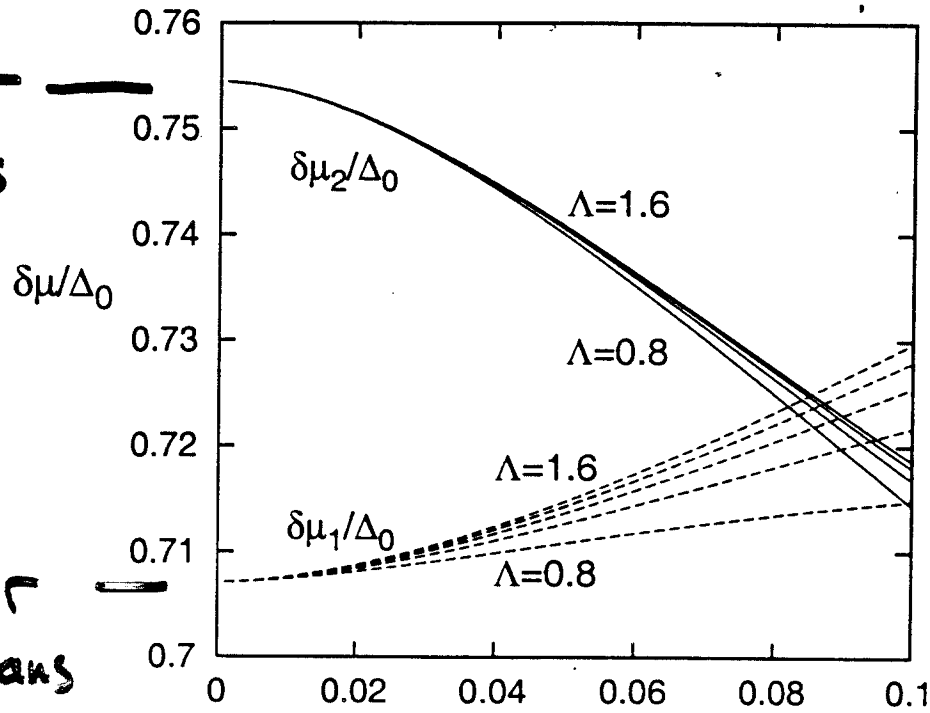
Normal

- 2nd Order
Phase Trans

LOFF

- 1st Order
Phase Trans

BCS



weak coupling — $\frac{\Delta_0}{\text{GeV}}$ — strong coupling

- stronger coupling favors BCS over LOFF
- results ~ Λ independent. (Good)

<u>Putting in #'s</u> (Just a game, without strange quarks)

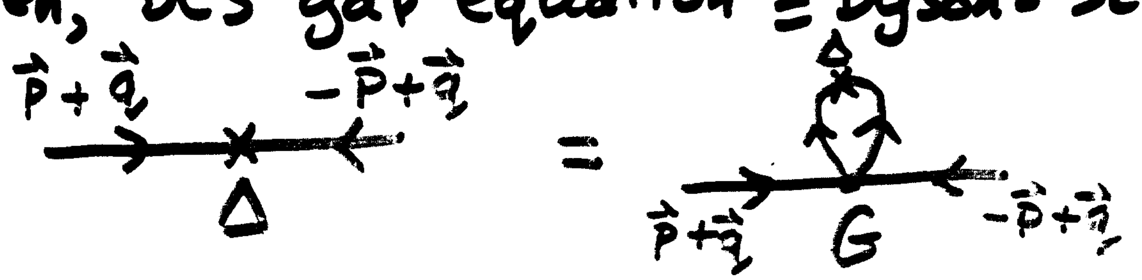
One reasonable example : $\delta\mu \sim 30$ MeV
$\Delta_0 \sim 40$ MeV

DIAGRAMMATIC REDERIVATION

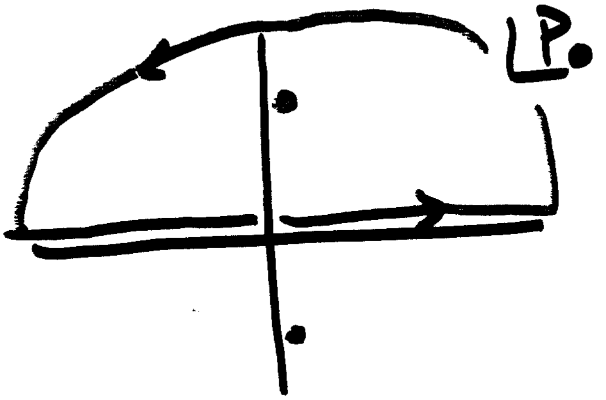
J Bowers, J Kundu, KR, E Shuster

Variational analysis provides much intuition, but is much less easily generalized than the equivalent diagrammatic analysis. Use Gorkov propagator, modified for $\vec{q} \neq 0$;

Then, BCS gap equation $\equiv$ Dyson-Schwinger:

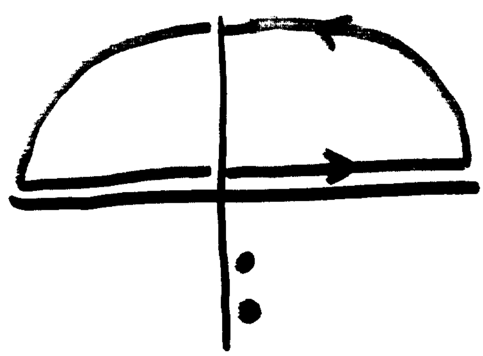


$$\rightarrow \cdots \rightarrow \quad 1 = G \int d^4p \, \frac{\sin^2(\beta/2)}{(p_0 - i E_1(\vec{p}))(p_0 + i E_2(\vec{p}))}$$



$E_1 > 0 \quad E_2 > 0$

vs.



$E_1 > 0 \quad E_2 < 0$

$\rightarrow$ blocking regions!

OPENING THE CRYSTALLINE COLOR SUPERCONDUCTIVITY WINDOW

A. Leibovich KR E. Shuster

Re-analyze, this time assuming quarks interact by gluon exchange

- small-angle scattering dominates
  → quasi-1D physics at F.S.
- in truly 1+1-dimensional calculation:

  $|\vec{q}| = \delta\mu$ → in 3D, spheres "kiss"

  and $\delta\mu_2/\Delta_0 \to \infty$!
- suggests LOFF window widens....
- note: this is a controlled, weak-coupling, first principles QCD calculation of a crystalline phase!
- What do we find? $\delta\mu_1$ unchanged and ....

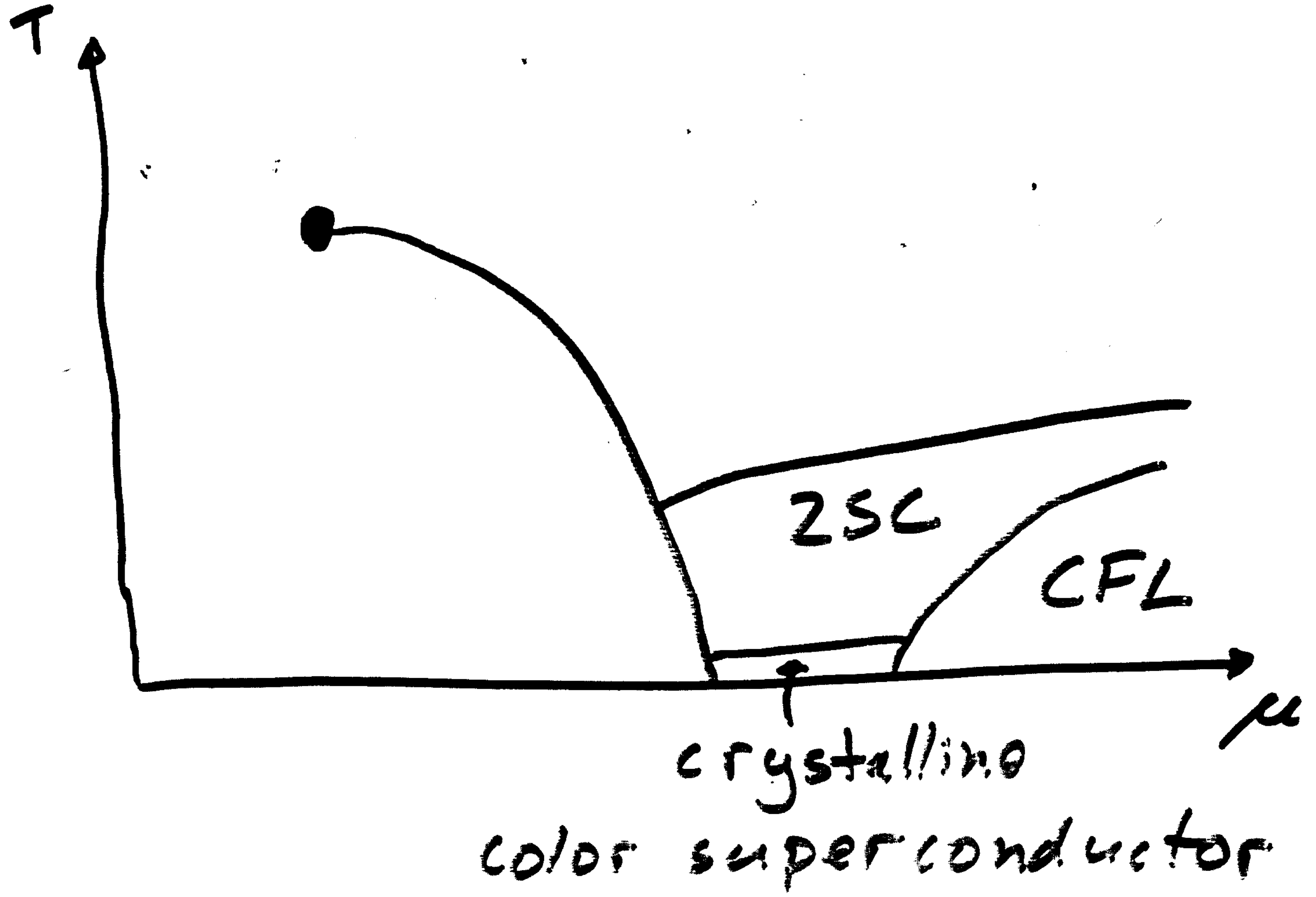
ecall: for point-like interaction, at
eak coupling: $\frac{|\vec{q}|}{\delta\mu} = 1.2$ $\underbrace{\frac{\delta\mu_1}{\Delta_0} = .71 \quad \frac{\delta\mu_2}{\Delta_0} = .75}_{\text{narrow window}}$

n contrast, with QCD interaction:

g	μ (MeV)	Δ_0 (MeV)	$\frac{\|\vec{q}\|}{\delta\mu}$	$\frac{\delta\mu_2}{\mu}$	$\frac{\delta\mu_2}{\Delta_0}$	$\frac{\delta\mu_1}{\Delta_0}$
43	400	107	1.01	.33	1.24	
25	10^3	38	1.001	.14	3.63	
.45	10^4	12	1	.025	20.6	
.98	10^6	8	1	.0017	225	0.71
.79	10^8	13	1	.00016	1254	
					! (cf. .75)	

ontrolled only for $g \lesssim 1$.
uggests greatly widened window, even
at much larger g, as at realistic μ.
ndicates crystalline color superconduct~~ivity~~
s GENERIC: occurs in any non-CFL
quark matter. Not just in narrow wind

PHASE DIAGRAM



- $\frac{m_s^2}{\mu\Delta}$ decreases at large μ; CFL wins their.
- No c.c.s. phase at all if there is "direct" nuclear → CFL transition
- T_c for c.c.s. phase is $\sim 0.49\Delta$, $\sim 0.1\Delta_0 \sim$ few MeV.
- c.c.s. window assumed not narrow in above figure, as suggested by asymptotic analysis.

CRYSTALLINE COLOR SUPERCONDUCT[ivity]
IN COMPACT (NEUTRON) STARS

1. If core of neutron star is quark matter, then it IS a color supercond.

   $T_{star} \sim \text{keV} \lll T_c$

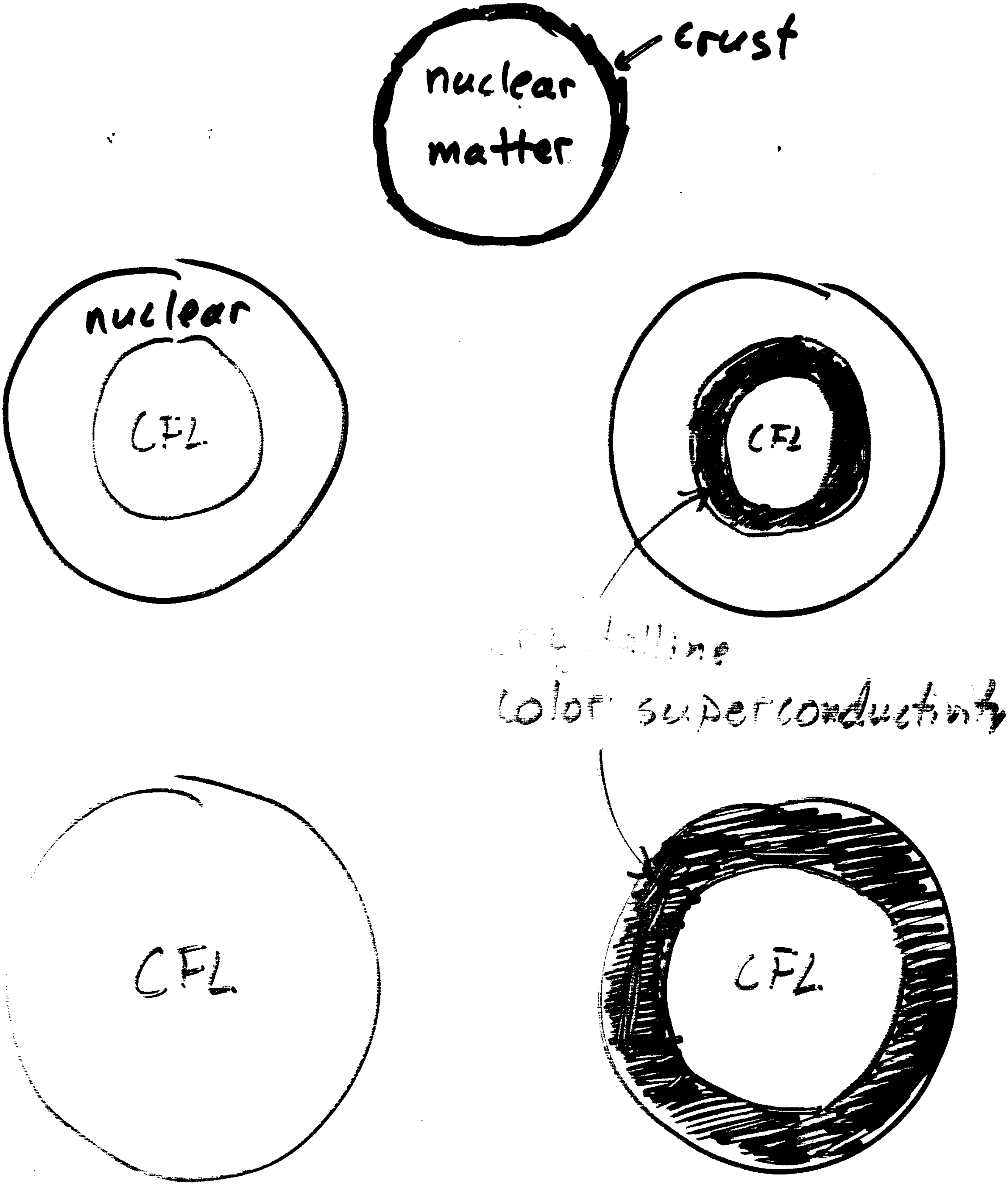
2. Not known whether neutron stars have quark matter cores.
3. GOAL: understand observational consequences, so we can find out.
4. I will not describe the half-dozen ideas various groups are studying. Focus today is LOFF. → GLITCHES?
5. As a function of increasing depth,

   μ rises
   $\delta\mu$ decreases
   Δ changes. (rises?)

   Numbers I gave before ~ reasonable.

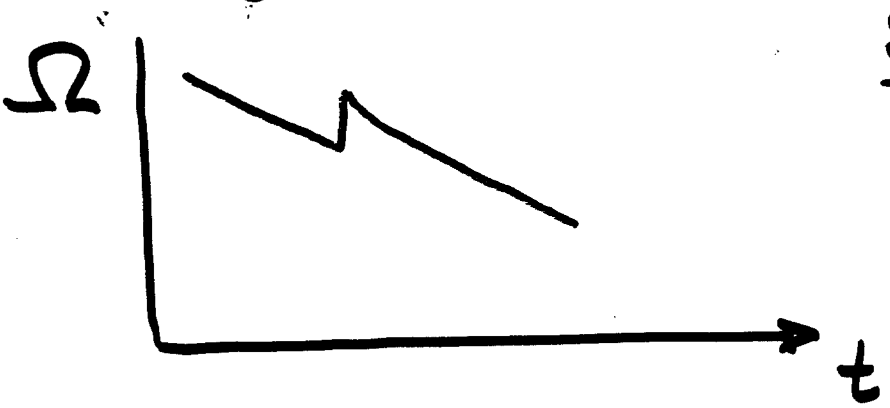
⇒ could have $\frac{\delta\mu}{\Delta} \in$ LOFF WINDOW in a shell.

SEVERAL SCENARIOS



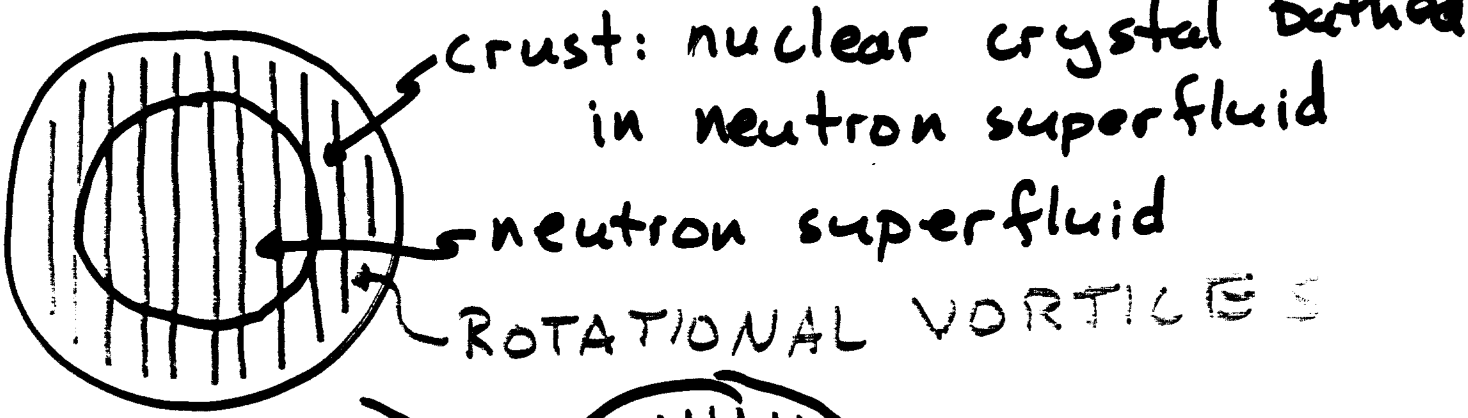
GLITCHES

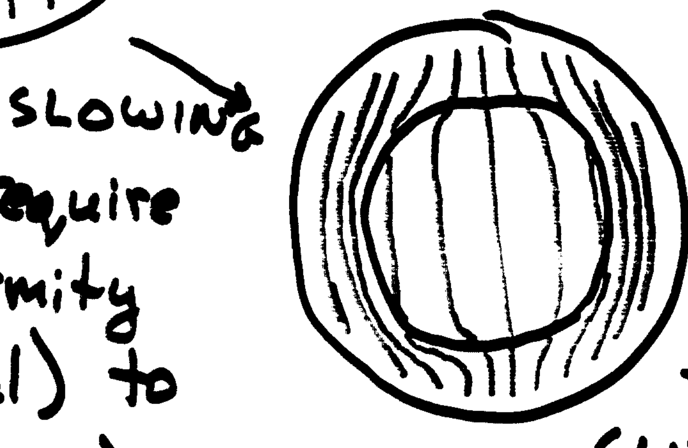
Pulsars glitch:



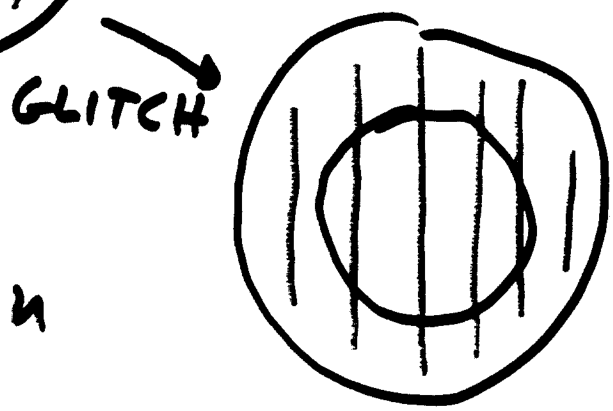
$$\frac{\delta\Omega}{\Omega} \sim 10^{-9} \to 10^{-6}$$

Conventional mechanism:





Glitches require non-uniformity (ie crystal) to impede (pin) motion of vortices.
∴ thought impossible in QM.

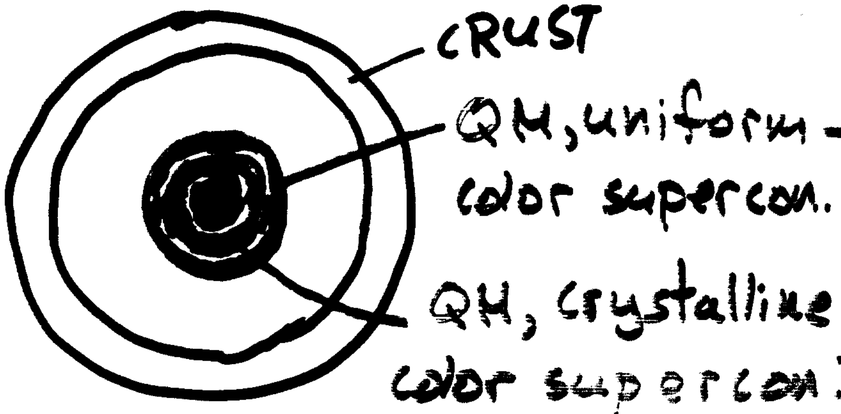


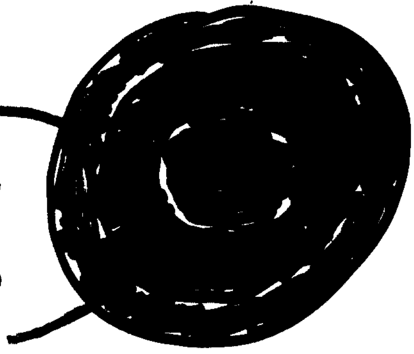
GLITCHES IN QUARK MATTER?

crystalline condensate will pin vortices.
vortices prefer nodes of LOFF crystal
crude estimates (dimensional analysis
using F_{LOFF} & q) suggest that
pinning force ~ comparable to that
in nuclear crust. (Real calculation: future

WO IDEAS:

neutron star | strange quark star





arger glitches from
rust; smaller glitches
from QM core.

May glitch!
Could pulsars be
strange quark stars
after all?

Real glitch phenomenology (eg are QM
glitches same or different from crustal
glitches) awaits better understanding of.

CONCLUDING REMARKS

More theoretical work needed before we can decide how best to use neutron star phenomena:

- observables related to equation of state
- supernova ν's time distribution
- LIGO wave forms
- ν-cooling of compact stars
- $\vec{B}$-evolution
- r-mode instability
- glitches

o learn whether they feature cores nade of:

- baryonic matter
- CFL quark matter
- less symmetrically paired quark matter with crystalline color superconductivity

Already now, studying cold
ense STRANGE quark matter
as provided us with a remarkable
ew theoretical laboratory:
A phase in which chiral symmetry
broken, the particles are those
f a confined phase, and
et, for large μ the coupling is
eak and we can calculate, e.g.,
arameters in effective theory
escribing Goldstone bosons,
assive vector bosons, and
uark/baryon quasiparticles.

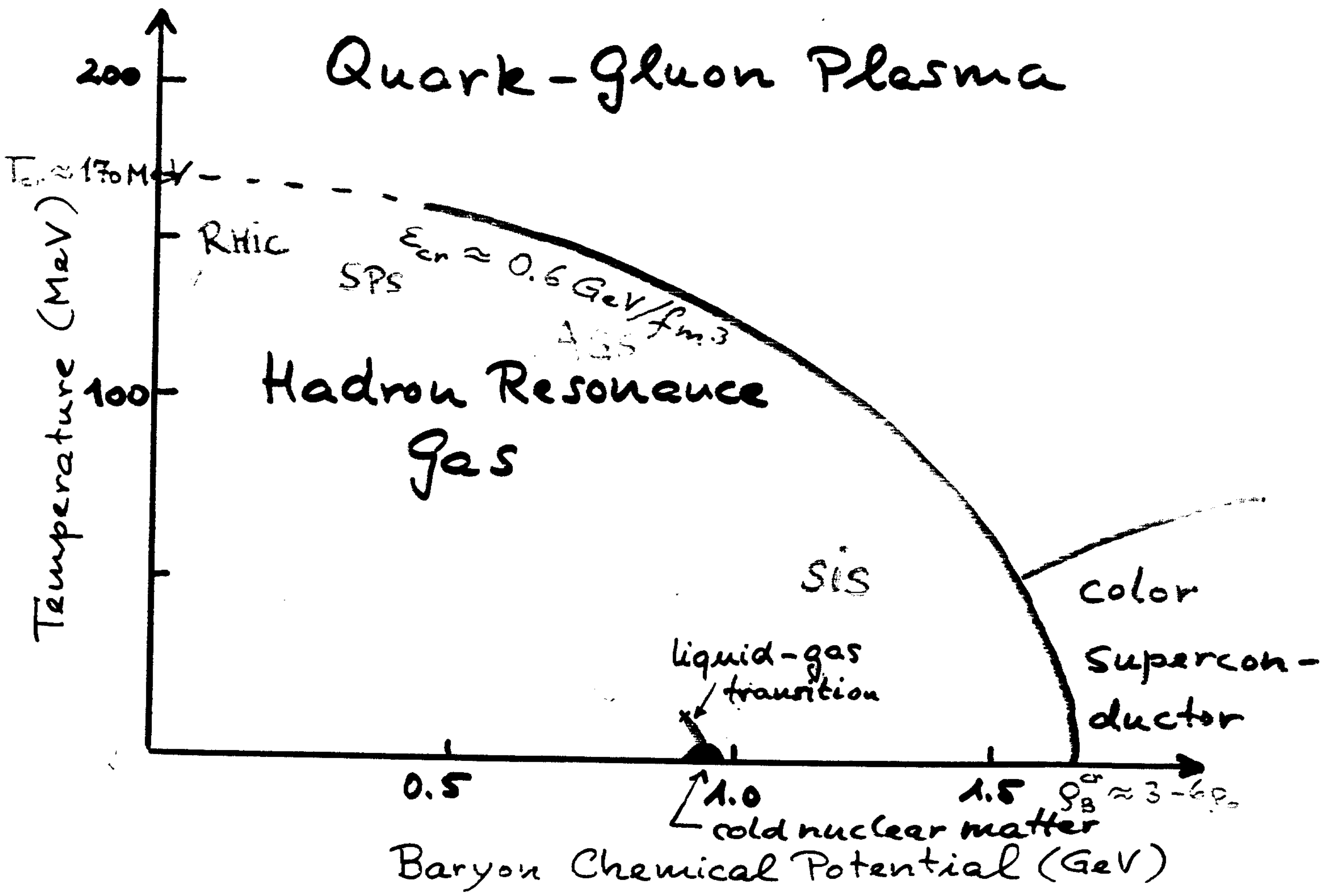
$m_{\pi, K, \eta}$ & $f_{\pi, K, \eta}$ now
alculated from first principles
for $g \to 0$, $\mu \to \infty$.

Casalbuoni, Gatto; Son, Stephanov; Rho, Wirzba, Zahed
Hong, Lee, Min; Manuel, Tytgat; Zarembo;
Beane, Bedaque, Savage; ....

QCD @ Work, Martina Franca, 17.6.2001

From SPS to RHIC: Breaking the Barrier to the Quark-Gluon Plasma

Ulrich Heinz
(Ohio State University)



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