Rare Hadronic B Decays from QCD

Matthias Neubert – Cornell University

(neubert@mail.lns.cornell.edu)

Based on collaborations with:
M. Beneke, G. Buchalla, C. Sachrajda
A. Kagan; J. Rosner
T. Becher, B. Pecjak

(Martina Franca – 18 June 2001)

Introduction

* observation of CP violation in K–$\bar{K}$ mixing (ϵ_K) in 1964 showed that CP is not a symmetry of Nature, but left open the question about the origin of CP violation (“superweak” interactions?)

* confirmation of CP violation in $K \to \pi\pi$ decays (“direct CP violation”, parameter ϵ') in 1999 proved that CP is violated in flavor-changing charged-current interactions, as predicted by the SM

* CKM mechanism relates all CP-violating quantities to a single parameter $\gamma = \delta_{\rm CKM}$

  → very predictive!

  → expect large CP asymmetries in some B decays

* important: B system admits a solid theoretical analysis, since $m_b \gg \Lambda_{\rm QCD}$

  → strong-interaction effects can be dealt with using heavy-quark expansions, i.e., expansions in powers of $\alpha_s(m_b) \ll 1$ and $\Lambda_{\rm QCD}/m_b \ll 1$

Cabibbo–Kobayashi–Maskawa matrix:

$$\boxed{\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}}$$

3×3 unitary matrix connecting mass eigenstates of down-type quarks with interaction eigenstates

* described by 4 real parameters (3 angles, 1 phase)

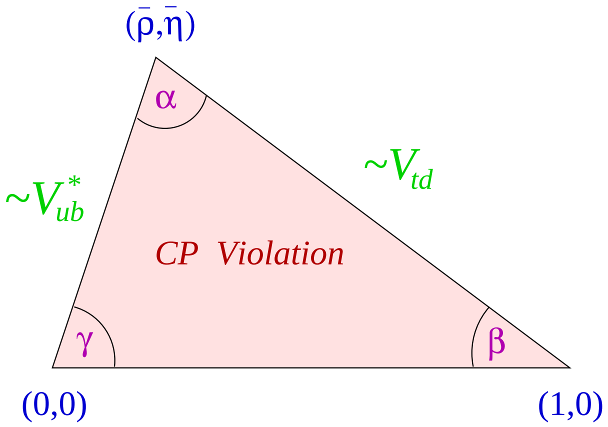
Wolfenstein parameterization:

$$V_{\text{CKM}} = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$

* accurately known: $|V_{us}|$ and $|V_{cb}|$ (λ and A)
* more uncertain: $|V_{ub}|$ and $|V_{td}|$ (ρ and η)
* complex phases appear in smallest matrix elements (requires at least 3 generations)

Unitarity triangle:

$$\boxed{V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0}$$

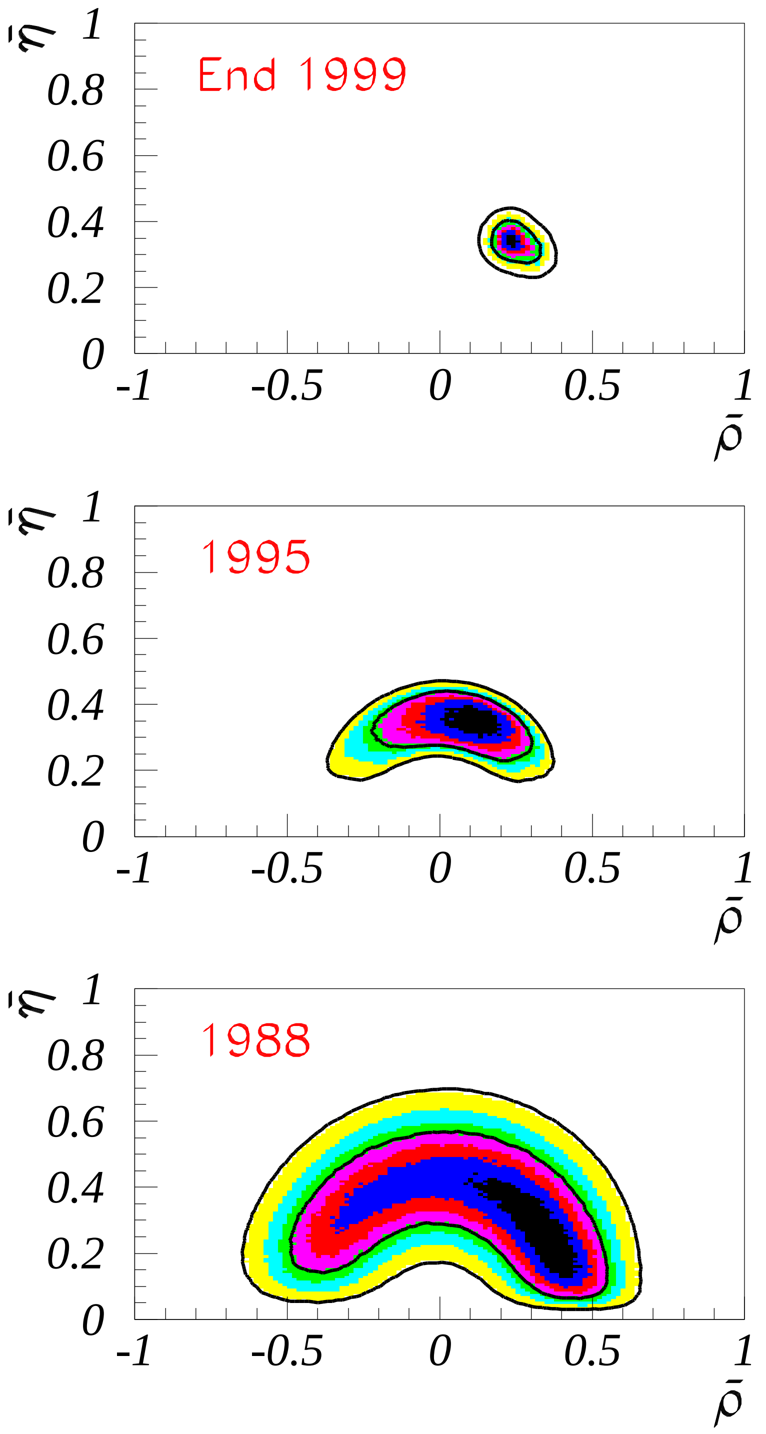


* combining measurements of $|V_{ub}|$ in semileptonic decays, $|V_{td}|$ in B–$\bar{B}$ mixing, and Im(V_{td}^2) in K–$\bar{K}$ mixing, the parameters of the unitarity triangle are determined already with great accuracy

  $\rightarrow$ has established existence of a complex phase in the top sector (V_{td})

* however, precise determination of $\bar\rho$ and $\bar\eta$ in itself is now only one of many goals; focus has shifted to testing the consistency of the CKM picture

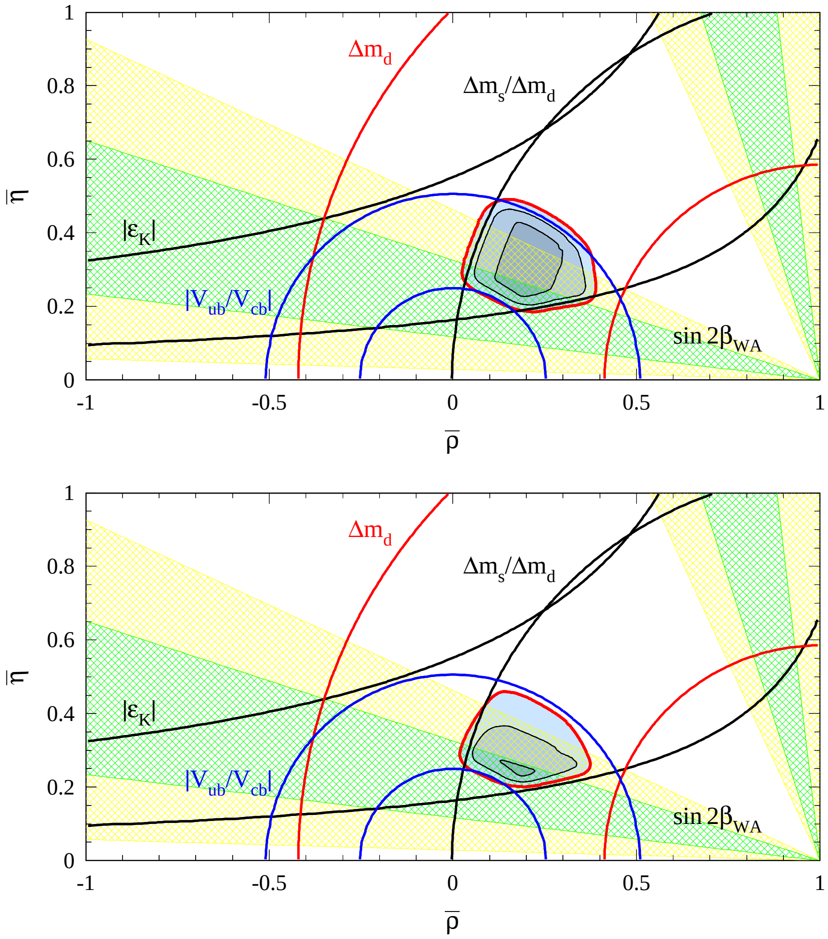
  $\rightarrow$ need to measure complex phase in the bottom sector (V_{ub})



(Caravaglios et al., 2000)

* measurements of $\sin 2\beta$ have been reported by OPAL ($3.2\,^{+1.8}_{-2.0} \pm 0.5$), CDF ($0.79\,^{+0.41}_{-0.44}$), ALEPH ($0.84\,^{+0.82}_{-1.04} \pm 0.16$), and recently by Belle ($0.58\,^{+0.32}_{-0.34}\,^{+0.09}_{-0.10}$) and BaBar ($0.34 \pm 0.20 \pm 0.05$)

$\rightarrow$ current world average $\sin 2\beta = 0.48 \pm 0.16$ is beginning to have an impact on the global fit of the unitarity triangle:



(Höcker et al., 2001)

Exploring New Flavor Physics:

* What if $\sin 2\beta_{\psi K}$ is low? What if there is New Physics affecting the particle–antiparticle mixing amplitudes of B_d, B_s and K mesons?

  $\rightarrow$ then most of the standard constraints on the unitarity triangle (ϵ_K, Δm_d, Δm_s, $\sin 2\beta_{\psi K}$) could be contaminated and would not determine the "true" CKM triangle!

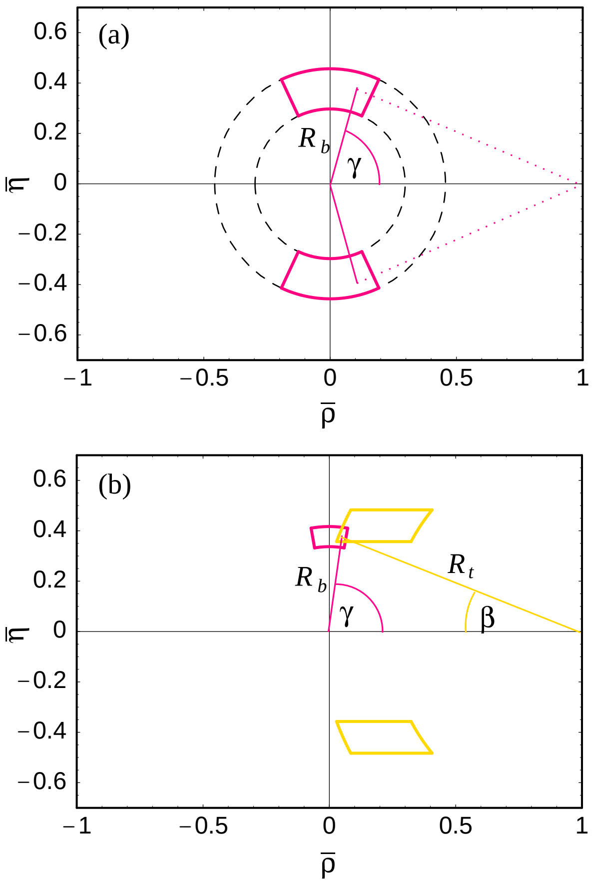
* need a reference triangle constructed independently of any information from mixing (Kagan, M.N., 2000)

  B system: measure $|V_{ub}|$ in semileptonic B decays, and $\gamma = \text{arg}(V_{ub}^*)$ using a variety of methods (e.g., charmless hadronic decays, $B \to DK$ decays, B_s decays)

  K system: extract $|V_{td}|$ and $\text{Im}(V_{td})$ from $K \to \pi\nu\bar{\nu}$ decays

* once reference triangle is known, one can explore separately potential New Physics contributions to mixing in the B_d, B_s and K systems

* knowledge of γ is the key ingredient that makes this strategy feasible and powerful!



Reference triangle in the near and long-term future:

(a) B-decay triangle with two-fold ambiguity, assuming uncertainties of 20% in $|V_{ub}/V_{cb}|$ and $\pm 25^\circ$ in γ (near-term).

(b) B-decay triangle (pink) with no ambiguity, assuming uncertainties of 10% in $|V_{ub}/V_{cb}|$ and $\pm 10^\circ$ in γ, and K-decay triangle (gold) with four-fold ambiguity, assuming 15% uncertainties in R_t and $|\eta|$ (long-term).

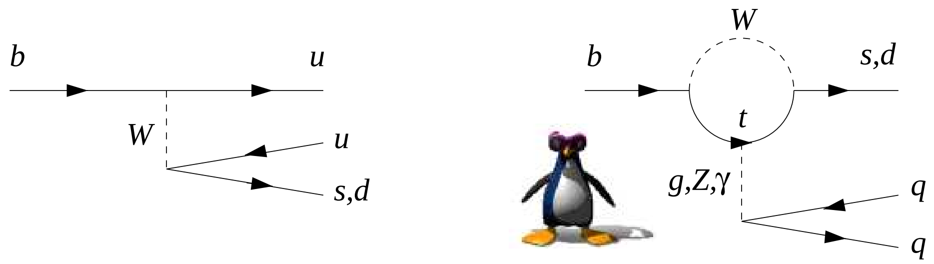
Charmless Hadronic B Decays

* recent experimental data on charmless hadronic B decays from CLEO, BaBar and Belle have caused a lot of excitement in the theory community

  $\rightarrow$ focus on $B \to \pi K$ and $B \to \pi\pi$ decays, which at present are best understood theoretically

* in general, sensitivity to CP-violating "weak" phases requires sizable interference of decay topologies which differ in their CKM parameters

* in charmless hadronic B decays, there is significant interference of tree and penguin topologies!



Tree	Penguin	Ratio
$V_{ub}V_{us}^* \sim \lambda^4\, e^{-i\gamma}$	$V_{tb}V_{ts}^* \sim \lambda^2$	$\lvert T/P\rvert \sim 0.2$
$V_{ub}V_{ud}^* \sim \lambda^3\, e^{-i\gamma}$	$V_{tb}V_{td}^* \sim \lambda^3\, e^{i\beta}$	$\lvert P/T\rvert \sim 0.3$

* implies potentially large CP asymmetries, e.g.:

$$A_{\mathrm{CP}}(B^\pm \to \pi^0 K^\pm) \approx \underbrace{2\left|\frac{T}{P}\right| \sin\gamma}_{\approx 0.4} \underbrace{\sin\delta_{\mathrm{st}}}_{\text{strong phase}}$$

* sensitivity to γ also in CP-averaged rates, e.g.:

$$R_* = \frac{\Gamma(B^\pm \to \pi^\pm K^0)}{2\Gamma(B^\pm \to \pi^0 K^\pm)} \approx 1 + 2\left|\frac{T}{P}\right| \cos\gamma \cos\delta_{\mathrm{st}}$$

* varying $-1 \le \cos\delta_{\mathrm{st}} \le 1$ yields bounds on $\cos\gamma$, provided that $R_* \ne 1$:

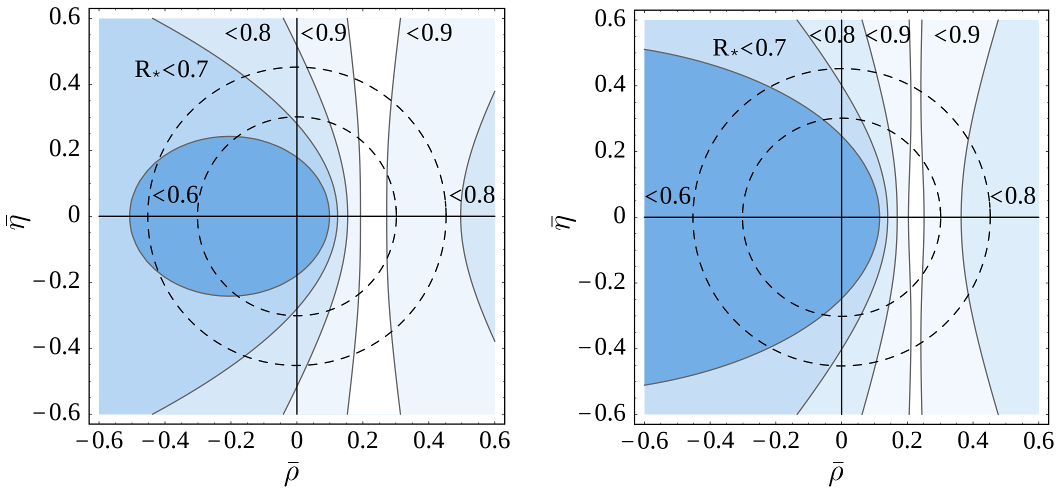
  $\to$ Fleischer–Mannel bound ($R_{\mathrm{FM}} = 1.06 \pm 0.18$)

  $\to$ Neubert–Rosner bound ($R_* = 0.71 \pm 0.14$)

* in some cases, one can use symmetries to eliminate hadronic uncertainties

NR bound:

- extract relevant ratio $\varepsilon = |T/P|$ from data ($\varepsilon_{\rm exp} = 0.22 \pm 0.03$)
- calculate small electroweak penguin contribution using Fierz relations and SU(3) symmetry



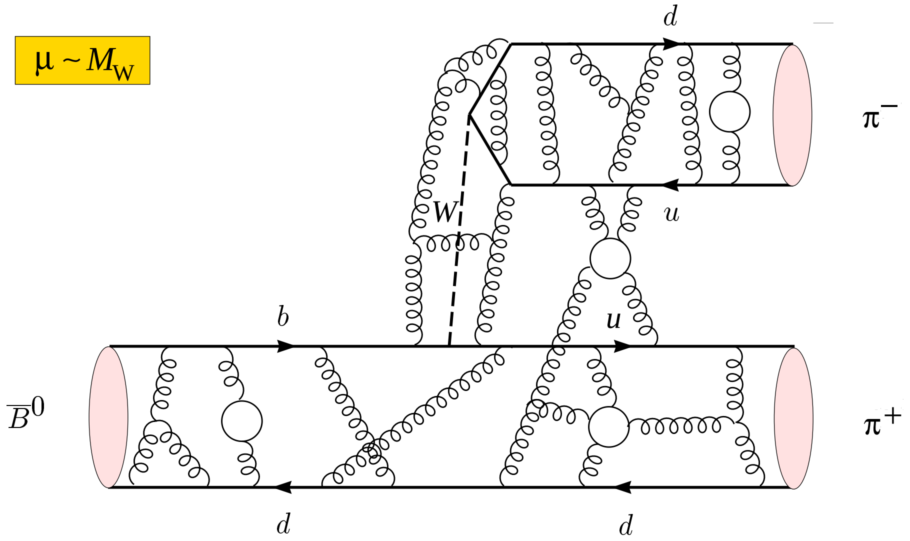
($\varepsilon_{\rm exp} < 0.19$) ($\varepsilon_{\rm exp} < 0.25$)

- to do better, need a theory of hadronic B decays

$\rightarrow$ recent progress: heavy-quark expansion for nonleptonic weak decays
(very active new field in heavy-quark theory!)

QCD Factorization Formula

- theoretical description of hadronic weak decays is difficult due to nonperturbative hadronic dynamics
- the problem:



- relevant mass scales:

M_W

↓ local OPE:
soft gluon effects are power suppressed in $1/M_W$

m_b

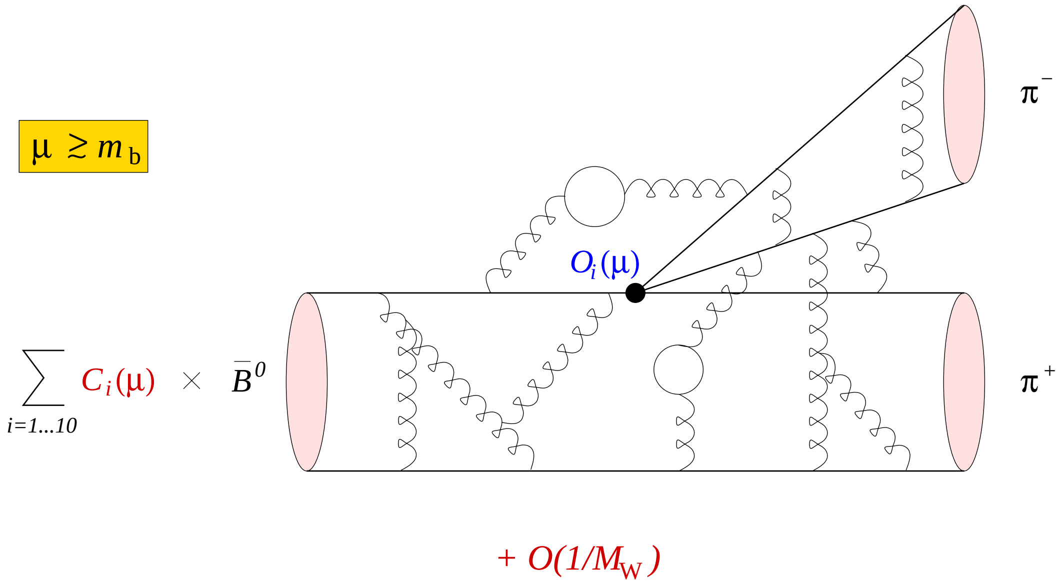
↓ QCD factorization: color transparency!
soft gluon effects are power suppressed in $1/m_b$

Λ_{QCD}

First step:

* hard gluon effects with $\mu \gg m_b$ can be calculated and factorized into the Wilson coefficients $C_i(\mu)$ of an effective weak Hamiltonian:

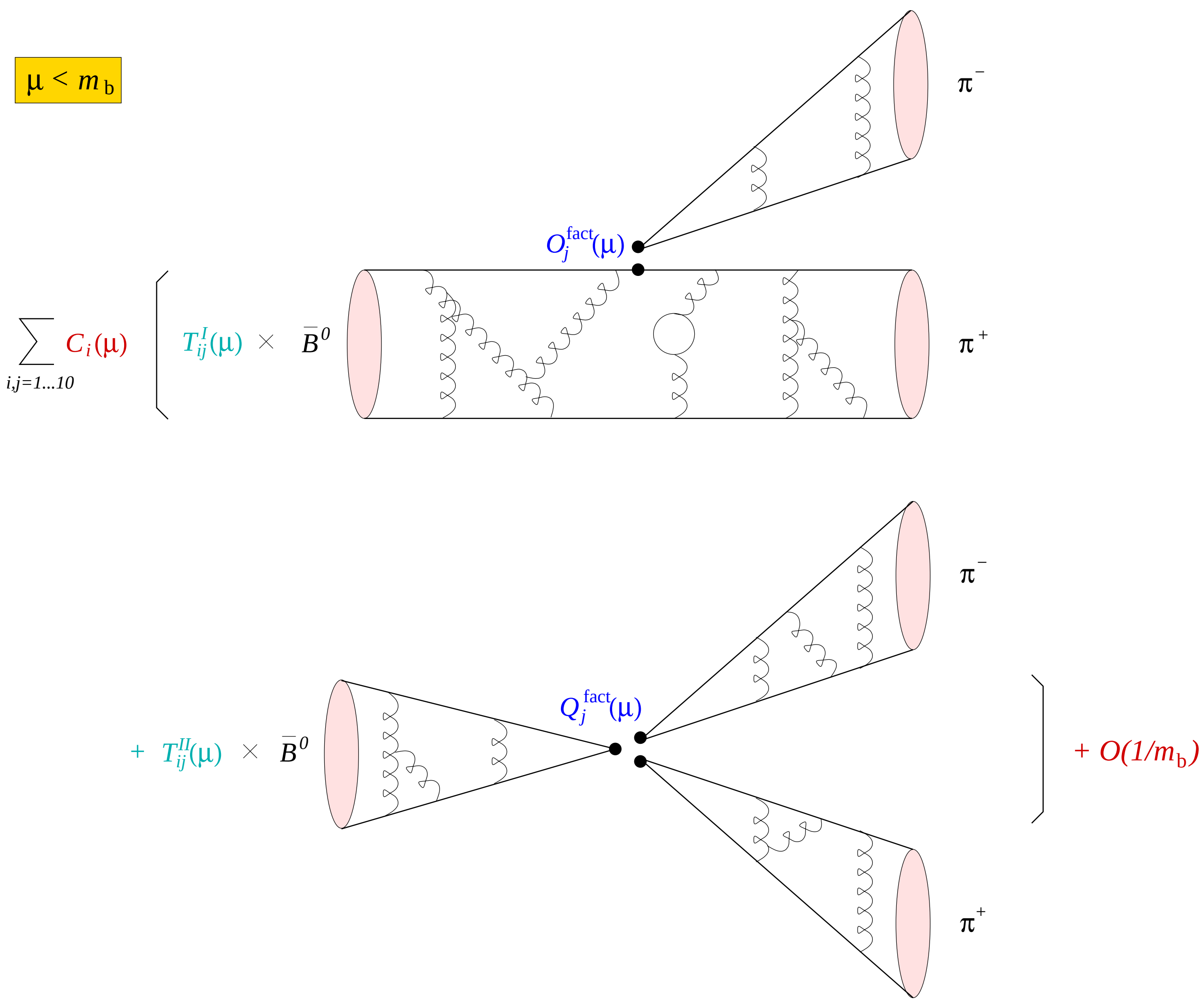
$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_i \lambda_i^{\text{CKM}}\, C_i(\mu)\, O_i(\mu)$$



* difficulty is to calculate hadronic matrix elements of local operators $O_i(\mu)$

Second step:

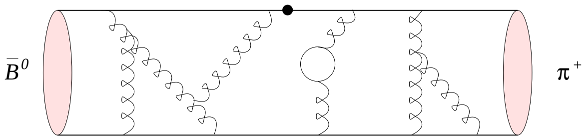
* hard gluon effects with $\mu \sim m_b$ can be calculated and factorized into hard-scattering kernels $T_{ij}(\mu)$

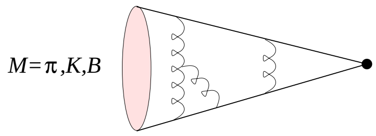


* soft (and collinear) nonfactorizable gluons decouple because of color transparency!

Nonperturbative input:

* semileptonic $B \to \pi, K$ form factors, meson decay constants and light-cone distribution amplitudes





$\to$ extract these quantities from experimental data on semileptonic $B \to M\, l\, \nu$ decay, leptonic $M \to l\, \nu$ decay, other hard exclusive processes, and theoretical calculations (lattice, QCD sum rules, ...)

QCD Factorization Formula:

* allows for a systematic, model-independent calculation of corrections to "naive" factorization, which emerges as leading term in heavy-quark limit
* strong FSI and rescattering phases are calculable and are perturbative or power suppressed (soft rescattering vanishes in the heavy-quark limit)

Important implication:

* fact that $\cos\delta_{\rm st} = 1$ up to second-order corrections to the heavy-quark limit can be used to turn the NR bound into a determination of γ
* theory predicts $\delta_{\rm st} = -(10 \pm 15)^\circ$ and $\cos\delta_{\rm st} > 0.9$

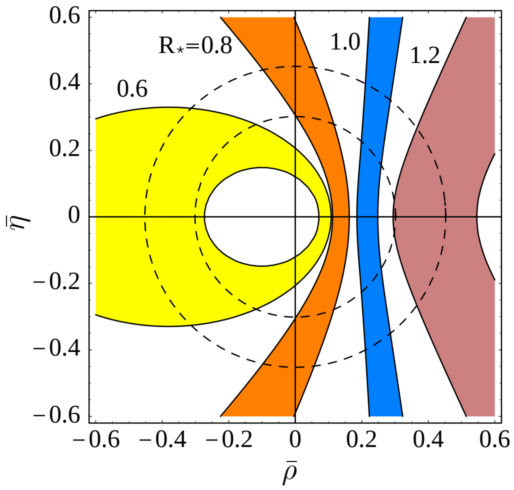
  $\rightarrow$ to be conservative, allow $0.8 \le \cos\delta_{\rm st} \le 1$
* factorization formula also allows a systematic calculation of "nonfactorizable" SU(3) violations to the NR bound, which are found to be small

  $\rightarrow$ residual corrections are highly suppressed:

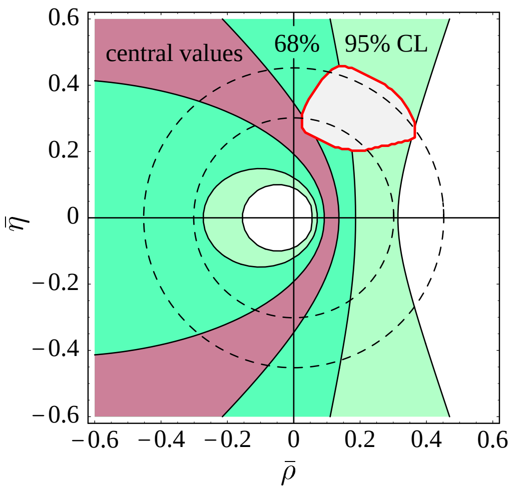
$$\mathcal{O}\bigg(\frac{1}{N_c}\cdot\frac{\Lambda_{\rm QCD}}{m_b}\cdot\frac{m_s - m_d}{\Lambda_{\rm QCD}}\bigg) \sim \text{few }\%$$

  $\rightarrow$ dominant power corrections can be estimated (see below)
* once R_* and $\varepsilon_{\rm exp}$ are measured with small errors, this will enable a determination of γ up to a sign ambiguity, and with small theoretical errors

$*$ Resulting constraint in the $(\bar{\rho}, \bar{\eta})$ plane:



$(\varepsilon_{\text{exp}} = 0.22)$



(current data)

$*$ for $0.8 < R_* < 1.1$ (range preferred by SM), the resulting band in the $(\bar{\rho}, \bar{\eta})$ plane is narrow

$\rightarrow$ theoretical uncertainties less than for all other constraints on $\bar{\rho}$ and $\bar{\eta}$ except for $\sin 2\beta$!

$*$ other values of R_* would indicate new flavour physics beyond the SM

$*$ with present data, the SM is still in good shape, although the central value of R_* prefers larger γ

Power Corrections

* factorization formula gives model-independent predictions for decay amplitudes and their phases in heavy-quark limit
  - valid to all orders of perturbation theory
  - corrections suppressed by powers of $\Lambda_{\mathrm{QCD}}/m_b$
* naive dimensional analysis suggest power corrections $< 10\%$, but they could be enhanced in some cases due to presence of small parameters other than $\Lambda_{\mathrm{QCD}}/m_b$ (CKM elements, Wilson coefficients,...)
* conceptual challenge:
  unlike for HQET, these power corrections cannot be analyzed using the operator product expansion
  (cf.: $e^+e^- \to$ jets, event shape variables)
* some classes of potentially large power corrections can be identified and their effects estimated

"Chirally-enhanced" power corrections:

* some power corrections scale like:

$$\frac{1}{m_b} \times \frac{2m_K^2}{m_u + m_s} \approx \frac{1}{m_b} \times \frac{2m_\pi^2}{m_u + m_d} \approx 0.8$$

  $\rightarrow$ this happens, e.g., for some matrix elements of penguin operators (important for $B \to \pi K$)

* all such terms can be identified and related to twist-3 light-cone distribution amplitudes

* these twist-3 penguin amplitudes factorize and can be consistently calculated!

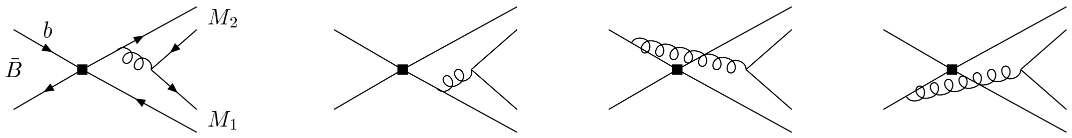
* however, related twist-3 corrections to twist-2 hard spectator contributions exist, which spoil factorization ($\rightarrow$ logarithmic endpoint divergences)

* this is entirely expected!

  $\rightarrow$ for phenomenology, calculate divergent integrals using a cutoff (e.g., transverse momentum), and assign a 100% (complex) theoretical uncertainty

Weak annihilation contributions:

* Keum, Li, Sanda have argued that annihilation topologies may be largely enhanced:



(a) (b) (c) (d)

* these effects are power suppressed and violate factorization

  → estimate "annihilation kernels" using a cutoff, and assign a conservative (complex) error

  → find moderate corrections ($\sim 10\%$), but with large uncertainty

Other power corrections:

* several recent estimates based on:
  - light-cone QCD sum rules (Khodjamirian, 2001)
  - renormalon calculus (Becher, M.N., Pecjak, 2001; Burrell, Williamson, 2001)

→ find corrections $< 10\%$, as suggested by naive dimensional analysis

Tests of QCD factorization

- important to test QCD factorization in decays of $B \to 2$ light mesons in a way that is insensitive to γ
- $B^\pm \to \pi^\pm \pi^0$ ($\simeq$ pure tree) and $B^\pm \to \pi^\pm K^0$ ($\simeq$ pure penguin) are best suited for this purpose, because they have negligible amplitude interference

Absolute predictions for branching ratios:

$$\text{Br}(B^\pm \to \pi^\pm \pi^0) = \left[5.3\,{}^{+0.8}_{-0.4}\,(\text{pars.}) \pm 0.3\,(\text{power})\right] \cdot 10^{-6} \times \left[\frac{|V_{ub}|}{0.0035}\,\frac{F_0^{B\to\pi}(0)}{0.28}\right]^2$$

$$\text{Exp.} = (5.6 \pm 1.5) \cdot 10^{-6}$$

$$\text{Br}(B^\pm \to \pi^\pm K^0) = \left[14.1\,{}^{+6.4}_{-4.0}\,(m_s)\,{}^{+8.1}_{-3.6}\,(\text{power})\right] \cdot 10^{-6} \times \left[\frac{F_0^{B\to\pi}(0)}{0.28}\right]^2$$

$$\text{Exp.} = (17.2 \pm 2.5) \cdot 10^{-6}$$

- sensitivity to semileptonic form factors can be eliminated by taking ratios

First test of factorization:

$$\frac{\Gamma(B^\pm\to\pi^\pm\pi^0)}{d\Gamma(\bar B^0\to\pi^+l^-\bar\nu)/dq^2|_{q^2=0}} = 3\pi^2 f_\pi^2 \underbrace{|a_1^{(\pi\pi)}+a_2^{(\pi\pi)}|^2}_{1.33\,{}^{+0.20}_{-0.11}\,(\text{pars.})\pm 0.07\,(\text{power})}$$

$$= (0.68\,{}^{+0.11}_{-0.06})\ \text{GeV}^2$$

$\to$ independent of weak annihilation and $|V_{ub}|$!

* probes factorization in $B\to\pi\pi$, including the sizeable ($\sim 25\%$) contribution of the hard scattering term in the factorization formula
* verification must await precise experimental data

Second test of factorization:

$$\varepsilon_{\rm exp} = \left|\frac{T}{P}\right| = \tan\theta_C\,\frac{f_K}{f_\pi}\left[\frac{2\text{Br}(B^\pm\to\pi^\pm\pi^0)}{\text{Br}(B^\pm\to\pi^\pm K^0)}\right]^{\frac{1}{2}} = 0.223\pm 0.034$$

$\to$ independent of form factors, but $\propto |V_{ub}/V_{cb}|$

* probes tree-to-penguin ratio and is sensitive to (penguin) annihilation contribution
* theoretical prediction agrees well with data:

$$\boxed{\varepsilon = 0.24\pm 0.04(\text{pars.})\pm 0.04(\text{power})\pm 0.05(V_{ub})}$$

Probing γ in $B \to \pi K, \pi\pi$ decays

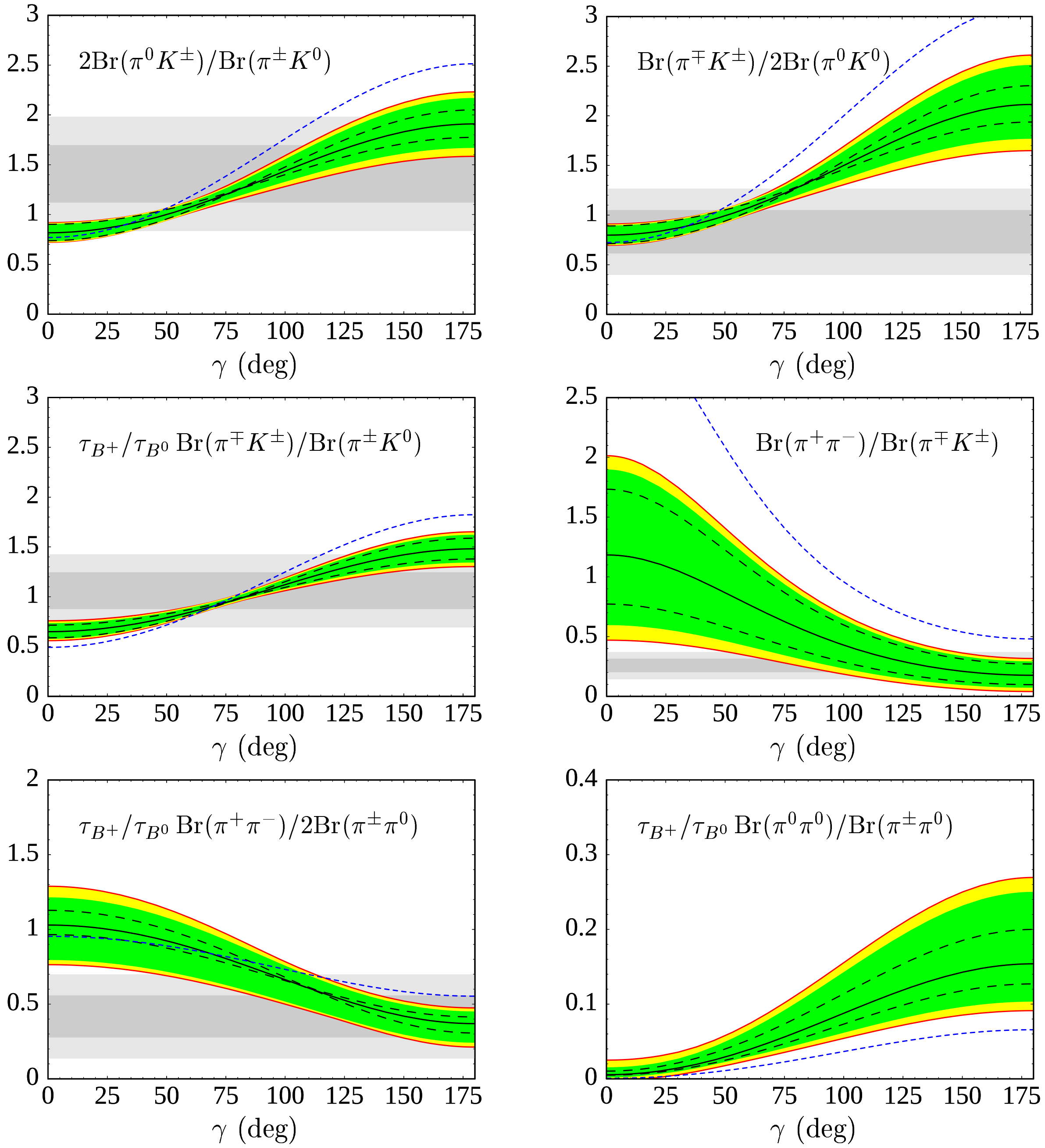
* factorization formula makes large class of rare, hadronic B decay modes accessible to a systematic, theoretical description

* basis for analysis of recent experimental data on CP-averaged branching ratios (in units of 10^{-6}):

Decay Mode	CLEO (PRL 2000)	BaBar (March 2001)	Belle (Feb. 2001)	Average
$B^0 \to \pi^+\pi^-$	$4.3\,^{+1.6}_{-1.4} \pm 0.5$	$4.1 \pm 1.0 \pm 0.7$	$5.9\,^{+2.4}_{-2.1} \pm 0.5$	4.4 ± 0.9
$B^\pm \to \pi^\pm\pi^0$	$5.6\,^{+2.6}_{-2.3} \pm 1.7$	$5.1\,^{+2.0}_{-1.8} \pm 0.8$	$7.1\,^{+3.6}_{-3.0}\,^{+0.9}_{-1.2}$	5.6 ± 1.5
	< 12.7 (90% C.L.)	< 9.0 (90% C.L.)	< 12.6 (90% C.L.)	
$B^0 \to \pi^\mp K^\pm$	$17.2\,^{+2.5}_{-2.4} \pm 1.2$	$16.7 \pm 1.6\,^{+1.2}_{-1.7}$	$18.7\,^{+3.3}_{-3.0} \pm 1.6$	17.2 ± 1.5
$B^\pm \to \pi^0 K^\pm$	$11.6\,^{+3.0}_{-2.7}\,^{+1.4}_{-1.3}$	$10.8\,^{+2.1}_{-1.9}\,^{+1.0}_{-1.2}$	$17.0\,^{+3.7}_{-3.3}\,^{+2.0}_{-2.2}$	12.1 ± 1.7
$B^\pm \to \pi^\pm K^0$	$18.2\,^{+4.6}_{-4.0} \pm 1.6$	$18.2\,^{+3.3}_{-3.0}\,^{+1.6}_{-2.0}$	$13.1\,^{+5.5}_{-4.6} \pm 2.6$	17.2 ± 2.5
$B^0 \to \pi^0 K^0$	$14.6\,^{+5.9}_{-5.1}\,^{+2.4}_{-3.3}$	$8.2\,^{+3.1}_{-2.7}\,^{+1.1}_{-1.2}$	$14.6\,^{+6.1}_{-5.1} \pm 2.7$	10.3 ± 2.5

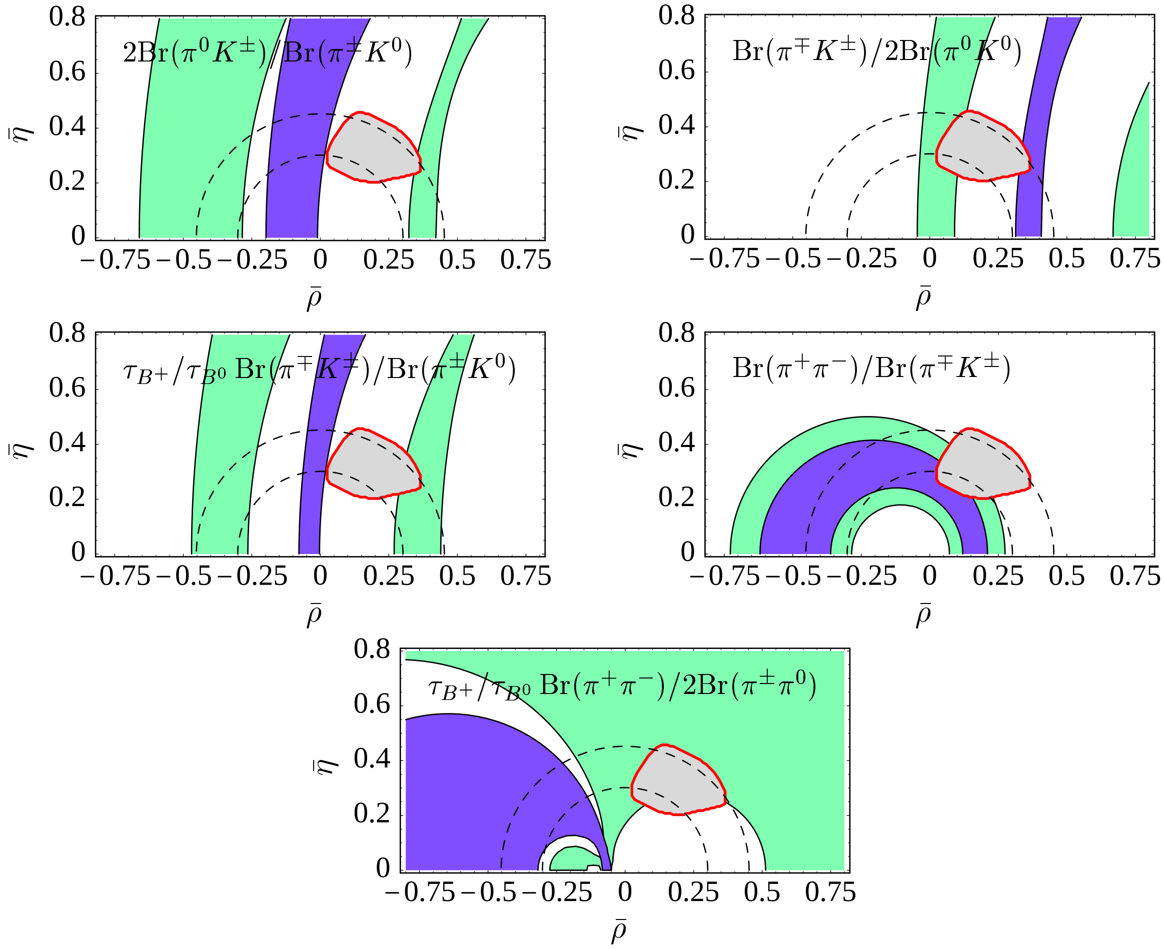
* experimental errors of order 10–20%, which will be reduced further by summer of 2001

* to match this precision theoretically, focus on ratios of decay rates, which are less sensitive to theory input parameters (e.g., semileptonic form factors)



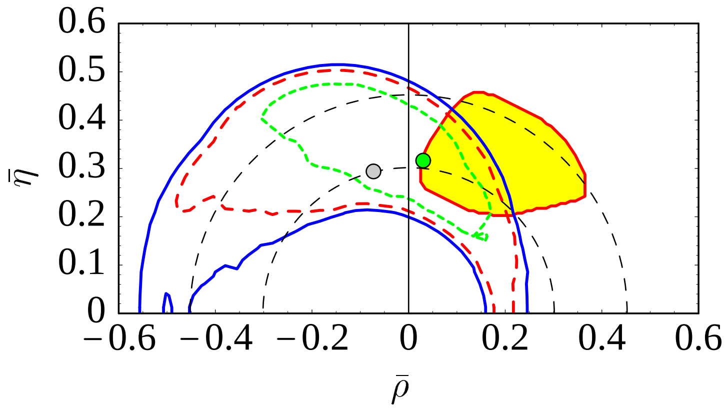
* present error on $|V_{ub}/V_{cb}| = 0.085 \pm 0.017$ is often a dominant contribution to the theoretical uncertainty (dashed lines)

$*$ unfold this dependence by presenting results in the $(\bar{\rho}, \bar{\eta})$ plane of Wolfenstein parameters:



$\rightarrow$ good agreement with standard CKM fit (gray area) at 1–2σ, but observe some intriguing tendencies...

* information from all $B \to \pi K, \pi\pi$ measurements can be combined into a global fit:



95%, 90% and 68% confidence level contours in the $(\bar{\rho}, \bar{\eta})$ plane obtained from rare hadronic B decays (green dot = overall best fit; gray dot = best fit for default parameter set). The yellow region shows the standard CKM fit.

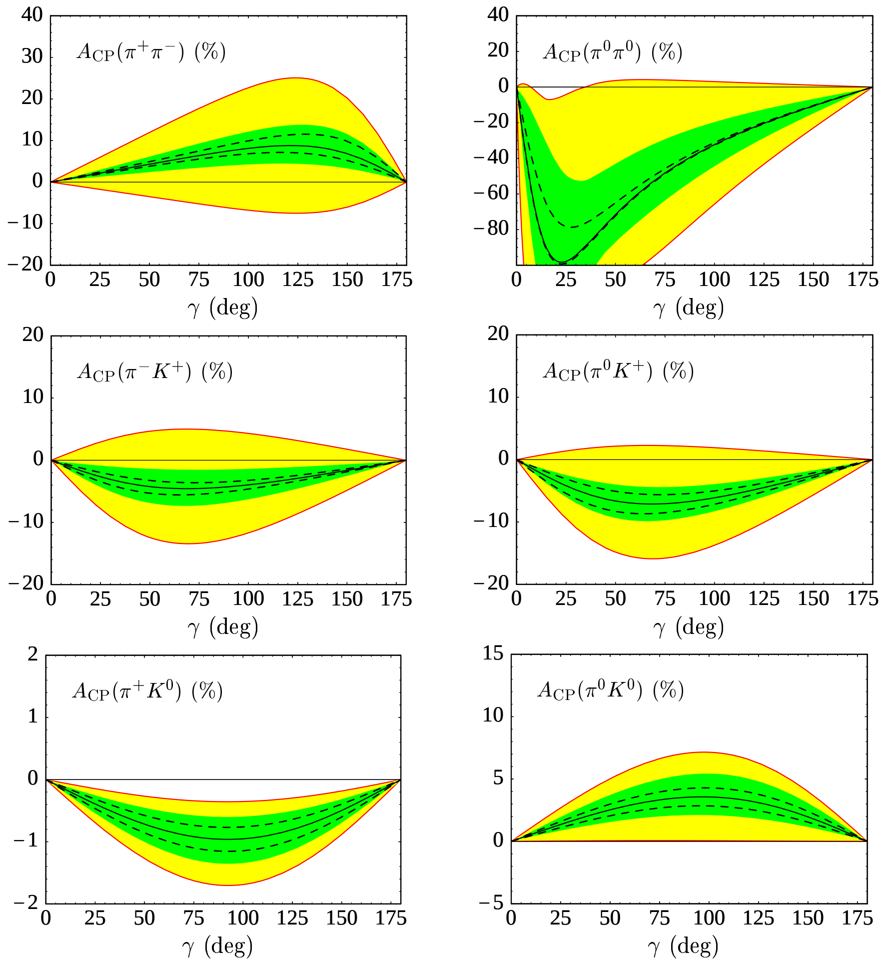
* results are compatible with the standard fit using information from semileptonic decays ($|V_{ub}|$, $|V_{cb}|$), K–$\bar{K}$ mixing ($\mathrm{Im}(V_{td})^2$), and B–$\bar{B}$ mixing ($|V_{td}|$, $\sin 2\beta$)

* best fit prefers larger γ and/or smaller $|V_{ub}|$, as well as $\mu \approx 3\,\mathrm{GeV}$ and $m_s(2\,\mathrm{GeV}) \approx 90\,\mathrm{MeV}$

* combination of results from rare hadronic B decays with $|V_{ub}|$ from semileptonic decays excludes $\bar{\eta} = 0$ at 95% C.L., thus establishing the existence of a CP-violating phase in the bottom sector!

Direct CP Asymmetries

* generic theoretical prediction:
  strong phases are suppressed (subleading in the heavy-quark expansion), except for very rare decays



* observing these asymmetries is a long-term goal
* this would determine the sign of γ and distinguish between the upper and lower half in the $(\bar\rho,\bar\eta)$ plane!

Outlook

- B physics has entered a new, exciting phase, where we will test the consistency of the CKM model, explore CP violation in the B system, and search for hints of flavor physics beyond the Standard Model
- rare decays play a central role in this program
  - systematic analysis possible due to recent theoretical developments:

    QCD factorization formula
  - heavy-quark expansion for nonleptonic decays
- expect many important results (and surprises?) in the very near future!