Pion Physics
at
Low Energy
and
High Accuracy

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QCD @ Work
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Standard Model at low energies

- For $E \ll M_{\mathrm{W}} c^2 \simeq 80\,\mathrm{GeV}$:
  weak interactions frozen, neutrini decouple
  Standard Model reduces to QED+QCD

- Precision theory for cold matter,
  atomic structure, solids, ...

  Bohr radius: $$a = \frac{4\pi}{e^2\, m_e}$$

- Effective theory, dynamical variables:
  gauge fields for photons and gluons
  Fermi fields for leptons and quarks

- Interaction fully determined by group geometry
  Lagrangian contains 3 coupling constants:

$$\boxed{g,\ e,\ \theta}$$

- Quark and lepton mass matrices can be brought to diagonal form, eigenvalues real, positive

$$\boxed{m_e,\, m_\mu,\, m_\tau,\, m_u,\, m_d,\, m_s,\, m_c,\, m_b,\, m_t}$$

- For bound states of quarks,
  e.m. interaction is a small perturbation

  Perturbation series in powers of $\dfrac{e^2}{4\pi}$ $\surd$

  Discuss only the leading term: set $e = 0$

- Lagrangian then reduces to QCD

$$\boxed{g\,,\, \theta\,,\, m_u\,, m_d\,,\, m_s\,,\, m_c\,,\, m_b\,,\, m_t}$$

- θ breaks CP

  Neutron dipole moment is very small

$\Rightarrow$ strong upper limit, $\theta \simeq 0$

Theoretical paradise

$m_u = m_d = m_s = 0$

$m_c = m_b = m_t = \infty$

$\boxed{\text{QCD with 3 massless quarks}}$

- Interactions of u, d, s are identical
  If the masses are the same,
  there is no difference at all

- More symmetry: For massless fermions,
  right and left do not communicate

⇒ Lagrangian of massless QCD is invariant under independent rotations of the right– and left–handed quark fields

$$\mathrm{SU(3)_R \times SU(3)_L}$$

⇒ 16 conserved "charges"

$Q^{\mathrm{V}}_1, \ldots, Q^{\mathrm{V}}_8$ (vector currents)

$Q^{\mathrm{A}}_1, \ldots, Q^{\mathrm{A}}_8$ (axial currents)

- Charges commute with the Hamiltonian:

$$[Q_i^{\mathsf{V}}, H_0] = 0 \qquad [Q_i^{\mathsf{A}}, H_0] = 0$$

"Chiral symmetry" of massless QCD

- Consequence of anomaly in singlet axial current: θ only enters in the combination

$$\det m \times e^{i\theta}$$

⇒ Massless theory is independent of θ

⇒ Lagrangian contains a single parameter: g

- g is net colour of a quark
depends on radius of the region considered
Colour contained within radius r

$$\frac{g^2}{4\pi} = \frac{2\pi}{9\,|\ln(r\,\Lambda_{\mathrm{QCD}})|}$$

- Intrinsic scale Λ_{QCD} is meaningful,
but not dimensionless

⇒ No dimensionless free parameter

All dimensionless physical quantities are pure numbers, determined by the theory
Cross sections can be expressed in terms of Λ_{QCD} or in the mass of the proton

Asymmetry of ground state

- Nambu 1961: state of lowest energy
  not invariant under chiral rotations
  $$Q_i^{\mathsf{A}}\,|0\rangle \neq 0$$
  For dynamical reasons, the state of
  lowest energy must be asymmetric

$\Rightarrow$ Chiral symmetry is spontaneously broken

- Very strong experimental evidence $\surd$
- Theoretical understanding on the basis
  of the QCD Lagrangian ?
- If $|0\rangle$ were invariant under chiral rotations

$\Rightarrow$ $\langle 0|\bar{u}u\,|0\rangle = \langle 0|\bar{u}_{\mathsf{R}}u_{\mathsf{L}}\,|0\rangle + c.c. = 0$

- Since ground state is not symmetric

$\Rightarrow$ no reason for $\langle 0|\bar{u}u\,|0\rangle$ to vanish
Order parameter of lowest dimension

- Vafa and Witten 1984: state of lowest energy
  invariant under the vector charges, $Q_i^{\mathsf{V}}\,|0\rangle = 0$

$\Rightarrow$ $\langle 0|\bar{u}\,u\,|0\rangle = \langle 0|\bar{d}\,d\,|0\rangle = \langle 0|\bar{s}\,s\,|0\rangle$

- Consequence of $Q_i^{\mathsf{A}}\,|0\rangle \neq 0$:

  $$H_0\,Q_i^{\mathsf{A}}\,|0\rangle = Q_i^{\mathsf{A}}\,H_0\,|0\rangle = 0$$

  Spectrum must contain 8 states

  $$Q_1^{\mathsf{A}}\,|0\rangle, \ldots, Q_8^{\mathsf{A}}\,|0\rangle \qquad \text{with} \quad E = 0,$$

  Spin 0, negative parity, octet of $\mathrm{SU}(3)_{\mathrm{V}}$
  Goldstone bosons

- Indeed, the 8 lightest mesons do have these quantum numbers:
  $\pi^+, \pi^0, \pi^-, K^+, K^0, \bar{K}^0, K^-, \eta$

  But massless they are not

- Real world $\neq$ paradise
  $m_u\,,\, m_d\,,\, m_s \neq 0$

  Quark masses break chiral symmetry,
  allow left to talk to right

- Chiral symmetry broken in two ways:

  spontaneously $\qquad \langle 0|\,\bar{q}_{\mathrm{R}}\, q_{\mathrm{L}}\,|0\rangle \neq 0$

  explicitly $\qquad m_u\,,\, m_d\,,\, m_s \neq 0$

- H_{QCD} only has approximate symmetry to the extent that m_u, m_d, m_s are small

Chiral perturbation theory

- Consequences of approximate chiral symmetry can be worked out by means of an effective field theory   Weinberg 1979
- Full Hamiltonian contains quark mass term

$$H_{\mathrm{QCD}} = H_0 + H_1$$
$$H_1 = \int d^3x \, \{m_u \bar{u}u + m_d \bar{d}d + m_s \bar{s}s\}$$

H_0 is the Hamiltonian of the massless theory

- Treat H_1 as a perturbation

$$\boxed{\begin{array}{c}\text{Expansion in} \\ \text{powers of } H_1\end{array}} \Longleftrightarrow \boxed{\begin{array}{c}\text{Expansion in} \\ \text{powers of } m_u, m_d, m_s\end{array}}$$

- To first order in H_1:

Gell-Mann, Oakes and Renner 1968

$$M_\pi^2 = (m_u \underset{\substack{\Uparrow \\ \text{explicit}}}{+} m_d) \times |\langle 0 | \underset{\substack{\Uparrow \\ \text{spontaneous}}}{\bar{q}\, q} | 0 \rangle| \times \frac{1}{F_\pi^2}$$

Coefficient: decay constant F_π

- $M_\pi^2 = (m_u + m_d)\, B + O(m^2)$

  $$B = \frac{|\langle 0|\, \bar{q}\, q\, |0\rangle|}{F_\pi^2}$$

- M_π disappears if the symmetry breaking is turned off, $m_u, m_d \to 0$ $\surd$

- Explains why the pseudoscalar mesons have very different masses

  $$M_{K^+}^2 = (m_u + m_s)\, B + O(m^2)$$

  $$M_{K^-}^2 = (m_d + m_s)\, B + O(m^2)$$

$\Rightarrow$ M_K^2 is about 13 times larger than M_π^2, because m_s happens to be large compared to m_u, m_d

- First order perturbation theory also yields

  $$M_\eta^2 = \tfrac{1}{3}\,(m_u + m_d + 4m_s)B + O(m^2)$$

$\Rightarrow$ $M_\pi^2 - 4M_K^2 + 3M_\eta^2 = O(m^2)$

Gell-Mann-Okubo formula for M^2 $\surd$

- Higher orders of the expansion in m_u, m_d, m_s ?
- Massless QCD contains infrared singularities due to the Goldstone bosons

$\Rightarrow$ Expansion is not a Taylor series, contains logarithms

- Chiral perturbation theory correctly reproduces the infrared singularities of QCD
- Quantities of interest are expanded in powers of external momenta and quark masses
- Results are exact, to any given order of this expansion
- Expansion has been worked out to next-to-leading order for many quantities Chiral perturbation theory to one loop
- In a few cases, the next-to-next-to leading order is also known

Illustration: Expansion of M_π^2

- Ignore isospin breaking, set $m_u = m_d = m$
- Keep m_s fixed at the physical value
- Replace m by $M^2 = 2\, m\, B$

$$M_\pi^2 = M^2 - \frac{1}{2}\,\frac{M^4}{(4\pi F)^2}\,\ln\frac{\Lambda_3^2}{M^2} + \frac{17}{18}\,\frac{M^6}{(4\pi F)^4}\left(\ln\frac{\Lambda_{\mathrm{M}}^2}{M^2}\right)^2 + k_{\mathrm{M}} M^6 + O(M^8)$$

F is pion decay constant for $m_u = m_d = 0$
ChPT to two loops Colangelo 1995

- Scale of the expansion is set by size of pion mass in units of decay constant:

$$\frac{M^2}{(4\pi F)^2} \simeq \frac{M_\pi^2}{(4\pi F_\pi)^2} = 0.0144$$

- Coefficients $\frac{1}{2}$ and $\frac{17}{18}$ determined by symmetry
- $\Lambda_3, \Lambda_{\mathrm{M}}$ and $k_{\mathrm{M}} \Longleftrightarrow$ coupling constants in $\mathcal{L}_{eff}$

- Some, but not all effective coupling constants can be determined from experiment
- Rely on theoretical estimates:
  Saturation of sum rules by lowest resonances
  Expansion in $1/N_c$
- Stern et al.: Standard analysis assumes that
  Quark condensate = leading order parameter
  Has not been checked experimentally

Role of quark condensate

- The low energy properties of the theory are governed by those of the ground state
- Among the operators that can have a nonzero vacuum expectation value $\overline{q}\,q$ is the one of lowest dimension

$\Rightarrow$ It is natural to assume that $\langle 0|\,\overline{q}\,q\,|0\rangle$ differs from zero and is the leading order parameter $\langle 0|\,\overline{q}\,q\,|0\rangle$ = quark condensate

- But do we know that this is true ?
- Analog of $\langle 0|\,\overline{q}\,q\,|0\rangle$ for magnets ?

Heisenberg model of ferromagnet

$$H = -g \sum_{\langle ij\rangle} \vec{s}_i \cdot \vec{s}_j$$

$g > 0$ $\Uparrow\Uparrow$ lower in energy than $\Uparrow\Downarrow$

- Ground state $= \Uparrow\Uparrow\Uparrow\Uparrow \cdots \Uparrow\Uparrow$
- Magnetization: $\vec{M} = \dfrac{\mu}{V}\sum_i \vec{s}_i$

$$\langle 0|\,\overline{q}\,q\,|0\rangle \iff \langle 0|\,\vec{M}\,|0\rangle \neq 0$$

Antiferromagnet

$$H = -g \sum_{\langle ij \rangle} \vec{s}_i \cdot \vec{s}_j$$

Same model, but with $g < 0$

$\Uparrow\Uparrow$ higher in energy than $\Uparrow\Downarrow$

- Ground state $= \Uparrow\Downarrow\Uparrow\Downarrow \cdots \Uparrow\Downarrow$
- Rotation symmetry also spontaneously broken
- Magnetization: $\vec{M} = \frac{\mu}{V} \sum_i \vec{s}_i$
- For dynamical reasons, the operator $\vec{M}$ does not pick up an expectation value

$$\langle 0 | \vec{M} | 0 \rangle = 0$$

$\Rightarrow$ In this case, $\langle 0 | \vec{M} | 0 \rangle$ does not represent the leading order parameter

- Instead can take "staggered magnetization": Include only the even sites in the sum over the magnetic moments $\Uparrow \Downarrow \Uparrow \Downarrow \cdots \Uparrow \Downarrow \Uparrow \Downarrow$

- What if the ground state of QCD is more like the one of an antiferromagnet ?

  What if $\langle 0|\bar{q}\,q\,|0\rangle$ is small or even vanishes ?

- Gell-Mann-Oakes-Renner relation:

  $$M_\pi^2 = (m_u + m_d) \cdot |\langle 0|\bar{q}\,q\,|0\rangle| \cdot \frac{1}{F_\pi^2} + \text{corrections}$$

  If $|\langle 0|\bar{q}\,q\,|0\rangle|$ is small $\Rightarrow$ the "corrections" could be comparable to the "leading" term

- Next–to–leading term:

  $$M_\pi^2 = M^2 \left\{ 1 - \frac{1}{2}\,\frac{M^2}{(4\pi F)^2}\,\ln\frac{\Lambda_3^2}{M^2} + O(M^4) \right\}$$

  $$M^2 \equiv (m_u + m_d) \cdot |\langle 0|\bar{q}\,q\,|0\rangle| \cdot \frac{1}{F_\pi^2}$$

  Λ_3 is intrinsic scale of $\ell_3 \in \mathcal{L}_{eff}^{(4)}$:

  $$\ln\frac{\Lambda_3^2}{M_\pi^2} = \bar{\ell}_3$$

- Indeed, scale Λ_3 not known well

- Crude estimate 1984

$$0.2 \text{ GeV} < \Lambda_3 < 2 \text{ GeV}$$

$\Rightarrow$ Next–to–leading term is small correction:

$$0.005 < \frac{1}{2}\,\frac{M_\pi^2}{(4\pi F_\pi)^2}\,\ln\frac{\Lambda_3^2}{M_\pi^2} < 0.04$$

- Not a test: Estimate assumes that $\langle 0|\,\bar{q}\,q\,|0\rangle$ is the leading order parameter
- Generalized ChPT: Use standard Lagrangian, but treat $\bar{\ell}_3 \Longleftrightarrow \Lambda_3$ as a free parameter
- Closely related issue: If the leading term does not dominate, then expect large corrections in

$$\frac{M_{K^+}^2}{M_{\pi^+}^2} = \frac{m_u + m_s}{m_u + m_d} + \ldots$$

- Results for quark mass ratios 1996

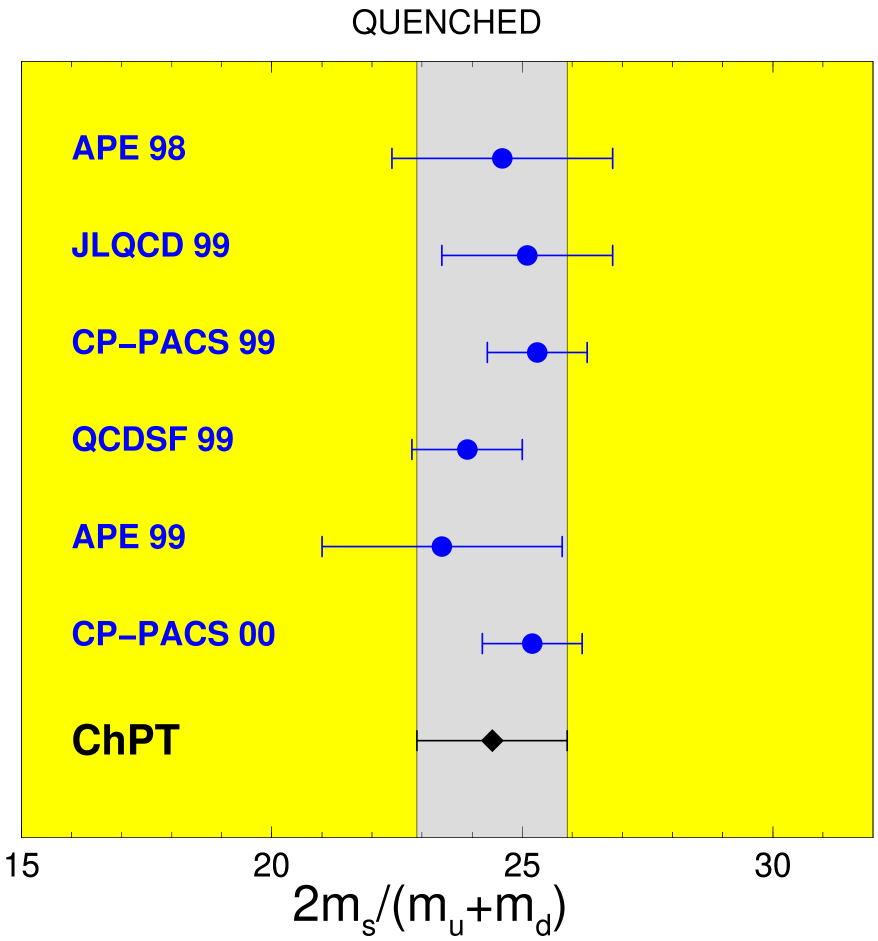
$$\frac{m_u}{m_d} = 0.553 \pm 0.043 \qquad \frac{m_s}{m_d} = 18.9 \pm 0.8$$

$$\frac{m_s}{\hat{m}} = 24.4 \pm 1.5 \qquad \hat{m} \equiv \tfrac{1}{2}(m_u + m_d)$$

Rely on the assumption that quark condensate is leading order parameter

Comparison with lattice results

V. Lubicz, talk at Lattice 2000



ChPT $$\frac{2m_s}{m_u+m_d} = 24.4 \pm 1.5$$

- In principle, the quark condensate can be measured on the lattice
- Quenched approximation or dynamical quarks with m_u and m_d of the order of m_s $\surd$
- Quarks with physical masses ?
- Extrapolations consistent with $M^2 \propto m_{\text{quark}}$
  No indication for $\langle 0|\bar{q}\,q\,|0\rangle$ to be small

- Value for $\langle 0|\bar{q}\,q\,|0\rangle$ in quenched approximation agrees with what is obtained from sum rules
- Results for $\langle 0|\bar{q}\,q\,|0\rangle$ with dynamical quarks are larger by about 20%

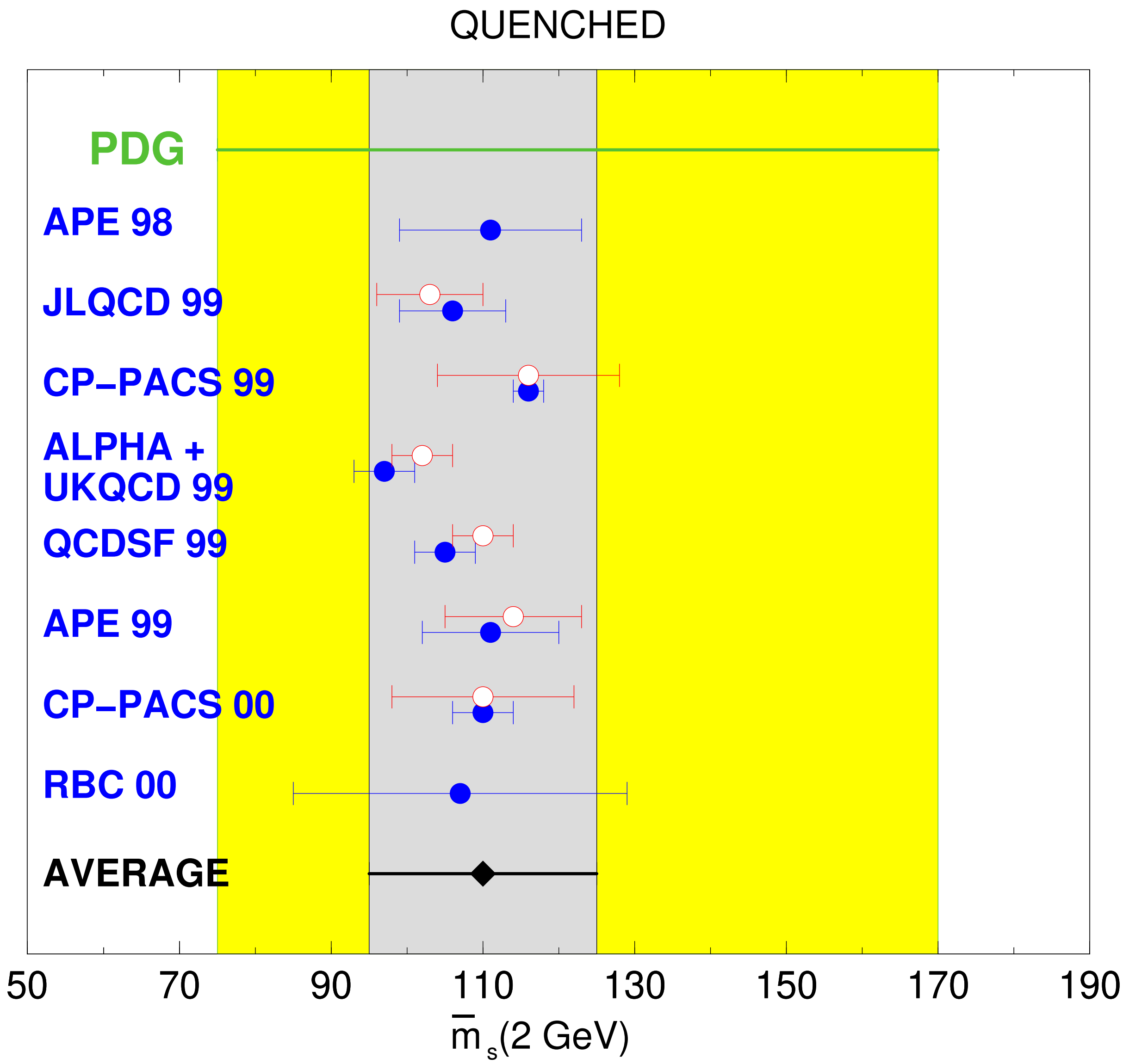
$\Rightarrow$ quark masses are correspondingly smaller

- Running $\overline{\text{MS}}$ masses at $\mu = 2\,\text{GeV}$:

sum rules, τ decay $\quad \Rightarrow m_s \simeq 115\,\text{MeV}$

recent lattice results $\quad \Rightarrow m_s \simeq 90\,\text{MeV}$

V. Lubicz, hep-ph/0010171



Values of the strange quark mass ($\overline{\mathrm{MS}}$, running scale 2 GeV), obtained from recent lattice calculations in the quenched approximation. The upper line is the estimate of m_s quoted by the PDG, and the lowest point represents the lattice (quenched) world average.

Expansion of F_π, scalar radius

$$F_\pi = F\left\{1 + \frac{M^2}{(4\pi F)^2}\ln\frac{\Lambda_4^2}{M^2} + O(M^4)\right\}$$

Λ_4 is intrinsic scale of $\bar{\ell}_4 \in \mathcal{L}_{eff}^{(4)}$:

$$\ln\frac{\Lambda_4^2}{M_\pi^2} = \bar{\ell}_4$$

- The scale Λ_4 also determines the leading term in the expansion of the scalar radius:

$$\langle r^2\rangle_s = \frac{6}{(4\pi F)^2}\left\{\ln\frac{\Lambda_4^2}{M^2} - \frac{13}{12} + O(M^2)\right\}$$

- Definition of scalar radius:

$$F_s(t) = \langle\pi(p')|\,\bar{q}\,q\,|\pi(p)\rangle \;, \qquad t = (p'-p)^2$$
$$F_s(t) = F_s(0)\left\{1 + \frac{1}{6}\langle r^2\rangle_s\, t + O(t^2)\right\}$$

- In massless QCD, the scalar radius diverges, because the density of the pion cloud only decreases with a power of the distance

- Same infrared singularity also occurs in the charge radius (e.m. current), but coefficient of the chiral logarithm is 6 times smaller:

$$\langle r^2\rangle_s = \frac{6}{(4\pi F)^2}\left\{\ln\frac{\Lambda_4^2}{M^2} - \frac{13}{12} + O(M^2)\right\}$$

$$\langle r^2\rangle_{em} = \frac{1}{(4\pi F)^2}\left\{\ln\frac{\Lambda_6^2}{M^2} - 1 + O(M^2)\right\}$$

$\Rightarrow$ $\langle r^2\rangle_s > \langle r^2\rangle_{em}$ if M small enough

- $\langle r^2\rangle_{em}$ can be determined experimentally

$$\langle r^2\rangle_{em} = 0.439 \pm 0.008\,\text{fm}^2$$

NA7 Collaboration, NP B277 (1986) 168

- For $\langle r^2\rangle_s$ need to rely on theory
  Analyticity relates the scalar form factor to the $I = 0$ S–wave phase shift

- Result of dispersive calculation

$$\langle r^2\rangle_s = 0.61 \pm 0.04\,\text{fm}^2$$

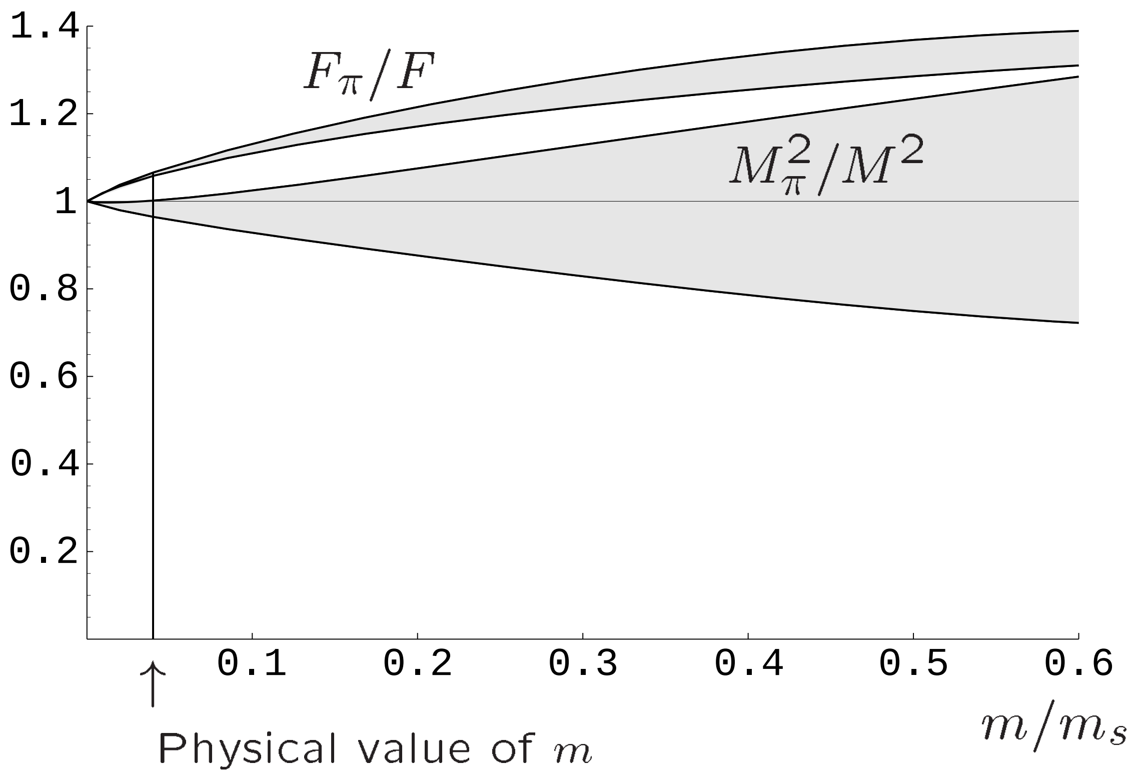
Colangelo et al. hep-ph/0103088

- Prediction $\langle r^2\rangle_s > \langle r^2\rangle_{em}$ is confirmed $\surd$
- Consequence for F_π in limit $m_u, m_d \to 0$

$$F = F_\pi \left\{ 1 - \frac{1}{6} M_\pi^2 \langle r^2\rangle_s - \frac{13}{12} \frac{M_\pi^2}{(4\pi F_\pi)^2} + O(M_\pi^4) \right\}$$
$$= (0.93 \pm 0.01)\, F_\pi$$

M_π, F_π versus quark masses

- Keep m_s fixed at the physical value
  Take $m_u = m_d = m$ and vary m
  $m = 0.5\, m_s$ corresponds to $M_\pi \simeq 500\,\text{MeV}$



- Uncertainties in M_π rapidly grow with m, because estimates for Λ_3 are very crude:

$$0.2 \text{ GeV} < \Lambda_3 < 2 \text{ GeV}$$

- Lattice data in the region $0.4\, m_s < m < 0.6\, m_s$ would suffice to accurately pin down Λ_3
- For F_π, the relevant scale is Λ_4, known rather well from scalar radius:

$$\Lambda_4 = 1.26 \pm 0.14\,\text{GeV}$$

- Curvature $\Rightarrow$ linear extrapolation fails

$\pi\pi$ scattering

Considerable progress recently

- Analyticity, unitarity and crossing symmetry impose strong constraints

$\Rightarrow$ Roy equations
Wanders, Gasser
Ananthanarayan et al., hep-ph/0005297

- New $K \to e\nu\pi\pi$ data from Brookhaven (E865)
Allow test of low energy theorem for a_0^0
Colangelo et al., hep-ph/0103063

Roy equations

Dispersion relations for partial wave amplitudes
At low energies, $S-$ and $P-$waves dominate

Input

- Value of the phase shifts at 0.8 GeV
(plus rough estimates for $E > 0.8$ GeV)
- 2 subtraction constants, can identify these with the $\pi\pi$ S-wave scattering lengths:

$$a_0^0\;,\;a_0^2 \begin{array}{l}\leftarrow \text{isospin}\\ \leftarrow \text{angular momentum}\end{array}$$

Output

- Full scattering amplitude in low energy region, including higher partial waves
- Noise due to experimental uncertainties at or above 0.8 GeV is small

$\Rightarrow$ Relevant parameters at low energy: a_0^0, a_0^2

Chiral symmetry

- Goldstone bosons of zero momentum do not interact

$\Rightarrow$ a_0^0, a_0^2 must vanish for $m_u, m_d \to 0$

$\Rightarrow$ $a_0^0, a_0^2 \sim M_\pi^2$ measure symmetry breaking

- Low Energy theorem Weinberg 1966:

$$a_0^0 = \frac{7M_\pi^2}{32\pi F_\pi^2} + O(M_\pi^4)$$

$$a_0^2 = -\frac{M_\pi^2}{16\pi F_\pi^2} + O(M_\pi^4)$$

$\Rightarrow$ Chiral symmetry predicts a_0^0, a_0^2

$\Rightarrow$ Roy equations fix scattering amplitude

- Accuracy is limited: Low energy theorem only specifies the first term in the expansion in powers of the quark masses
  Corrections from higher orders ?

- Next term in the expansion:

$$a_0^0 = \frac{7M_\pi^2}{32\pi F_\pi^2}\left\{1 + \frac{9}{2}\frac{M_\pi^2}{(4\pi F_\pi)^2}\ln\frac{\Lambda_0^2}{M_\pi^2} + O(M_\pi^4)\right\}$$

- Coefficient of chiral logarithm unusually large
  Strong, attractive final state interaction

- Scale Λ_0 is determined by the coupling constants of $\mathcal{L}_{eff}^{(4)}$:

$$\frac{9}{2}\ln\frac{\Lambda_0^2}{M_\pi^2} = \frac{20}{21}\bar{\ell}_1 + \frac{40}{21}\bar{\ell}_2 - \frac{5}{14}\bar{\ell}_3 + 2\,\bar{\ell}_4 + \frac{5}{2}$$

- Information about $\bar{\ell}_1, \ldots, \bar{\ell}_4$?

$\bar{\ell}_1, \bar{\ell}_2$	$\Longleftrightarrow$	momentum dependence of scattering amplitude

$\Rightarrow$ Can be determined phenomenologically

$\bar{\ell}_3, \bar{\ell}_4$	$\Longleftrightarrow$	dependence of scattering amplitude on quark masses

Masses cannot be varied experimentally
Cannot fix $\bar{\ell}_3, \bar{\ell}_4$ with $\pi\pi$ phenomenology

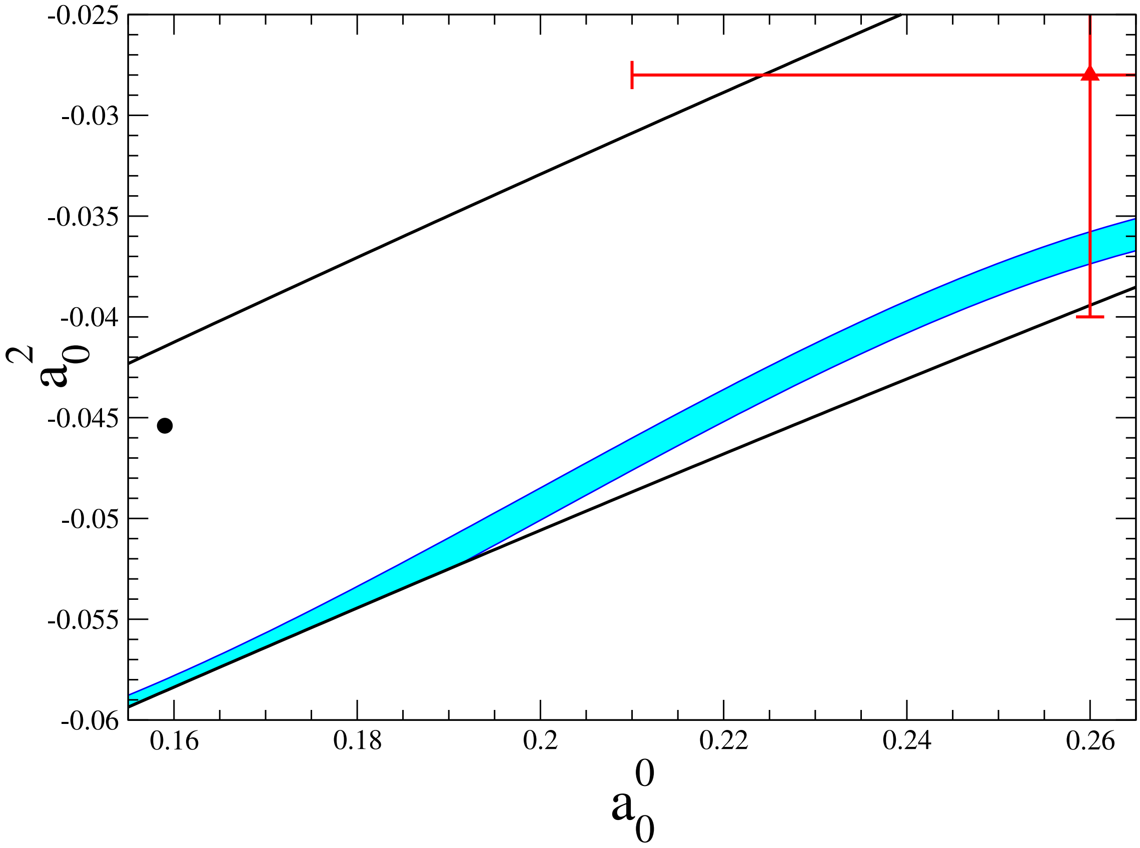
- $\bar{\ell}_3, \bar{\ell}_4$ also determine dependence of M_π, F_π on quark mass

$$\bar{\ell}_3 = \ln \frac{\Lambda_3^2}{M_\pi^2} \qquad \bar{\ell}_4 = \ln \frac{\Lambda_4^2}{M_\pi^2}$$

- Λ_4 known from scalar radius
- Treat $\Lambda_3 \Longleftrightarrow \bar{\ell}_3$ as a free parameter

$\Rightarrow$ No prediction for a_0^0, a_0^2 individually, but correlation: $2a_0^0 - 5a_0^2$ fixed by scalar radius

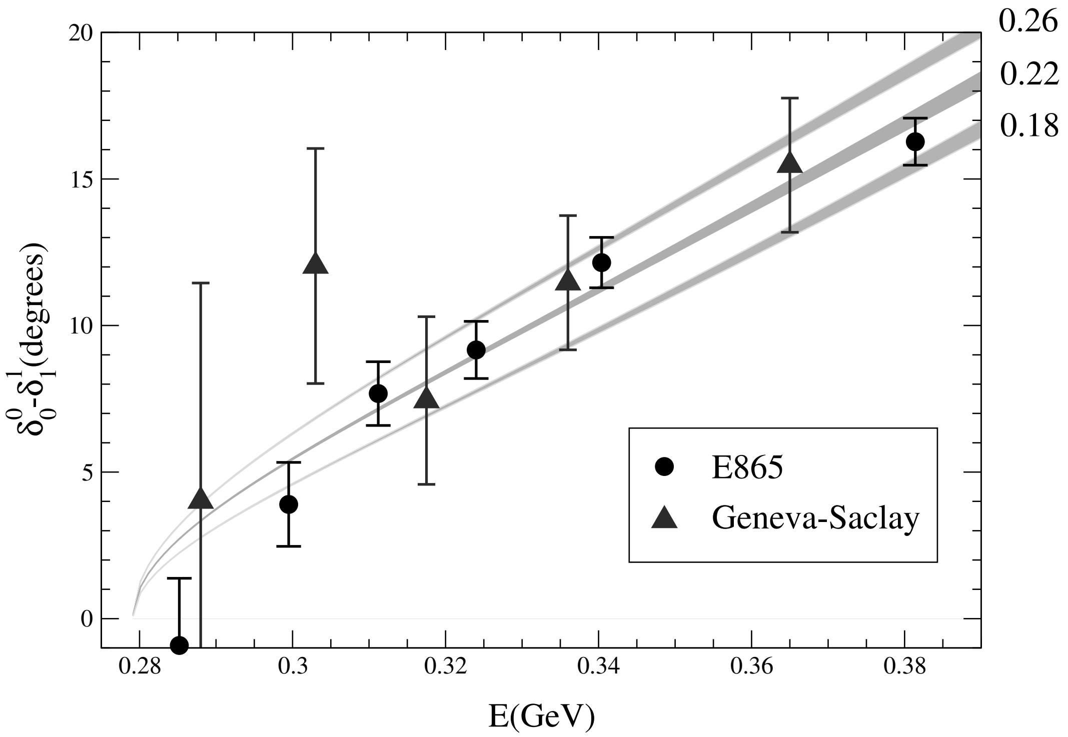
$$2a_0^0 - 5a_0^2 = \frac{3M_\pi^2}{4\pi F_\pi^2}\left\{1 + \frac{M_\pi^2 \langle r^2 \rangle_s}{3} + \frac{41 M_\pi^2}{192\pi^2 F_\pi^2} + O(M_\pi^4)\right\}$$



The Roy equations only admit solutions in the "universal band" spanned by the two tilted lines. Blue strip: correlation between a_0^0 and a_0^2 imposed by the scalar radius. Point on the left: Weinbergs leading order formulae. Triangle with error bars: range permitted by the old data, $a_0^0 = 0.26 \pm 0.05\,, \quad a_0^2 = -0.028 \pm 0.012$

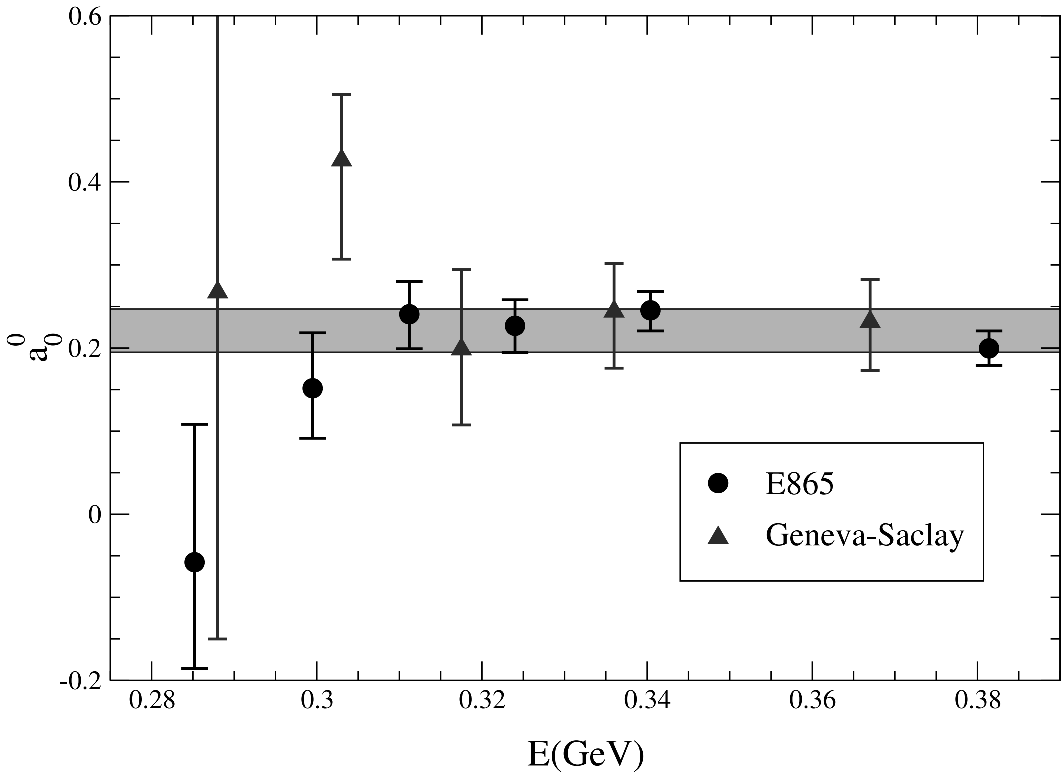
G. Colangelo, J. Gasser and H. L., hep-ph/0103088

- New $K \to e\nu\pi\pi$ data from Brookhaven
  Yield a measurement of $\delta_0^0(E) - \delta_1^1(E)$
  $2M_\pi < E < M_K$ is energy of $\pi\pi$



The three bands show the prediction for the phase difference in generalized chiral perturbation theory, for $a_0^0 = 0.18, 0.22, 0.26$, respectively (width of the bands $\Longleftrightarrow$ uncertainties in the prediction).

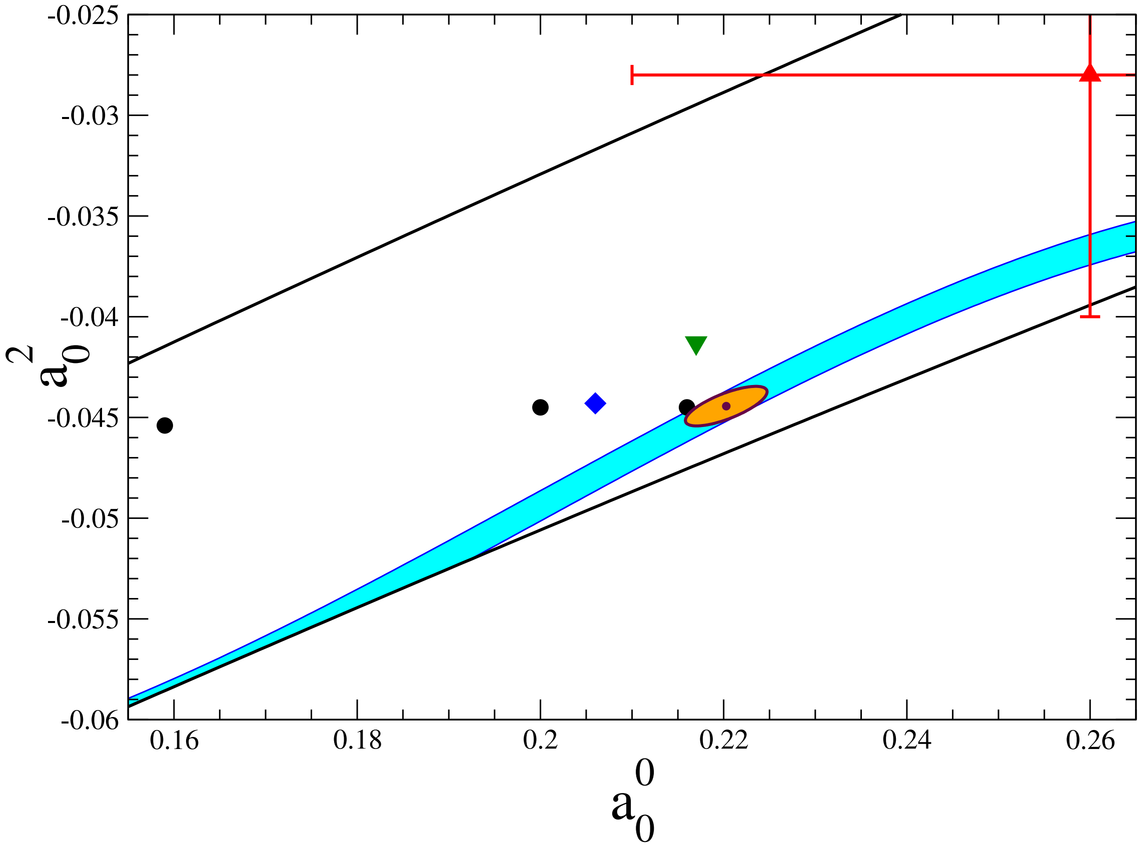
Taken from Colangelo, Gasser, L., hep-ph/0103063



Fit to the data yields $a_0^0 = 0.221 \pm 0.026$
at 95 % confidence

- To be compared with the prediction of standard ChPT: $a_0^0 = 0.220 \pm 0.005$
Amoros, Bijnens, Talavera, hep-ph/0003258
Colangelo, Gasser, L., hep-ph/0007112

$\Rightarrow$ More than 94 % of the pion mass originates in the quark condensate term
$M_\pi^2 \simeq (m_u + m_d) \times |\langle 0 | \bar{q}\, q\, | 0 \rangle| \times \dfrac{1}{F_\pi^2}$ $\surd$



Convergence of the perturbation series in standard ChPT. It is assumed that the quark condensate is the leading order parameter. Black points: tree level, one loop and two loop, respectively. Error ellipse: prediction obtained by matching the solution of the Roy equations with the two loop representation of ChPT. Triangle and diamond near the center of the figure: set I and II of Bijnens, Colangelo, Ecker, Gasser and Sainio, hep-ph/9707291.

Taken from Colangelo et al., hep-ph/0103088.

- Lattice result:
  $a_0^2 = -0.0374 \pm 0.0049$ or $R_2 = 0.82 \pm 0.11$
  JLQCD Collaboration, S. Aoki et al. 2000

  To be compared with the prediction
  $a_0^2 = -0.0444 \pm 0.0010$ or $R_2 = 0.98 \pm 0.02$

- On the lattice, can vary m_u and m_d
  Difficult to deal with light quarks
  Must use extrapolation to reach physical values

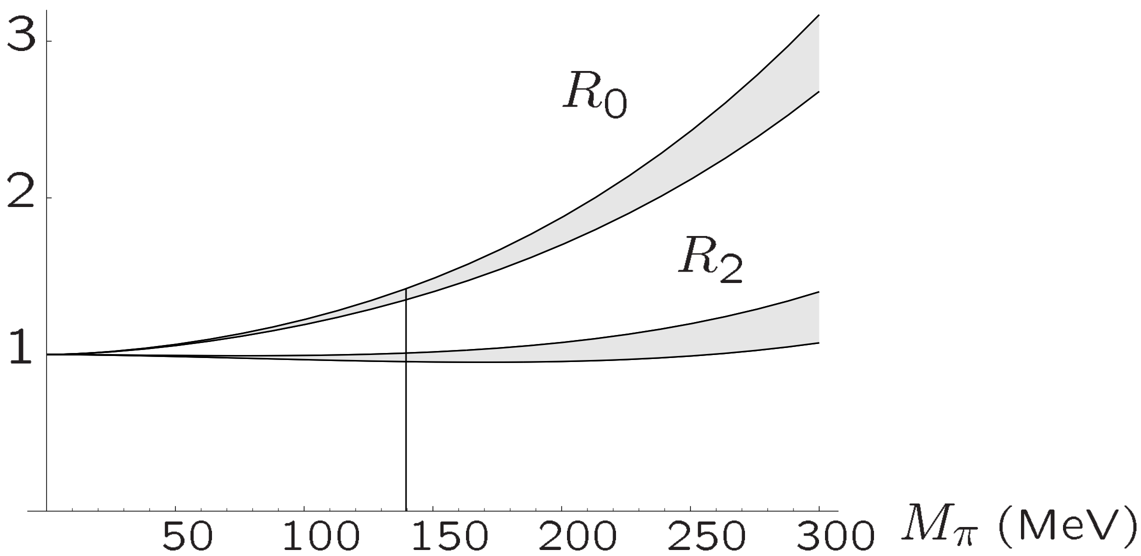
- ChPT predicts the dependence on m_u, m_d in terms of effective coupling constants

$\Rightarrow$ Brings the extrapolation under control

$\Rightarrow$ Can determine some of the effective coupling constants by means of lattice data

Scattering lengths versus M_π

$$a_0^0 = \frac{7M_\pi^2}{32\pi F_\pi^2} R_0 \,, \qquad a_0^2 = -\frac{M_\pi^2}{16\pi F_\pi^2} R_2$$



Strong curvature in $I = 0$ channel due to final state interaction

$\Rightarrow$ Naive extrapolation fails

Outlook

- Chiral symmetry broken in two ways:
  spontaneously and *explicitly*
- Splendid experimental evidence for symmetry and spontaneous breakdown thereof
- Evidence for the quark mass term in H_{QCD} to properly account for the explicit symmetry breaking is scarce
- Sensitive test: Low energy theorems for a_0^0, a_0^2
- a_0^0 and a_0^2 are small $\Rightarrow$ difficult to measure
  For once in strong interaction physics,
  theory is ahead of experiment
- New data on $K \to e\nu\pi\pi$ decay offer test at the 10 % level

  Confirm the Gell-Mann-Oakes-Renner relation
  Quark condensate = leading order parameter $\surd$

- Beautiful experiment under way at CERN
  $\Rightarrow$ dirac.web.cern.ch/DIRAC   L. Nemenov
  Annecy, Basel, Bern, Genève, . . .
- Electromagnetically bound states of pions
  $$\pi^+\pi^- \text{ "atoms"}$$
- Atoms decay through the strong interaction
  $$\pi^+\pi^- \to \pi^0\pi^0$$
  Decay rate $\propto (a_0^0 - a_0^2)^2$
- Interference of e.m. and strong interactions in bound state and decay is now well understood
  Gasser, Rusetsky et al.
- Measurement of lifetime to 10% $\Rightarrow$ clean test of symmetry breaking due to m_u and m_d

- What is high accuracy in pion physics good for ?

  a. Tests of QCD: m_u, m_d are very small
  Quark mass effects visible only
  at high precision

  b. Example for physics beyond QCD:
  Magnetic moment of the muon
  Discrepancy between theory and experiment
  $\sim 5\%$ of the hadronic contribution
  Experimental accuracy goal is 1%
  Meaningful only if can calculate hadronic
  contribution to comparable accuracy