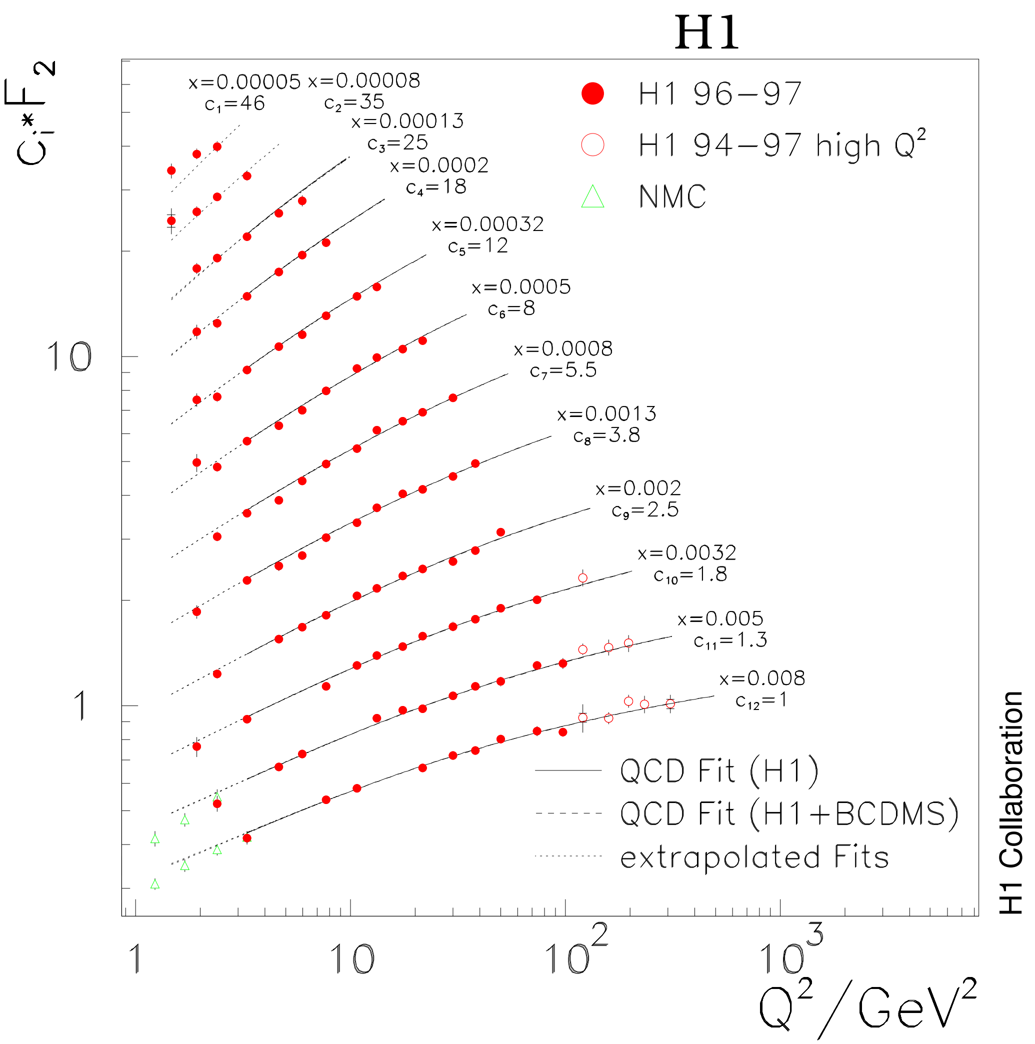
THEORETICAL ASPECTS
OF
HERA PHYSICS

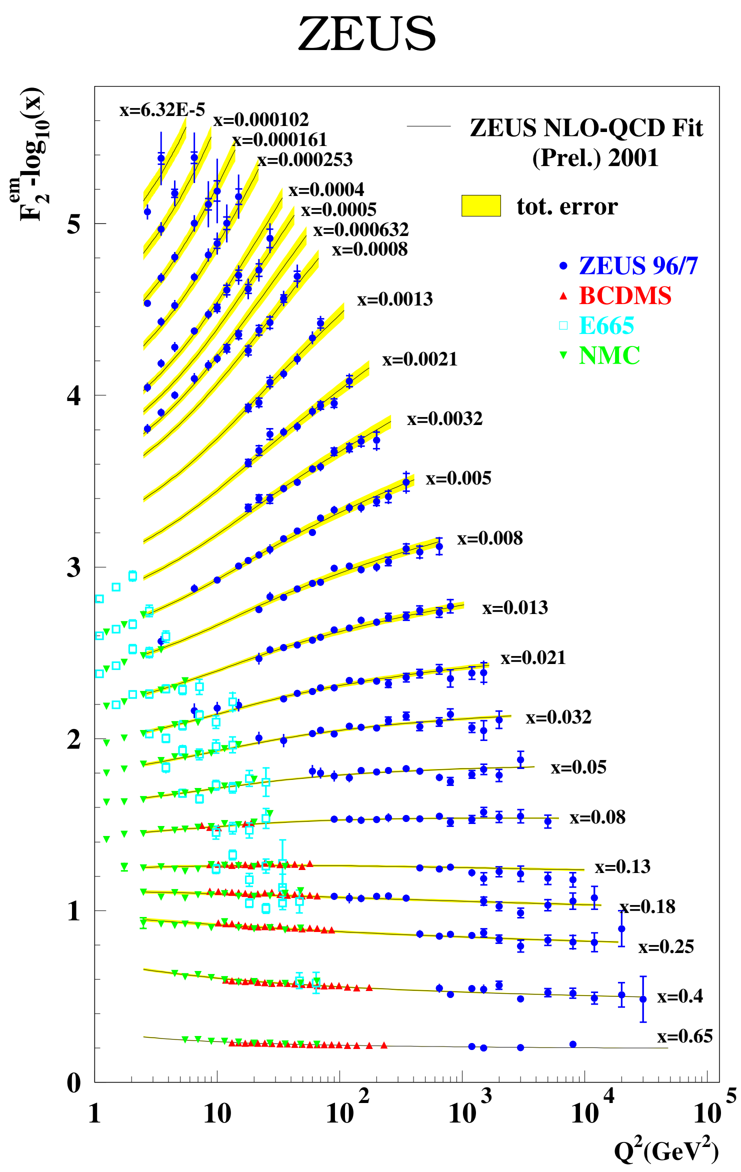
STEFANO FORTE
I.N.F.N. ROMA III

QCD@WORK MARTINA FRANCA, JUNE 17 2001

Fixed–order perturbative QCD is extremely successful in describing HERA data:

scaling violations of structure functions





. . . INDEPENDENT OF DETAILS OF PARTON DISTRIBUTIONS:

DOUBLE ASYMPTOTIC SCALING

$\ln F_2$ predicted in NLO QCD to rise linearly as a function of the scaling variable σ, indep. of value of scaling variable ρ, up to universal rescaling factor R_F

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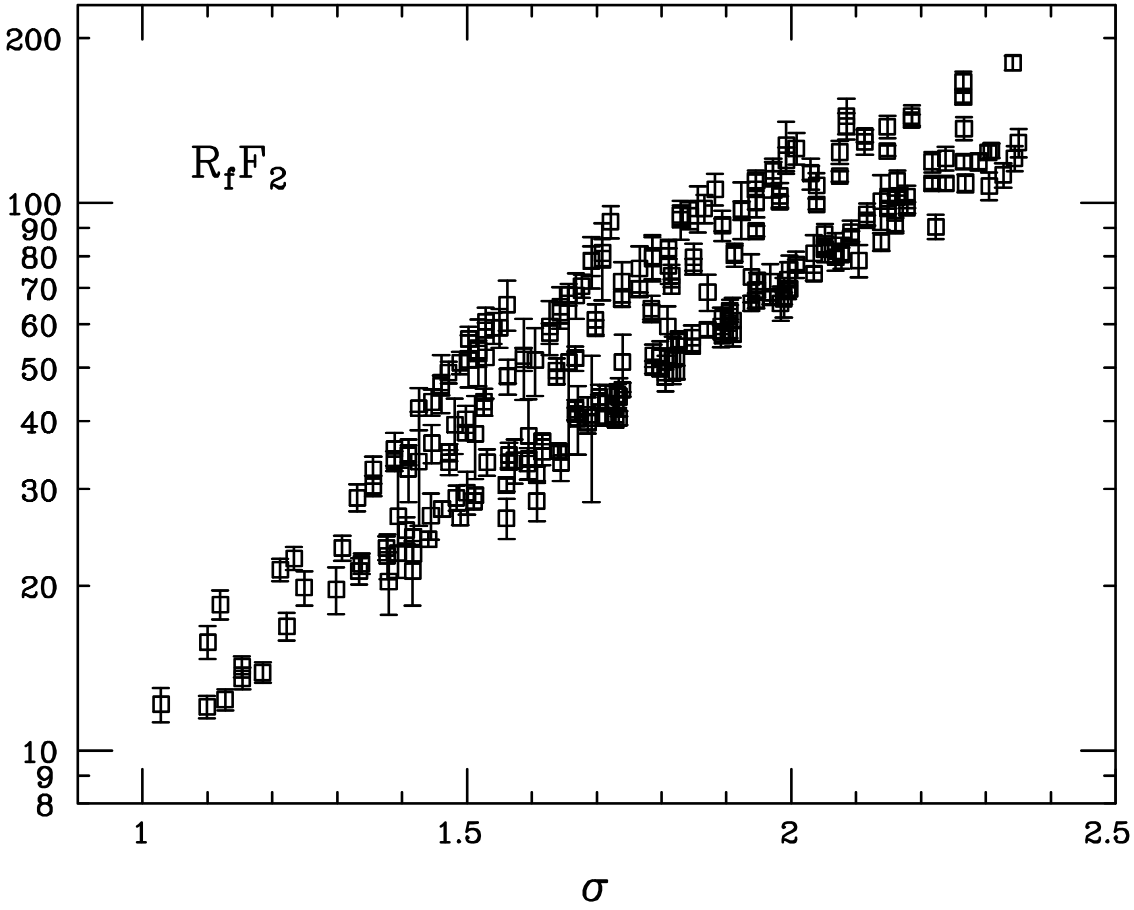

SCALING VARIABLES

$$\sigma = \sqrt{\ln \frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)} \ln \frac{x_0}{x}}$$

$$\rho = \sqrt{\ln \frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)} / \ln \frac{x_0}{x}}$$

RESCALING FACTOR

$$R_F = \rho\sqrt{\sigma} \exp\left(\delta\sigma/\rho\right)$$

$$\delta = (11 + 2n_f/27)/(11 - 2n_f/3)$$

SLOPE

$$2\gamma = 12/\sqrt{33 - 2n_f}$$

H1 94-97 DATA, $10^{-5} \leq x \leq 10^{-1}$, $1 < Q^2 < 1500$ GeV^2;

$\rho \geq 1$, $\sigma \geq 1$, $Q^2 \geq (1.8 \text{ GeV})^2$; $x_0 = 0.1$, $Q_0 = 1$ GeV

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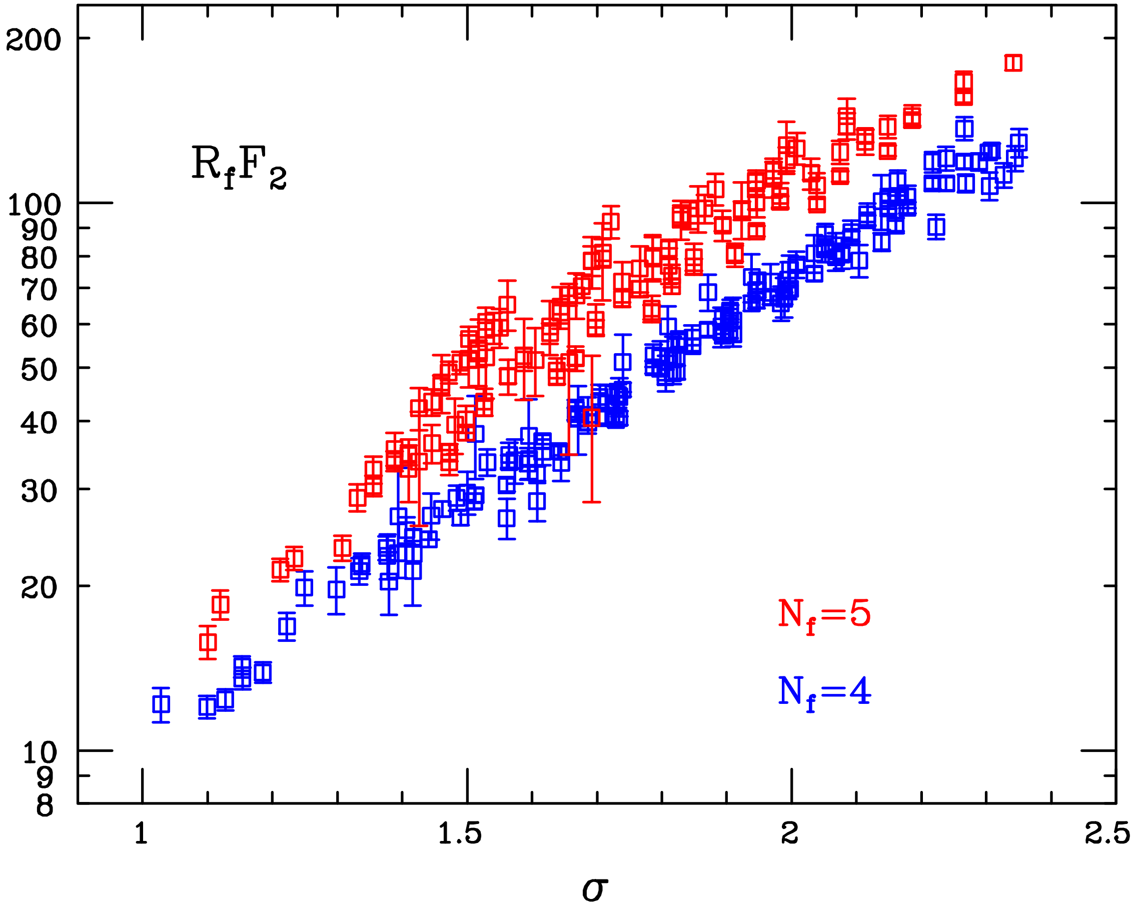

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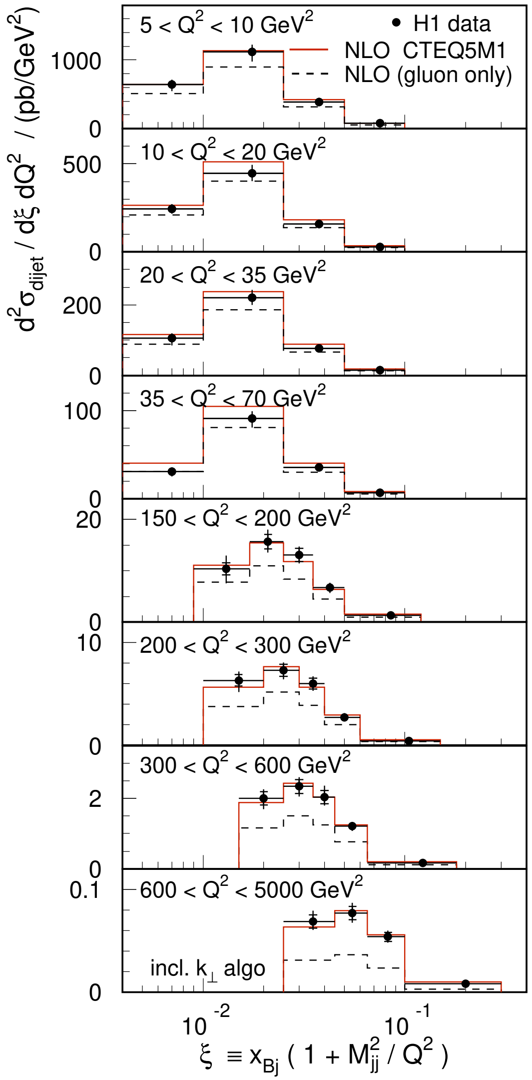
The agreement is equally good for less inclusive observables:

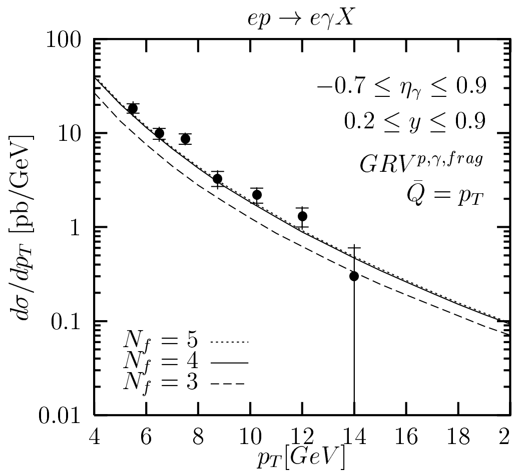
Dijet cross sections (H1)



$x \sim \xi$

PROBED DOWN TO $x \sim 10^{-2} \div 10^{-3}$

Photoproduction of isolated photons (ZEUS)



$x \sim p_T/\sqrt{s};\ \sqrt{s} = 300\ \mathrm{GeV}$

A PROBLEM. . .

LARGE $\ln 1/x \Rightarrow$ EXPECT LARGE CORRECTIONS AT SMALL x

SCALING VIOLATIONS

$$\frac{d}{dt} G(N,Q^2) = \gamma_N G(N,Q^2)$$

$$t \equiv \ln Q^2/\Lambda^2;\ G(N,Q^2) \equiv \int_0^1 x^{N-1} G(x,Q^2)$$

$$\gamma_N \sim \sum_{k=1}^{\infty} c_k \left(\frac{\alpha_s}{N}\right)^k \quad \Leftrightarrow P(x) \sim \frac{1}{x} \sum_{k=0}^{\infty} \frac{c_{k+1}}{k!} \left(\alpha_s \ln \frac{1}{x}\right)^k$$

SERIES FOR SPLITTING FUNCTION CONVERGES FOR ALL $x > 0$

BUT NEED $\sim 2.7 \frac{12 \ln 2}{2\pi} \alpha_s \ln \frac{1}{x}$ TERMS:

@HERA NEED $2 \div 10$ TERMS!

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MORE PROBLEMS

$$c_k = \quad c_k^{(0)} \quad +\alpha_s c_k^{(1)}$$

$\downarrow$ BFKL $\quad\quad \downarrow$ FADIN-LIPATOV

NL SMALL x CORRECTION GROWS LINEARLY WITH PERT. ORDER:

$$\lim_{k\to\infty} c_k^{(1)}/c_k = -Ck;\ C \sim 6\div 16$$

according to factn. scheme

@HERA, ($\alpha_s\times$NLO)/LO$\sim -1.5\div 15$!

BFKL?

SHOULD WE DESCRIBE PERTURBATIVE EVOLUTION AT SMALL x THROUGH A DIFFERENT EVOLUTION EQUATION, WHICH RESUMS LEADING LOGS OF $s \sim \frac{Q^2}{x}$?

EVOLUTION IN $\xi \equiv \frac{1}{x}$:

$$\frac{d}{d\xi} G(\xi, M) = \chi(M, \alpha_s)\; G(\xi, M),$$

$$\xi \equiv \ln \tfrac{1}{x},\; G(\xi, M) \equiv \textstyle\int_{-\infty}^{\infty} dt\, e^{-Mt}\; G(\xi, t),$$

BFKL?

Should we describe perturbative evolution at small x through a different evolution equation, which resums leading logs of $s \sim \frac{Q^2}{x}$?

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"Consistency of 'Altarelli-Parisi' and 'BFKL' evolution and the respective factorizations are established at leading twist. They are related by a set of perturbative duality relations." ALTARELLI, BALL, S.F, 1999-2001 $\Rightarrow$ THIS TALK

SEE ALSO WORK BY CIAFALONI, SALAM AND COLLABORATORS

DUALITY

THE ALTARELLI-PARISI EQN IS AN INTEGRO–DIFFERENTIAL EQUATION $\Rightarrow$ IT CAN BE EQUIVALENTLY VIEWED AS Q^2–EVOLUTION EQUATION FOR x–MOMENTS, OR x–EVOLUTION EQUATION FOR Q^2–MOMENTS

EVOLUTION IN $t = \ln Q^2$

$$\frac{d}{dt} G(N,t) = \gamma(N, \alpha_s)\; G(N,t)$$

MELLIN x–MOMENTS

$$G(N,t) = \int_0^\infty d\xi\, e^{-N\xi}\; G(\xi,t)$$

EVOLUTION IN $\xi = \ln 1/x$

$$\frac{d}{d\xi} G(\xi, M) = \chi(M, \alpha_s)\; G(\xi, M)$$

MELLIN Q^2–MOMENTS

$$G(\xi, M) = \int_{-\infty}^\infty dt\, e^{-Mt}\; G(\xi,t)$$

THE TWO EQUATIONS HAVE THE SAME SOLUTIONS

PROVIDED THE EVOLUTION KERNELS ARE RELATED BY

$$\boxed{\begin{aligned} \chi(\gamma(N,\alpha_s),\alpha_s) &= N \\ \gamma(\chi(M,\alpha_s),\alpha_s) &= M \end{aligned}}$$

& BOUNDARY CONDITIONS RELATED BY

$H_0[M] \to G_0(N) = H_0[\gamma(N,\alpha_s)]$ I.E. $M \to \gamma(N)$ IN B.C.

Running Coupling and Factorization

- The running of the coupling $\alpha(t) = \alpha_{\mu^2}[1 - \beta_0 \alpha_{\mu^2} t + \ldots]$ ($t \equiv \ln \frac{Q^2}{\mu^2}$) is Leading Log Q^2, but Next–to–Leading Log $\frac{1}{x}$
- Upon M-Mellin transformation ($\ln x$ evolution) $\alpha_s(t)$ becomes an operator:

$$\alpha_s(M) = \alpha_{\mu^2}[1 + \beta_0 \alpha_{\mu^2} \frac{d}{dM} + \ldots]$$

$\Rightarrow$ Evolution equation

$\frac{d}{d\xi} G(\xi, M) = \{\alpha_{\mu^2}[1 + \beta_0 \alpha_{\mu^2} \frac{d}{dM} + \ldots](\chi_0(M) + \alpha_{\mu^2}\chi_1(M)) + \ldots\} G(\xi, M)$

Is this compatible with factorization?

What about the boundary condition? $G_0(N) = H_0[\gamma(N, \alpha_s)]$

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What about the boundary condition? $G_0(N) = H_0[\gamma(N, \alpha_s)]$

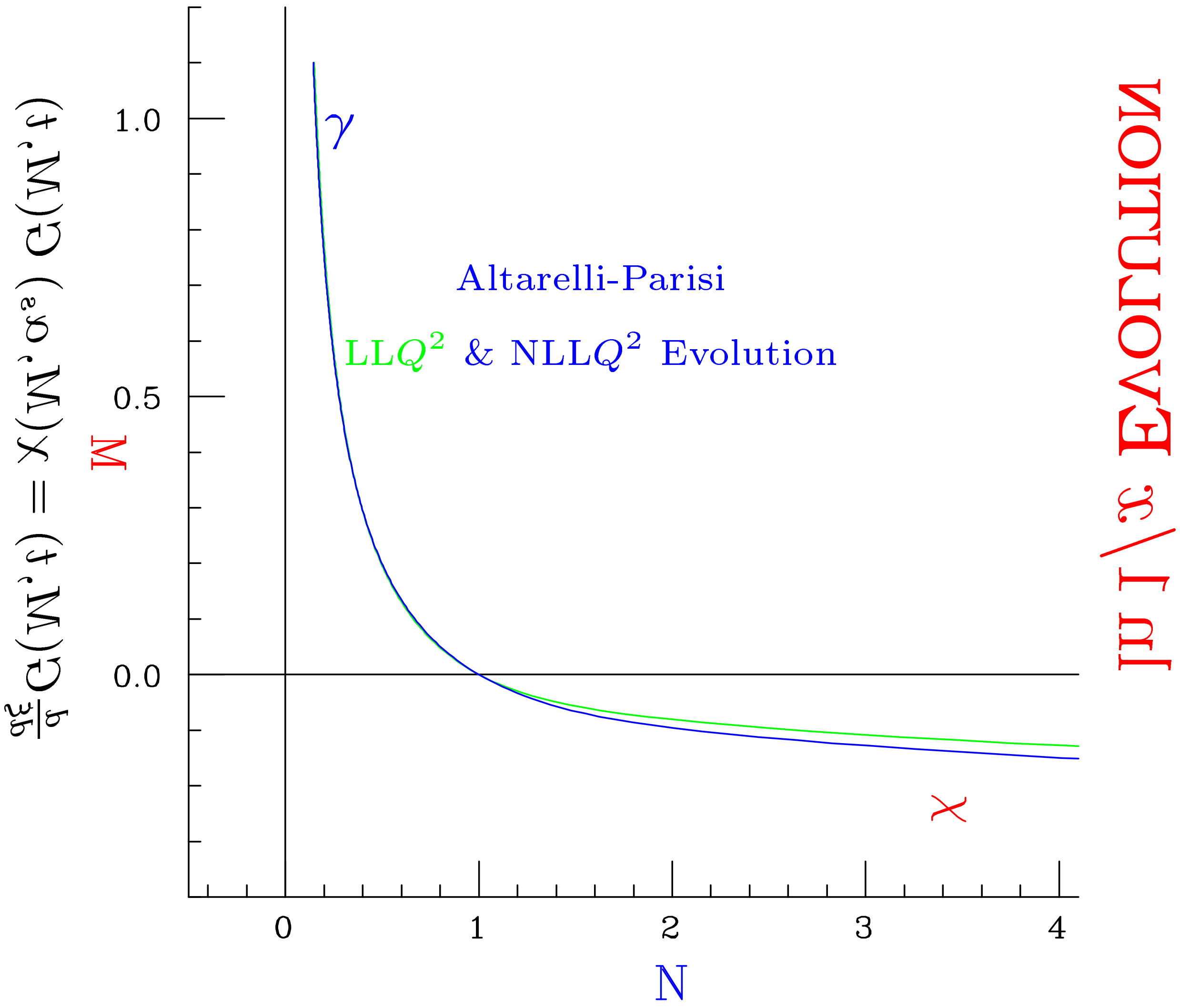
YES!:

- Duality remains valid, provided χ is supplemented by a 'running coupling' contribn.: $\chi_0^{\rm eff} = \chi_0 + \alpha_s \beta_0 \frac{\chi_0'' \chi_0}{{\chi_0'}^2} + \ldots;$
- Matched boundary condition remains Q^2–independent
- Results hold perturbatively up to NNLx (Ball, S.F. 1999) can be proven to all perturbative orders (Altarelli, Ball, S.F. 2001)

THE SAME SOLUTION CAN BE FOUND BY EVOLVING IN $t \equiv \ln Q^2/\mu^2$

AN N-DEPENDENT BOUNDARY CONDITION. . .

$\ln Q^2$ EVOLUTION

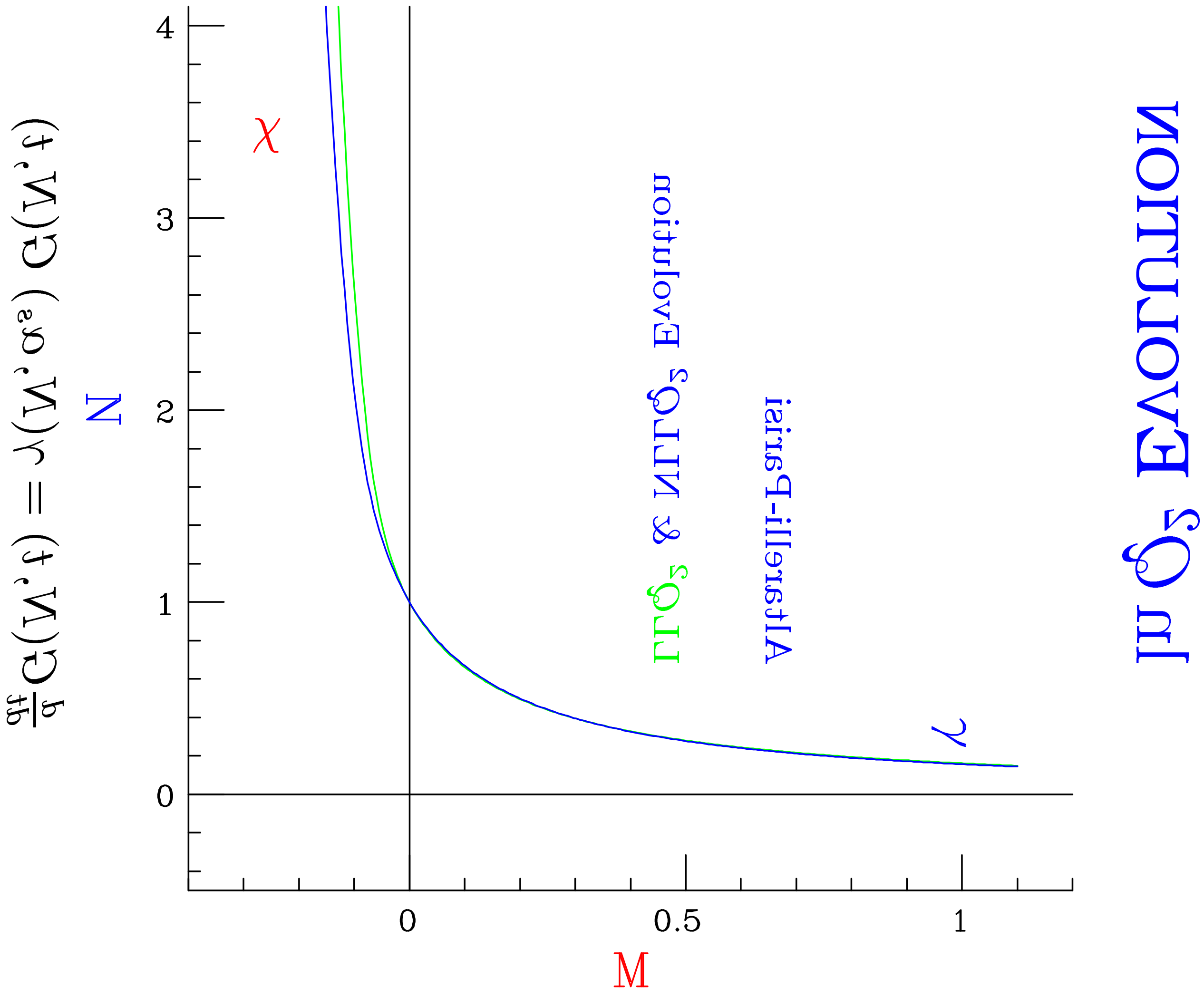


$$\frac{d}{dt} G(N,t) = \gamma(N,\alpha_s)\ G(N,t)$$

. . . OR EVOLVING IN $\xi \equiv \ln 1/x$

AN M-DEPENDENT BOUNDARY CONDITION

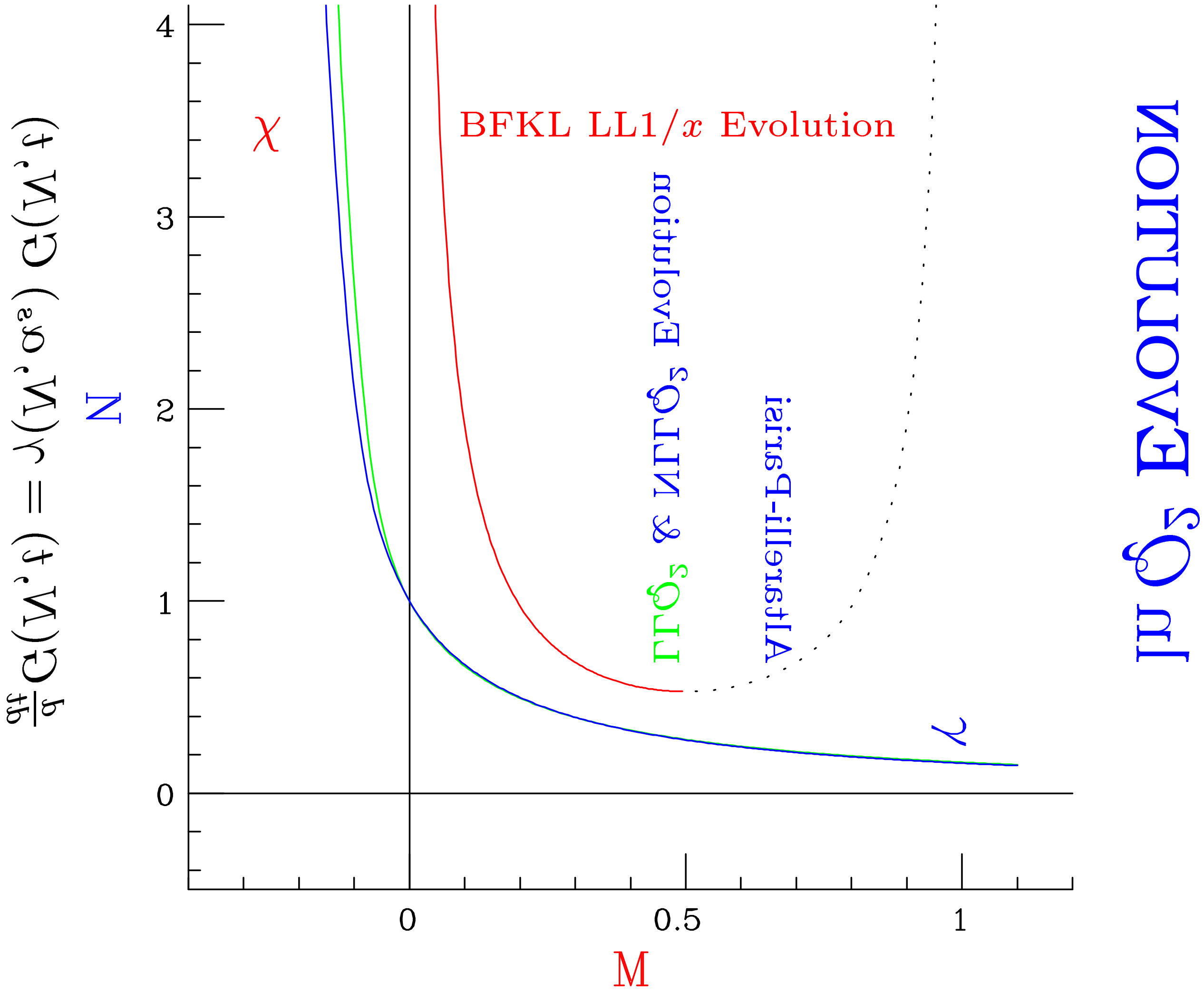
$\ln 1/x$ EVOLUTION



$$\frac{d}{d\xi}G(M,t) = \chi(M,\alpha_s)\ G(M,t)$$

… but one can switch from LLQ^2 to LL$1/x$ by choice of the Evolution Kernel

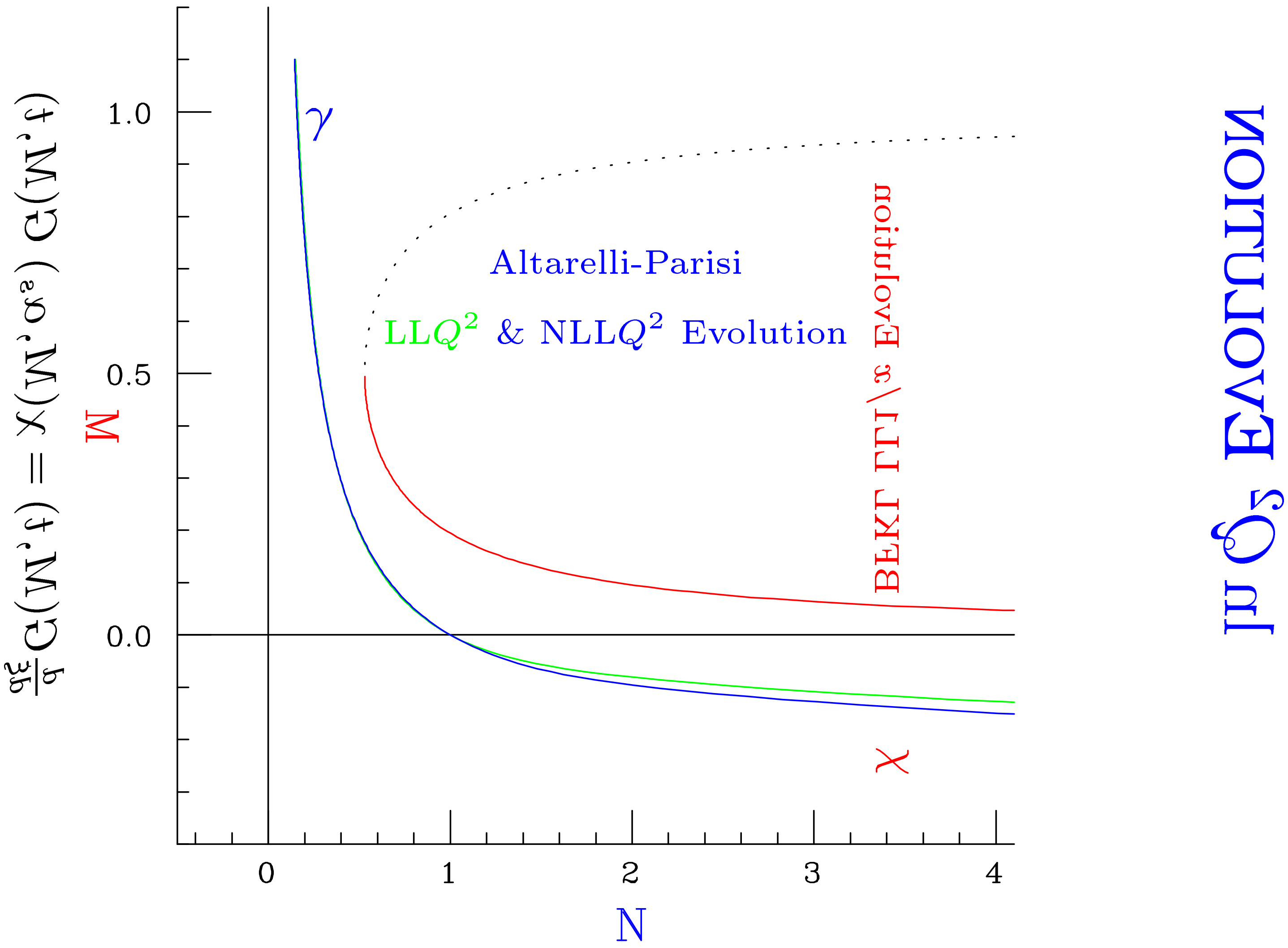
$\ln 1/x$ Evolution



$$\frac{d}{d\xi}G(M,t) = \chi(M,\alpha_s)\ G(M,t)$$

. . . IN EITHER EQUATION!

$\ln 1/x$ Evolution



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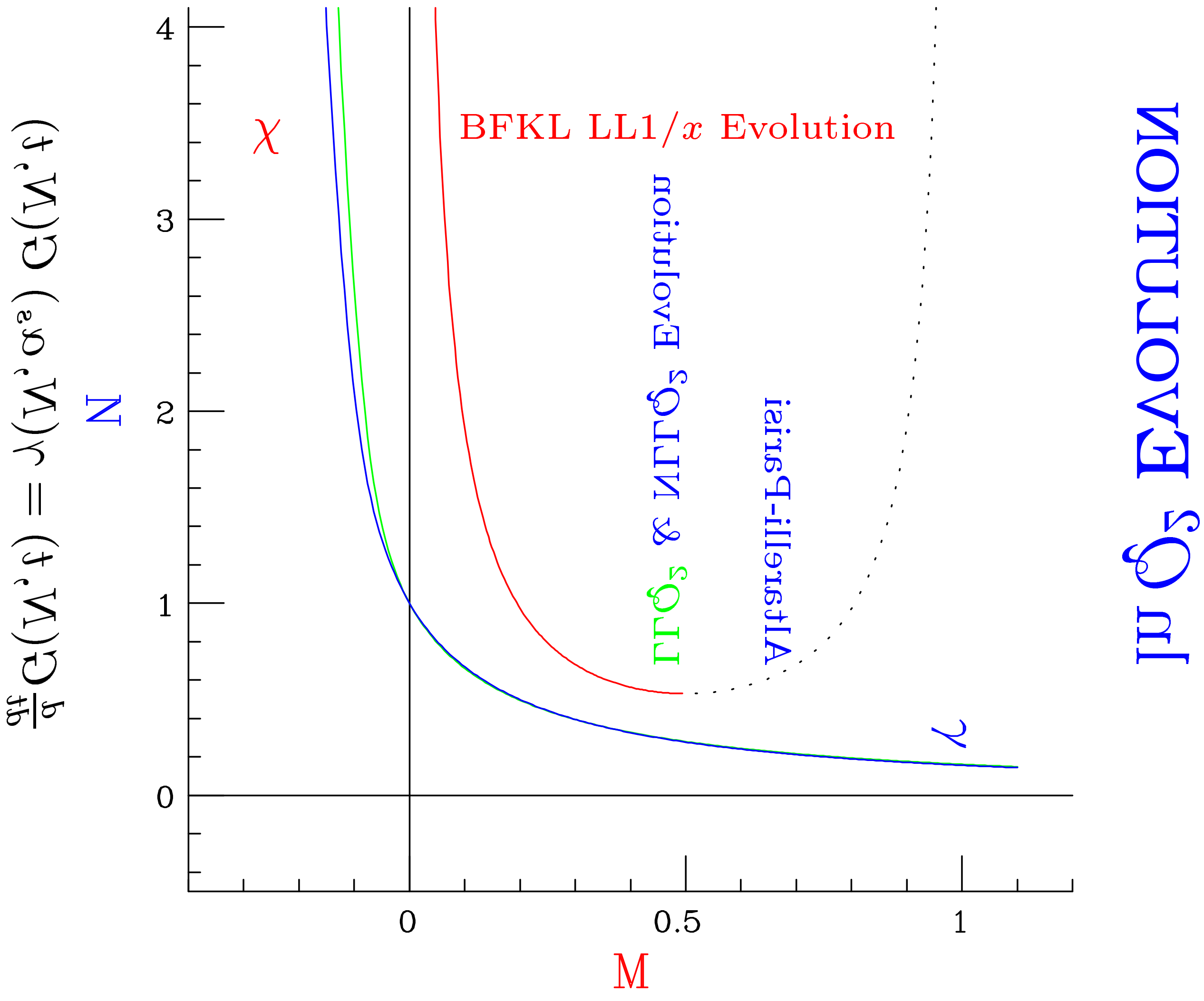
$\ln 1/x$ EVOLUTION

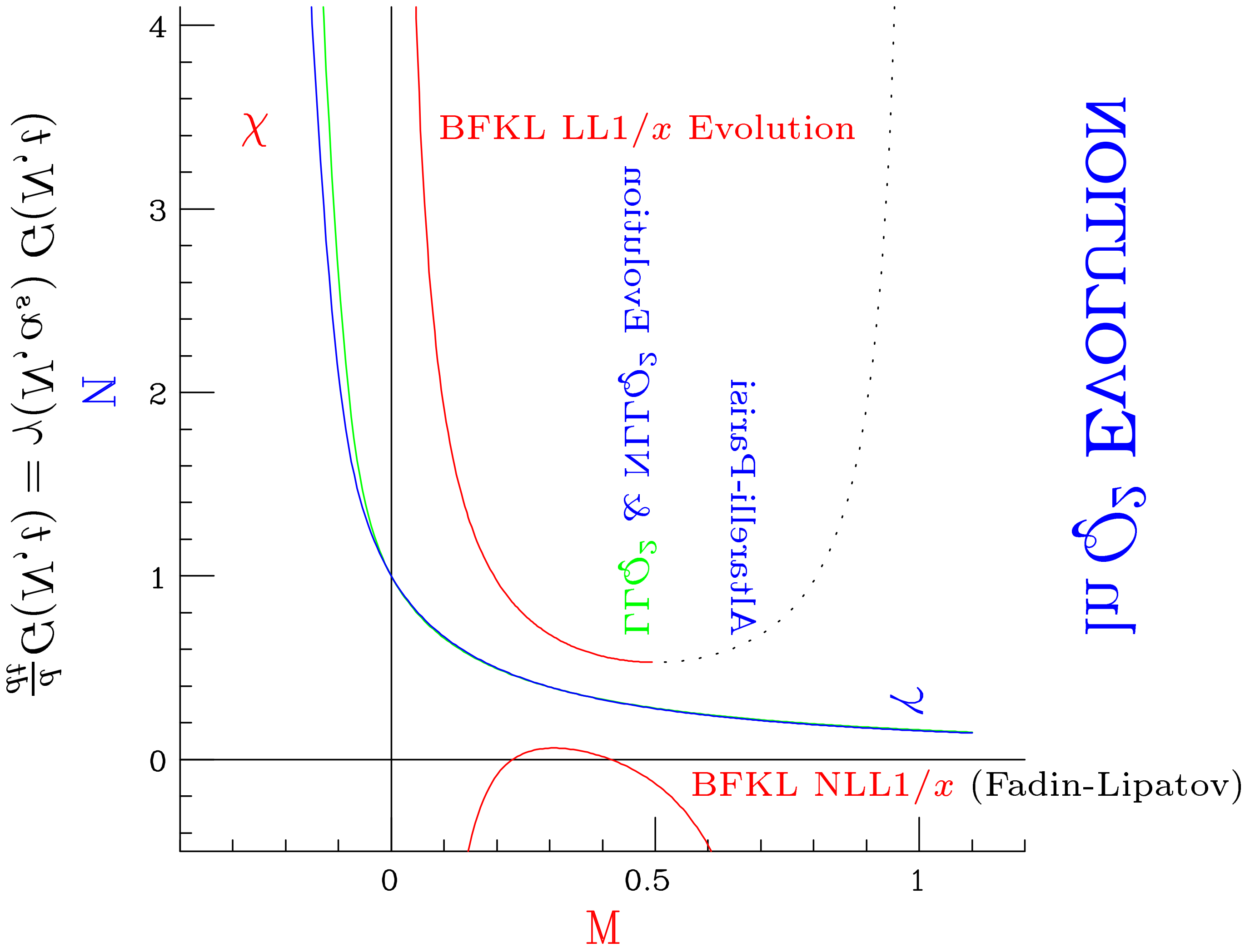


$$\frac{d}{d\xi}G(M,t) = \chi(M,\alpha_s)\;G(M,t)$$

A closer look to LL$1/x$ Evolution: ...

NL corrections!

$\ln 1/x$ Evolution



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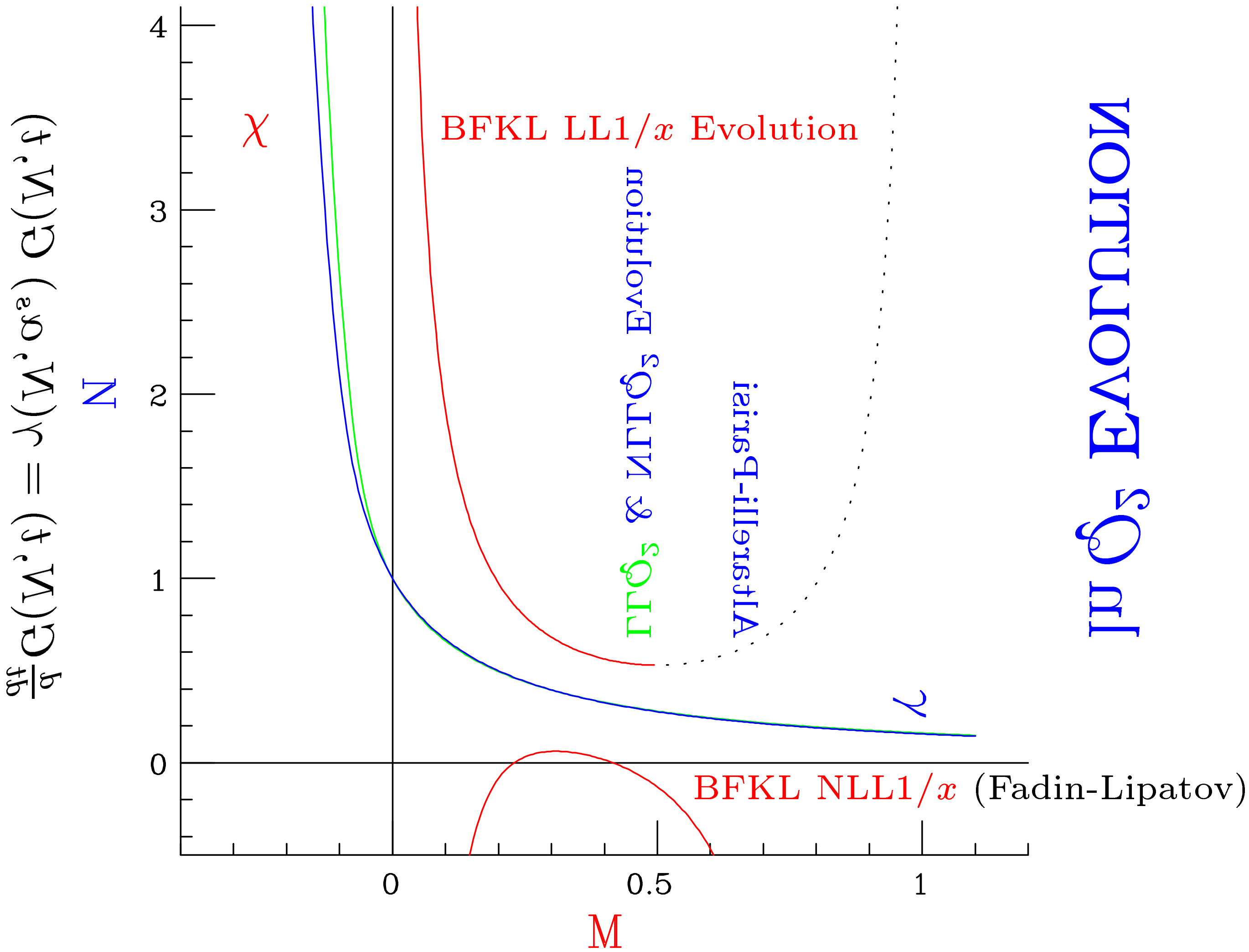
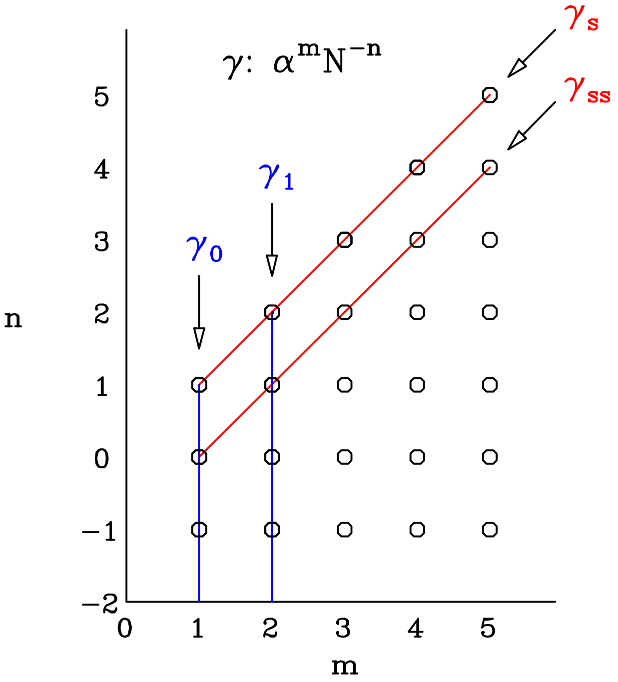
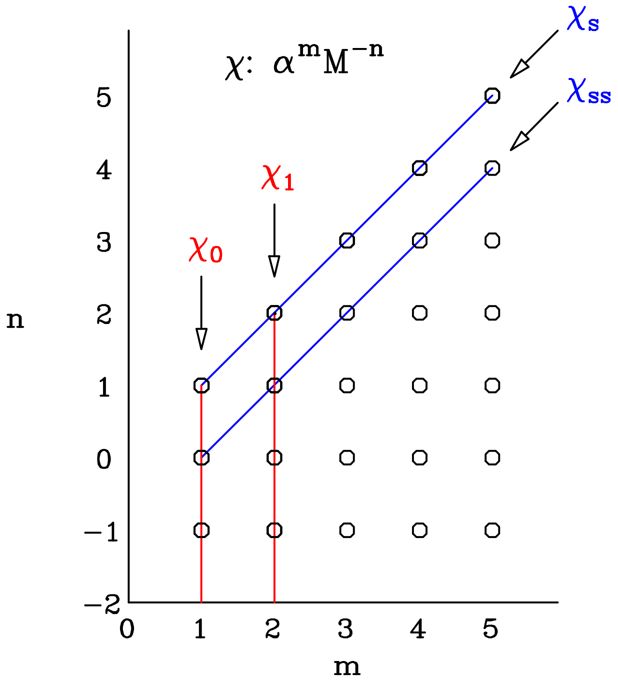


$$\frac{d}{d\xi}G(M,t) = \chi(M,\alpha_s)\ G(M,t)$$

- The LLQ^2 and LL$1/x$ kernels greatly differ from each other
- The expansion of the LL$1/x$ kernel looks very unstable

DUALITY: A CLOSER LOOK





$\ln Q^2$ EVOLUTION

$\ln 1/x$ EVOLUTION

$$\gamma(N) = \alpha\left(\frac{c^{(1)}_{-1}}{N} + c^{(1)}_0 + \ldots\right) + \alpha^2\left(\frac{c^{(2)}_{-2}}{N^2} + \frac{c^{(2)}_{-1}}{N} + \ldots\right)$$

$$\chi(M) = \alpha\left(\frac{\tilde{c}^{(1)}_{-1}}{M} + \tilde{c}^{(1)}_0 + \ldots\right) + \alpha^2\left(\frac{\tilde{c}^{(2)}_{-2}}{M^2} + \frac{\tilde{c}^{(2)}_{-1}}{M} + \ldots\right)$$

$$\gamma_s(N) = c^{(1)}_{-1}\frac{\alpha}{N} + c^{(2)}_{-2}\frac{\alpha^2}{N^2} + \ldots$$

$$\chi_s(M) = \tilde{c}^{(1)}_{-1}\frac{\alpha}{M} + \tilde{c}^{(2)}_{-2}\frac{\alpha^2}{M^2} + \ldots$$

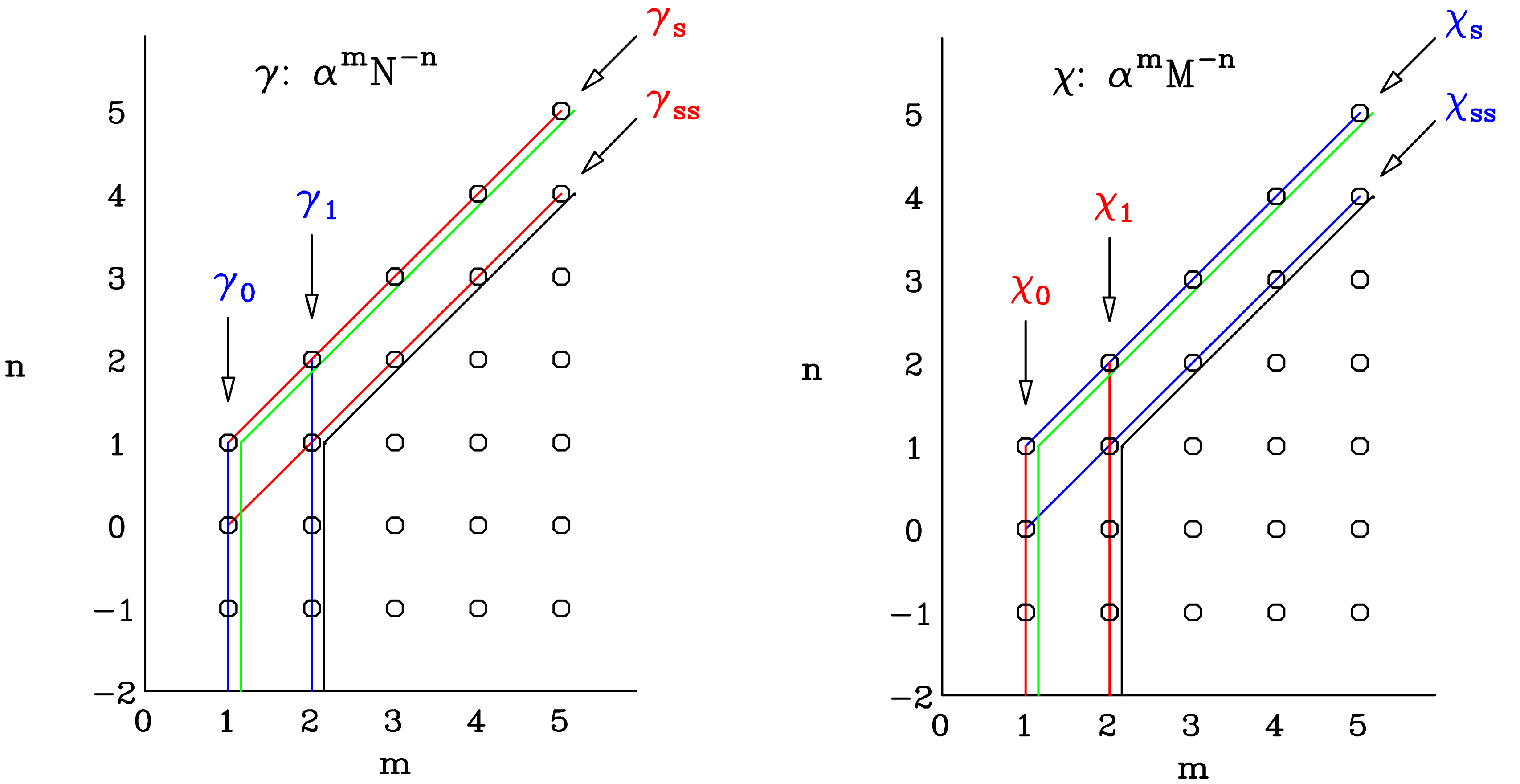
$1/N$ POLES $\Leftrightarrow \ln 1/x$

$1/M$ POLES $\Leftrightarrow \ln Q^2$

$$\gamma_0(N) \Longleftarrow\!\!\Longrightarrow \chi_s(\alpha_s/M)$$

$$\gamma_s(\alpha_s/N) \Longleftarrow\!\!\Longrightarrow \chi_0(M)$$

THE DOUBLE–LEADING EXPANSION



$$\gamma(N,\alpha_s) = \left[\alpha_s\gamma_0(N) + \gamma_s\left(\frac{\alpha_s}{N}\right) - \frac{n_c\alpha_s}{\pi N}\right] + \alpha_s\left[\alpha_s\gamma_1(N) + \gamma_{ss}\left(\frac{\alpha_s}{N}\right) - \alpha_s\left(\frac{e_2}{N^2} + \frac{e_1}{N}\right) - e_0\right] + \cdots$$

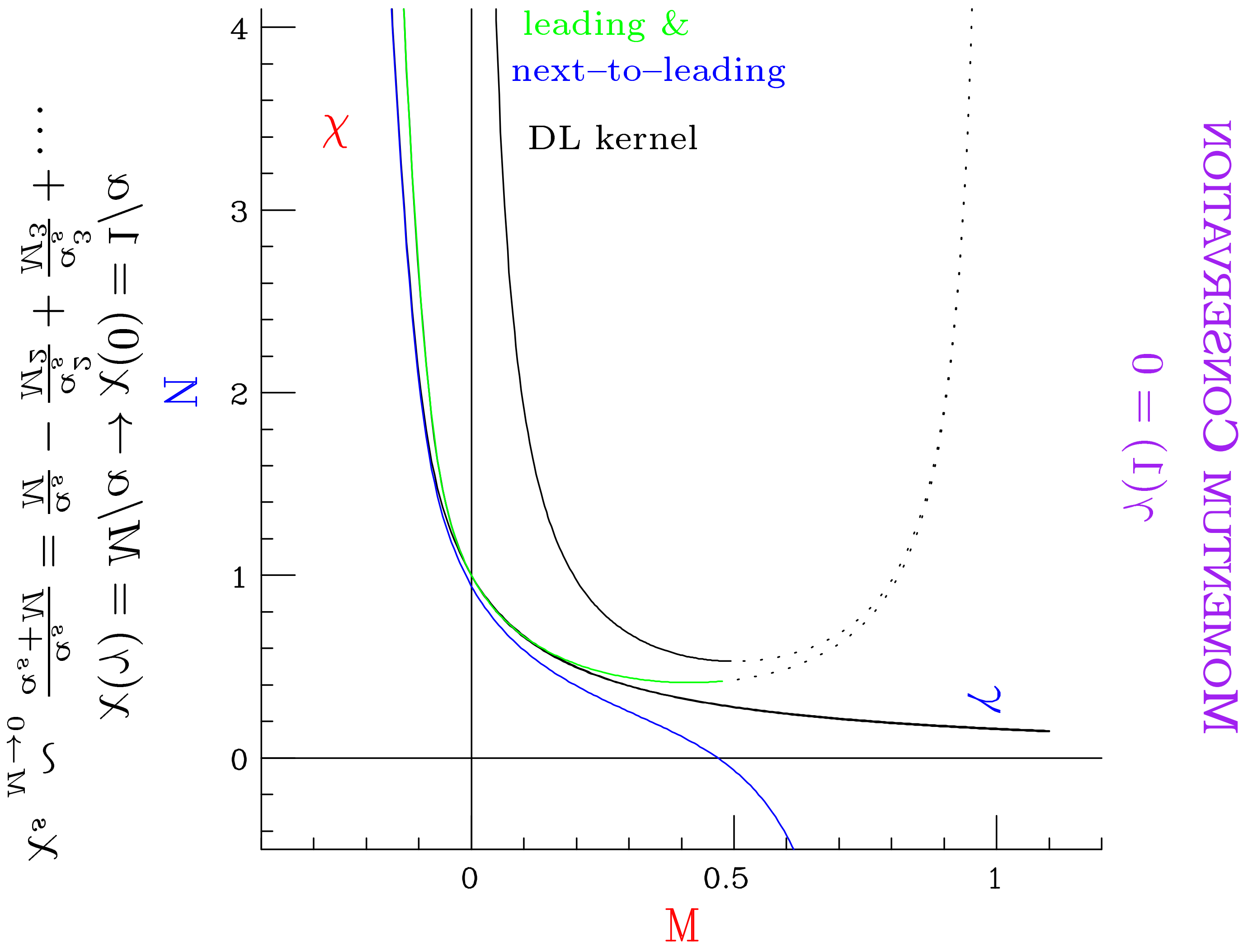
$$\Leftarrow \Rightarrow$$

$$\chi(M,\alpha_s) = \left[\alpha_s\chi_0(M) + \chi_s\left(\frac{\alpha_s}{M}\right) - \frac{n_c\alpha_s}{\pi M}\right] + \alpha_s\left[\alpha_s\chi_1(M) + \chi_{ss}\left(\frac{\alpha_s}{M}\right) - \alpha_s\left(\frac{f_2}{M^2} + \frac{f_1}{M}\right) - f_0\right] + \cdots$$

DUALITY HOLDS ORDER–BY–ORDER IN THE DOUBLE–LEADING EXPANSION:

the dual of $\chi_{\rm DL}^{\rm LO}$ is $\gamma_{\rm DL}^{\rm LO}$ up to terms of order $\gamma_{\rm DL}^{\rm NLO}$, and conversely

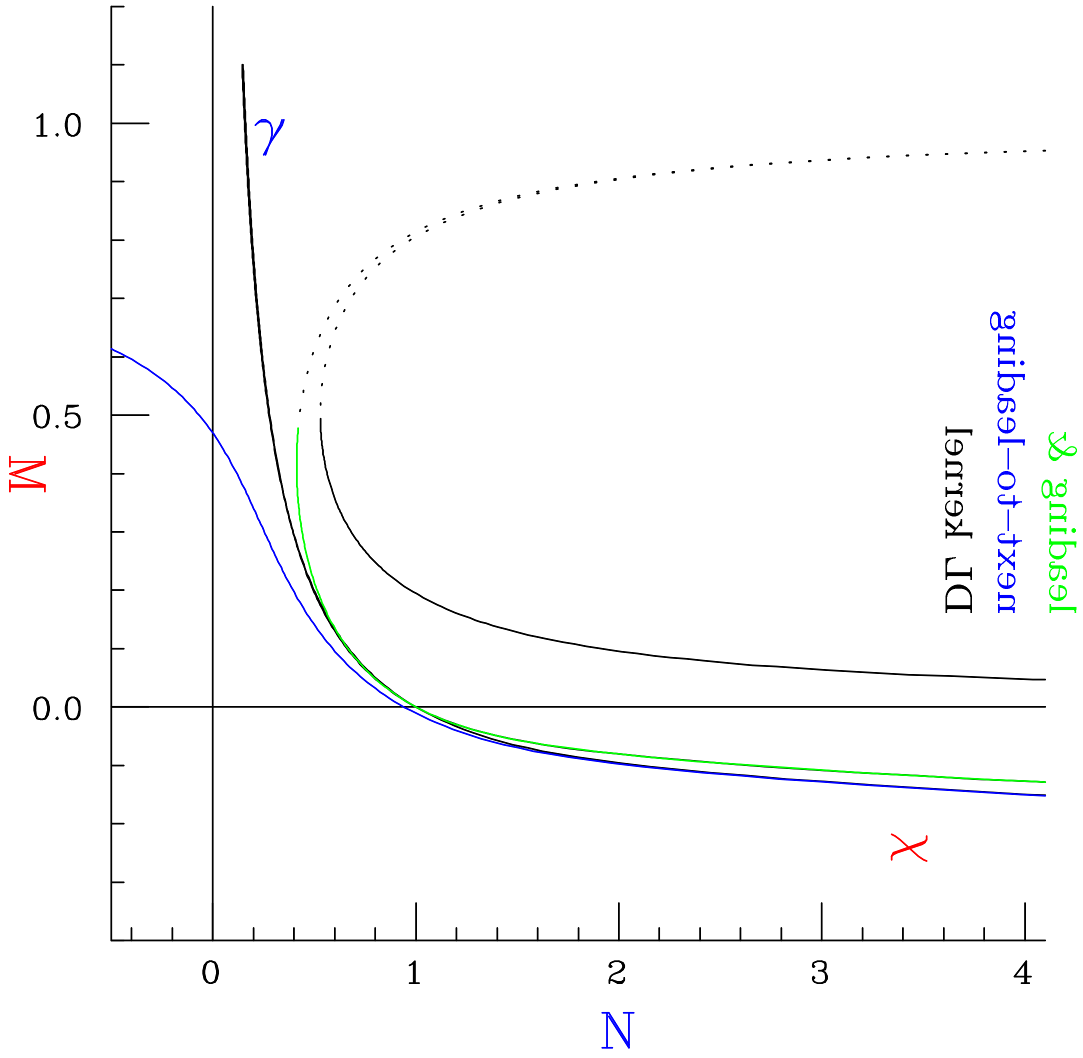
Double–Leading Evolution



- The DL kernel has a well–behaved perturbative expansion
- DL is almost everywhere close to the LLQ^2 result, close to LL$1/x$ only when $M \sim 1/2$

Double–Leading Evolution

Momentum Conservation

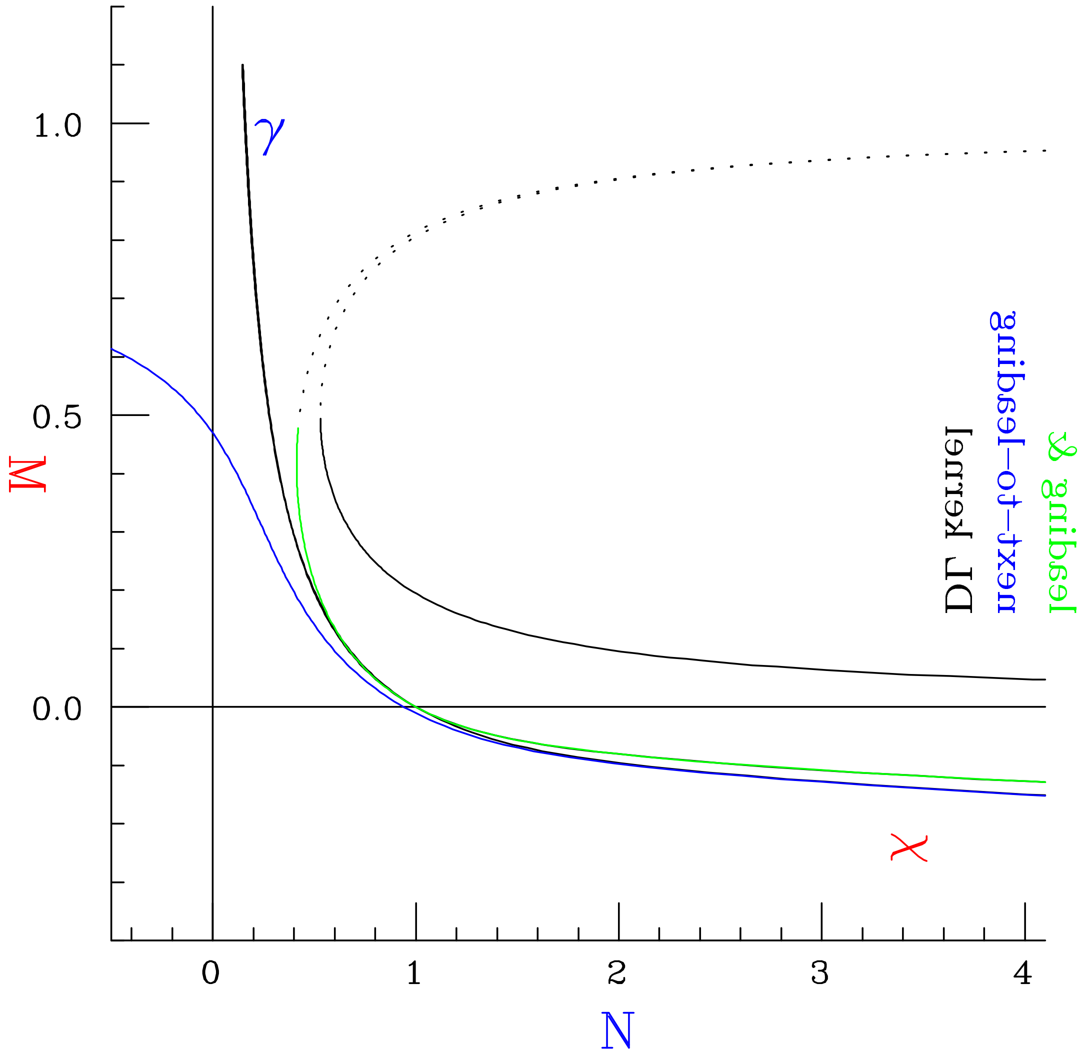


$$\chi(\gamma) = M$$

Double–Leading Evolution

Momentum Conservation

$\gamma(1) = 0$



$$\chi(\gamma) = M \rightarrow \chi(0) = 1$$

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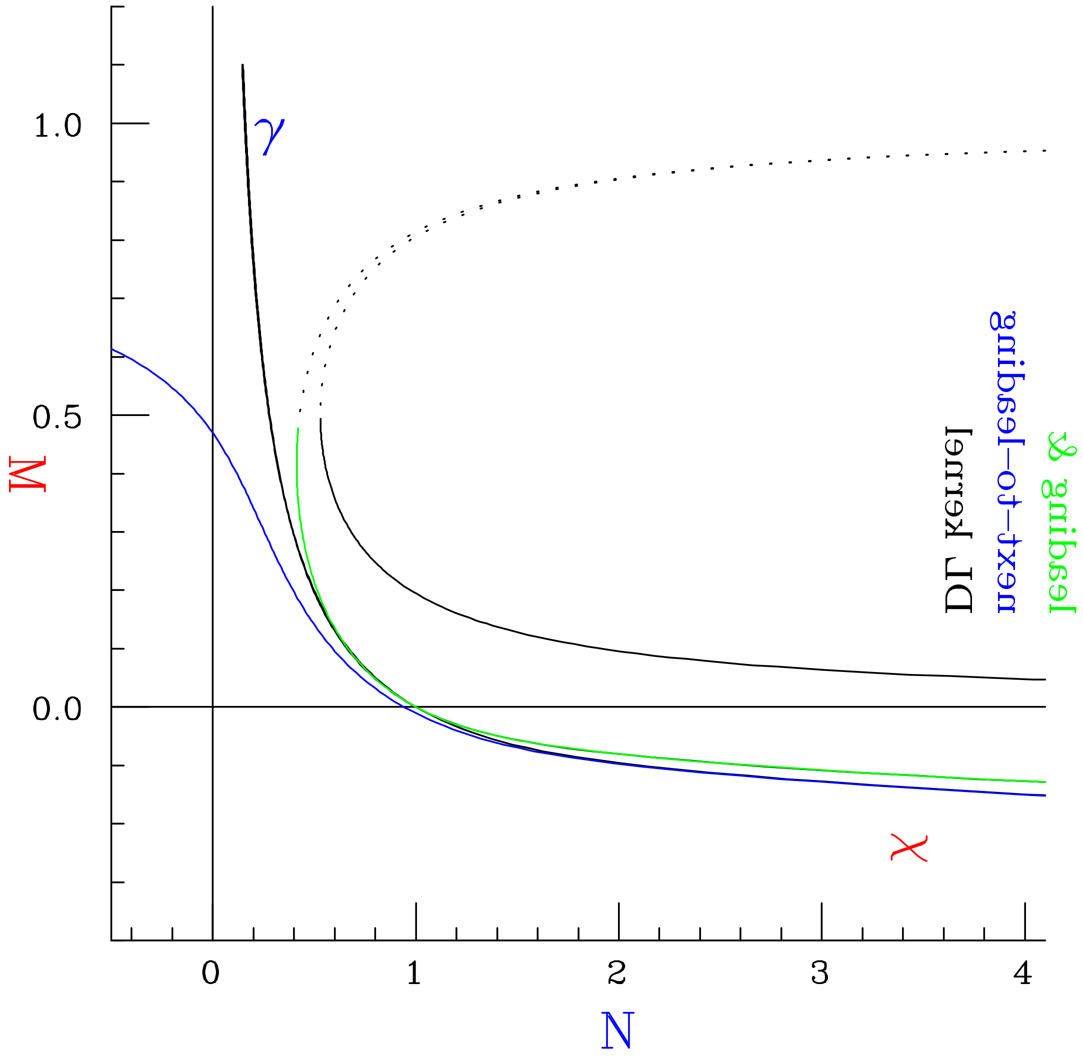
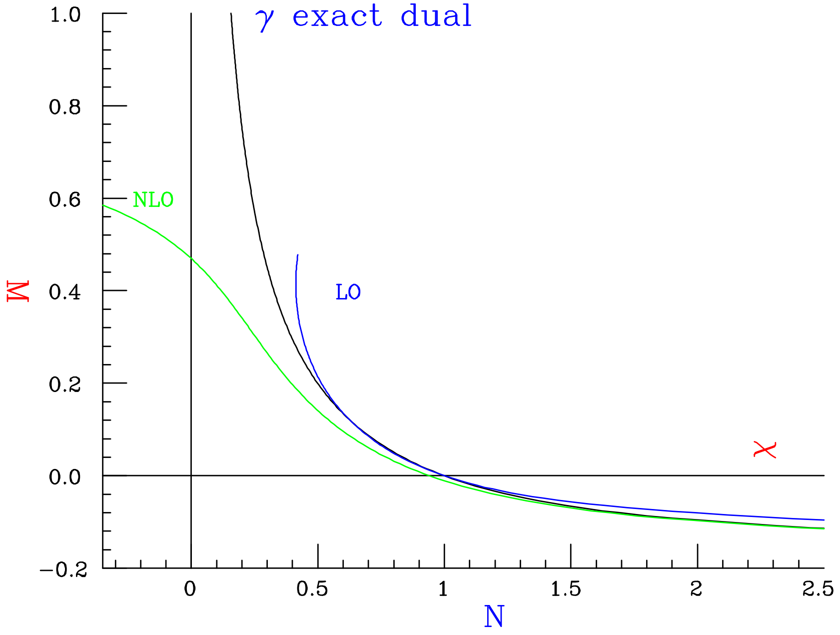


$$\chi(\gamma) = M \rightarrow \chi(0) = 1$$

$$\chi(M) \underset{M \to 0}{\sim} \frac{\alpha}{\alpha + M} = \frac{\alpha}{M} - \frac{\alpha^2}{M^2} + \frac{\alpha^3}{M^3} + \dots$$

RESUMMATION



- THE NLO TERM IN THE DL EXPANSION IS A SMALL CORRECTION TO χ AT FIXED M FOR ALL M, BUT FOR SMALL $N \sim 0.5$ IT IS A LARGE CORRECTION TO γ AT FIXED N
- NAIVE PERTURBATIVE EXPANSION OF γ_s, γ_{ss} DUAL TO χ_0, χ_1 IS UNSTABLE

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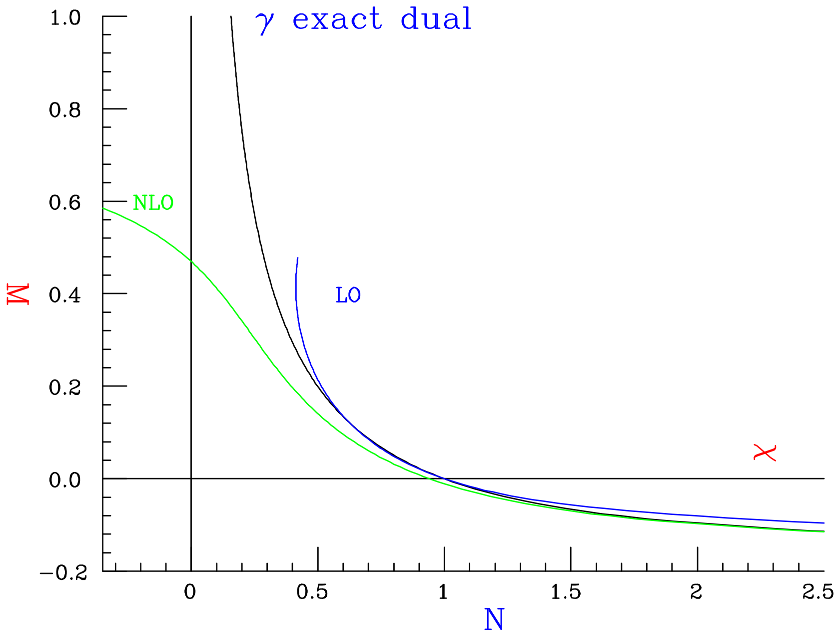

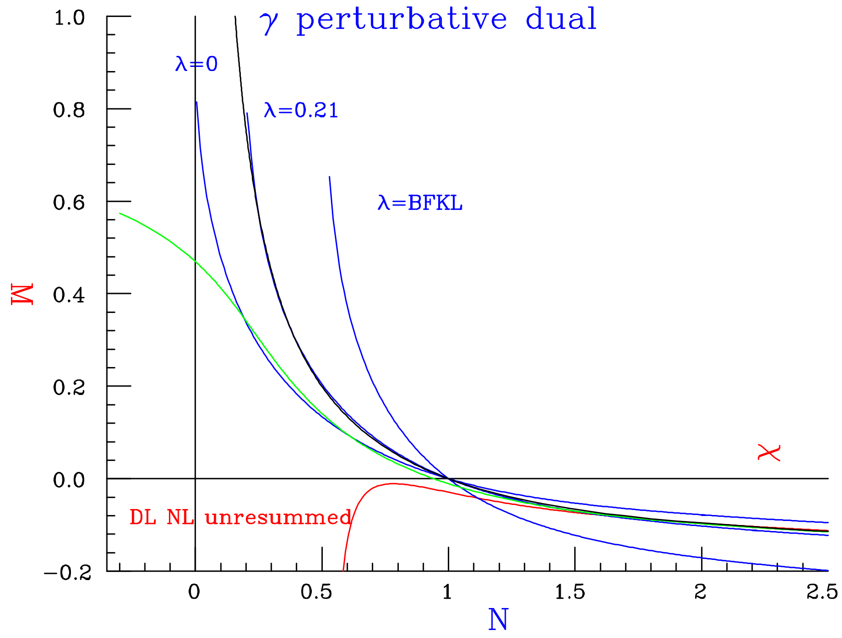

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- MUST RESUM ALL–ORDER MINIMUM INTO THE LEADING ORDER:

$$\chi(M) = (\chi_0(M) + \alpha\chi_1(1/2) + \ldots) \quad \Leftarrow \text{Resummed LO}$$
$$+\alpha\left(\chi_1(M) - \alpha\chi_1(1/2) + \ldots\right) + \ldots$$

- RESUMMATION PARAMETRIZED BY ASYMPTOTIC SMALL x BEHAVIOUR

$$\chi_0^{\mathrm{res}}(M_{\mathrm{min}}) = \lambda \Leftrightarrow F_2 \underset{x \to 0}{\sim} x^\lambda$$

PHENOMENOLOGY

Can resum the full set of anomalous dimensions and coefficient functions at NLO in the double-leading expansion

- Resummed results depend on unknown all-order asympt. small x behaviour, parametrized by λ
- Does the perturbative $N = 0$ sing. survive resummation?
  $\Rightarrow$ Further uncertainty: S (pole survives) vs. R (pole resummed to cut)

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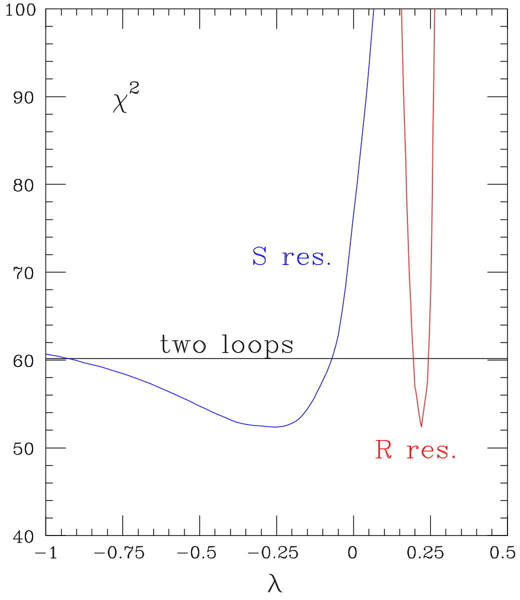

- RESUMMED RESULTS DEPEND ON UNKNOWN ALL-ORDER ASYMPT. SMALL x BEHAVIOUR, PARAMETRIZED BY λ
- DOES THE PERTURBATIVE $N = 0$ SING. SURVIVE RESUMMATION? $\Rightarrow$ FURTHER UNCERTAINTY: S (pole survives) VS. R (pole resummed to cut)
- QUALITY OF THE RESUMMED FIT AS GOOD AS OR BETTER THAN TWO LOOPS
- SMALL λ ("SOFT POMERON") FAVOURED; EXPONENT MUST BE FINE–TUNED IF PERT. SINGULARITY KILLED BY RESUMMATION (R-RES.)

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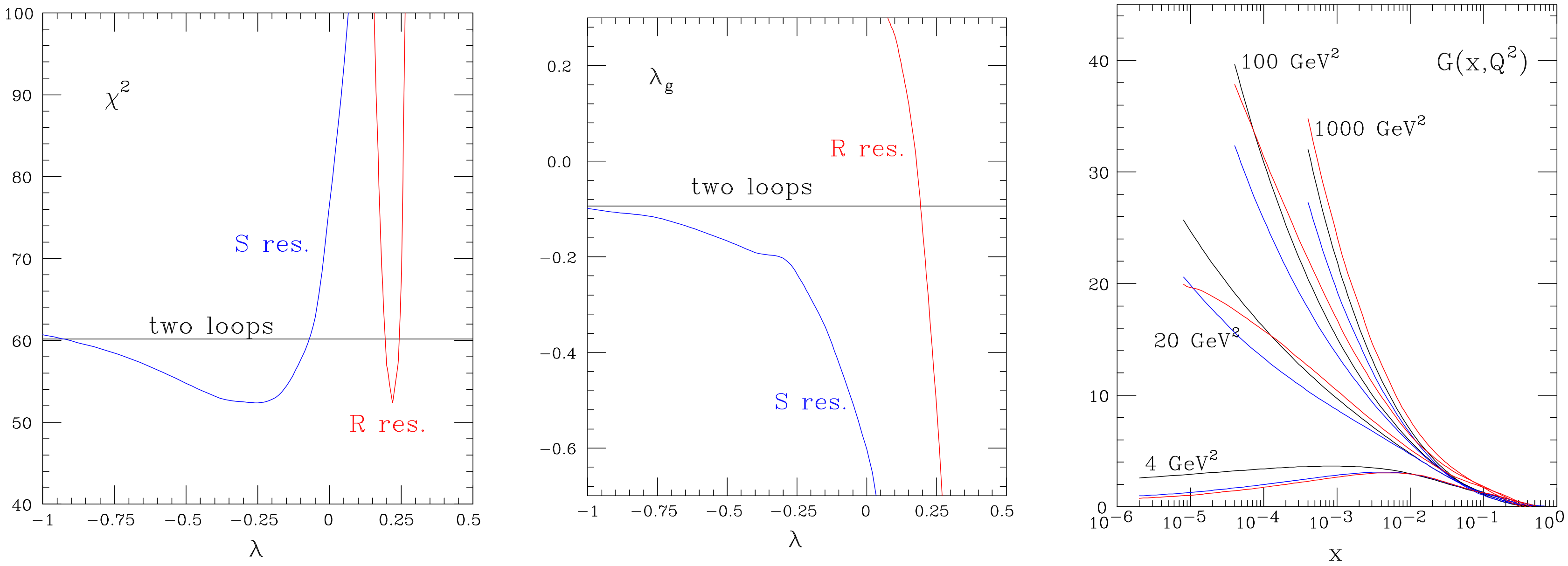

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- Does the perturbative $N = 0$ sing. survive resummation?
  $\Rightarrow$ Further uncertainty: S (pole survives) vs. R (pole resummed to cut)
- Quality of the resummed fit as good as or better than two loops
- Small λ ("soft pomeron") favoured; exponent must be fine–tuned if pert. singularity killed by resummation (R-res.)
- Significant impact on gluon: "softer" (valencelike) at small x, small Q^2

CONCLUSIONS

- The matching of Altarelli-Parisi and BFKL evolution is now fully understood in leading twist, large Q^2 factorization.
- When all available info on perturbative evolution is combined, the small–x instabilities go away
- The Regge limit is perturbatively stable only after an all–order resummation, which necessarily depends on (at least) one nonpert. parameter
- Resummed results can be very close to fixed–order, but gluon and α_s sizeably affected

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- THEORY: The success of NLO perturbation theory poses stringent constraints on the unknown higher–order structure of anomalous dimensions
- PHENOMENOLOGY: Resummation is crucial for reliable phenomenology @ small x