QCD Analysis of Polarized Structure Functions

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Outline:

- **Formalism**
- **World Data**
- **Parton Distributions**
- **New Parton Parametrizations with Errors**
- $\Lambda_{\rm QCD}$ **and** $\alpha_s(M_Z)$
- **Factorization Scheme Invariant Evolution**
- **Conclusions**

Martina Franca, June 17th 2000

Polarized Structure Function $g_1(x)$

- Quark Parton Model (LO) :

$$g_1(x) = \frac{1}{2}\sum_{i=1}^{n_f} e_i^2 \left[\Delta q_i(x) + \Delta\overline{q}_i(x)\right]$$

$$\Delta q_i(x) \quad = \quad \left(q_i^+(x) - q_i^-(x)\right) \ ,$$

$q_i^{+(-)}$ is the quark density with helicity aligned(anti-aligned) to the helicity of the parent nucleon (for $\Delta\overline{q}_i(x)$ accordingly)

- QCD Improved Quark Parton Model (NLO) :

$$\begin{aligned} g_1(x, Q^2) \quad &= \quad \frac{1}{2}\left(\frac{1}{n_f}\sum\nolimits_{i=1}^{n_f} e_i^2\right)\left[\delta C_S \otimes \Delta\Sigma\right. \\ &\qquad \left. + \delta C_G \otimes \Delta G + \delta C_{NS} \otimes \Delta q^{NS}\right] \end{aligned}$$

The symbol $\otimes$ denotes convolution w.r.t. x of the polarized parton densities $\Delta q_i(x, Q^2)$ with the Wilson coeffient functions $\delta C_i(x, \alpha_s(Q^2))$:

$$f(x) \otimes g(x) = \int_0^1 dx_1 \int_0^1 dx_2 \delta(x - x_1 x_2) f(x_1) g(x_2)$$

Polarized Structure Function $g_1(x)$

– Singlet Polarized Quark Distribution :

$$\Delta\Sigma(x,Q^2) = \sum_{i=1}^{n_f} \Delta^+ q_i(x,Q^2)$$

with

$$\Delta^+ q_i(x,Q^2) = \left[\Delta q_i(x,Q^2) + \Delta\bar{q}_i(x,Q(2)\right]$$

– Polarized Gluon Distribution : $\Delta G(x,Q^2)$

– Non-Singlet Polarized Quark Distribution :

$$\Delta q^{NS}(x,Q^2) = \frac{\sum_{i=1}^{n_f}\left(e_i^2 - \frac{1}{n_f}\sum_{k=1}^{n_f} e_k^2\right)\Delta^+ q_i(x,Q^2)}{\frac{1}{n_f}\sum_{k=1}^{n_f} e_k^2}$$

- For $n_f = 3\,(u,d,s,\bar{u},\bar{d},\bar{s})$:

$$\begin{aligned} \Delta\Sigma &= (\Delta u + \Delta\bar{u}) + (\Delta d + \Delta\bar{d}) + (\Delta s + \Delta\bar{s}) \\ \Delta q^{NS}_{p(n)} &= +(-)\frac{3}{4}\Delta q_3 + \frac{1}{4}\Delta q_8 \end{aligned}$$

with

$$\begin{aligned} \Delta q_3 &= (\Delta u + \Delta\bar{u}) - (\Delta d + \Delta\bar{d}) \\ \Delta q_8 &= (\Delta u + \Delta\bar{u}) + (\Delta d + \Delta\bar{d}) - 2(\Delta s + \Delta\bar{s}) \end{aligned}$$

Evolution Equations

- Evolution Equations to go from Q_0^2 to Q^2 $(t \equiv lnQ^2)$
  - for $\Delta\Sigma$ and ΔG :

$$\frac{d}{dt}\begin{pmatrix} \Delta\Sigma(x,t) \\ \Delta G(x,t) \end{pmatrix} = \frac{\alpha_s(t)}{2\pi}\begin{pmatrix} \Delta P_{qq}(x) & \Delta P_{qG}(x) \\ \Delta P_{Gq}(x) & \Delta P_{GG}(x) \end{pmatrix} \otimes \begin{pmatrix} \Delta\Sigma(x,t) \\ \Delta G(x,t) \end{pmatrix}$$

  - for Δq^{NS} :

$$\frac{d}{dt}\Delta q^{NS}(x,t) = \frac{\alpha_s(t)}{2\pi}\Delta P_{qq}^{NS}(x) \otimes \Delta q^{NS}(x,t)$$

with $\Delta P_{ij}(x)$ the polarized splitting functions.

- The polarized Wilson coefficient functions $\delta C_i(x, \alpha_s(Q^2))$ and the polarized splitting functions $\Delta P_{ij}(x, \alpha_s(Q^2))$ are known in the $\overline{MS}$ scheme up to $\mathcal{O}(\alpha_s^2)$. [E.B. Zijlstra and W.L. van Neerven, Nucl. Phys. B417 (1994) 61, R. Mertig and W.L. van Neerven, Z. Phys. C70 (1996) 637, W. Vogelsang, Phys. Rev. D54 (1996) 2023]

$\Longrightarrow$ NLO QCD Analysis: Determination of $\Lambda_{\rm QCD}$ and the Parton densities with Errors.

Parametrization

- General choice for the parametrization of the polarized parton distributions at Q_0^2:

$$x\Delta q_i(x, Q_0^2) = \eta_i A_i x^{a_i}(1-x)^{b_i}(1+\gamma_i x+\rho_i x^{\frac{1}{2}})$$

- Normalization:

$$A_i^{-1} = \left(1+\gamma_i \frac{a_i}{a_i+b_i+1}\right)\frac{\Gamma(a_i)\Gamma(b_i+1)}{\Gamma(a_i+b_i+1)} + \rho_i \frac{\Gamma(a_i+0.5)\Gamma(b_i+1)}{\Gamma(a_i+b_i+1.5)}$$

such that

$$\int_0^1 dx \Delta q_i(x, Q_0^2) = \eta_i$$

are the first moment of $\Delta q_i(x, Q_0^2)$

- The polarized parton distributions to be fitted are:

$$\Delta u_v \;,\; \Delta d_v \;,\; \Delta\bar{q} \;,\; \Delta G$$

where the index v denotes the $valence$ quark.

Note that: $\Delta q + \Delta\bar{q} = \Delta q_v + 2\Delta\bar{q}$.

Choice of Parameters

- $Q_0^2 = 4.0\, GeV^2$
- SU(3) flavour symmetry assumed

  η_{u_v} and η_{d_v} determined from F and D the SU(3) parameters involved in the matrix elements describing the neutron and hyperon β-decays:

  $\eta_{u_v} = 2F = 0.926$; $\eta_{d_v} = F - D = -0.341$
- Flavor symmetric sea assumed

  $\Delta\bar{u}(x, Q_0^2) = \Delta\bar{d}(x, Q_0^2) = \Delta\bar{s}(x, Q_0^2) = \Delta\bar{q}(x, Q_0^2)$
- No assumption made concerning positivity and helicity retention
- For u_v and d_v : $\rho_{u_v} = \rho_{d_v} = 0$
- For the gluon : $\gamma_G = \rho_G = 0$ (Gluon A)
- For the sea : $\gamma_{\bar{q}} = \rho_{\bar{q}} = 0$ (Sea A)
- The normalizations of the different data sets against each other were fitted and fixed afterwards
- The remaining 12 parameters to be determined are:

  Δu_v: a_u , b_u , γ_u $\quad$ Δd_v: a_d , b_d , γ_d

  $\Delta\bar{q}$: $\eta_{\bar{q}}$, $a_{\bar{q}}$, $b_{\bar{q}}$ $\quad$ ΔG: η_G , a_G , b_G

Note:

$$x\Delta q_i(x, Q_0^2) = \eta_i A_i x^{a_i}(1-x)^{b_i}(1 + \gamma_i x + \rho_i x^{\frac{1}{2}})$$

The World Data: $g_1/F_1(x,Q^2)$ or $A_1(x,Q^2)$

Published Experimental Data above $Q^2 = 1.0\ GeV^2$

Experiment	type	x–range	Q^2–range $[GeV^2]$	ΔN [%]	# data points
E143	g_1/F_1	0.027 – 0.749	1.17 – 9.52	3.7	82
HERMES	g_1/F_1	0.028 – 0.660	1.13 – 7.46	3.0	39
E155	g_1/F_1	0.015 – 0.750	1.22 – 34.72	7.6	24
SMC	A_1	0.005 – 0.480	1.30 – 58.0	4.0	59
EMC	A_1	0.015 – 0.466	3.50 – 29.5	14.0	10
proton					214
E143	g_1/F_1	0.027 – 0.749	1.17 – 9.52	4.9	82
E155	g_1/F_1	0.015 – 0.750	1.22 – 34.79	4.0	24
SMC	A_1	0.005 – 0.479	1.30 – 54.8	4.0	65
deuteron					171
E142	A_1	0.035 – 0.466	1.10 – 5.50	3.0	28
HERMES	A_1	0.033 – 0.464	1.22 – 5.25	3.0	9
E154	A_1	0.017 – 0.564	1.20 – 15.0	3.0	11
neutron					48
total					433

$$g_1/F_1 \approx (1+\gamma^2)A_1, \quad where \quad \gamma^2 = Q^2/\nu^2$$

$$F_1 = \frac{(1+\gamma^2)}{2x(1+R)} F_2$$

F_2–Parametrization: NMC, M. Arneodo et al., Phys. Lett. **B364** (1995) 107.

R–Parametrization: SLAC, L. Withlow et al., Phys. Lett. **B250** (1990) 193.

The World Data: $g_1(x, Q^2)$

Published Experimental Data above $Q^2 = 1.0\ GeV^2$

Experiment	x-range	Q^2-range $[GeV^2]$	ΔN [%]	# data points
E143	0.031 – 0.749	1.27 – 9.52	3.7	28
HERMES	0.028 – 0.660	1.13 – 7.46	3.0	39
E155	0.015 – 0.750	1.22 – 34.73	7.6	24
SMC	0.005 – 0.480	1.30 – 58.0	4.0	12
proton				103
E143	0.031 – 0.749	1.27 – 9.52	4.9	28
E155	0.015 – 0.750	1.22 – 34.79	4.0	24
SMC	0.005 – 0.479	1.30 – 54.8	4.0	12
deuteron				64
E142	0.035 – 0.466	1.10 – 5.50	3.0	8
HERMES	0.033 – 0.464	1.22 – 5.25	3.0	9
E154	0.017 – 0.564	1.20 – 15.0	3.0	17
neutron				34
total				201

Parameter Values at $Q_0^2 = 4.0\,GeV^2$

8+1 Parameter Fit based on the Asymmetry Data:

Parameter	LO		NLO	
	value	error	value	error
η_{u_v}	0.926	fixed	0.926	fixed
a_{u_v}	0.189	0.025	0.313	0.065
b_{u_v}	2.506	0.190	3.361	0.318
γ_{u_v}	21.34	fixed (*)	27.22	fixed (*)
η_{d_v}	-0.341	fixed	-0.341	fixed
a_{d_v}	0.115	0.058	0.130	0.085
b_{d_v}	2.690	1.135	3.116	1.457
γ_{d_v}	38.50	fixed (*)	22.95	fixed (*)
$\eta_{\bar{q}}$	-0.520	0.189	-0.553	0.125
$a_{\bar{q}}$	0.281	0.202	0.476	0.279
$b_{\bar{q}}$	4.98	fixed (*)	5.82	fixed (*)
η_G	1.692	0.786	1.162	0.590
a_G	2.101	0.776	1.668	0.694
b_G	4.65	fixed (*)	4.91	fixed (*)
χ^2 / NDF	0.96		0.89	

$\Rightarrow$ The parameters marked by (*) have been fitted first and then fixed since the present data do not constrain its values well enough.

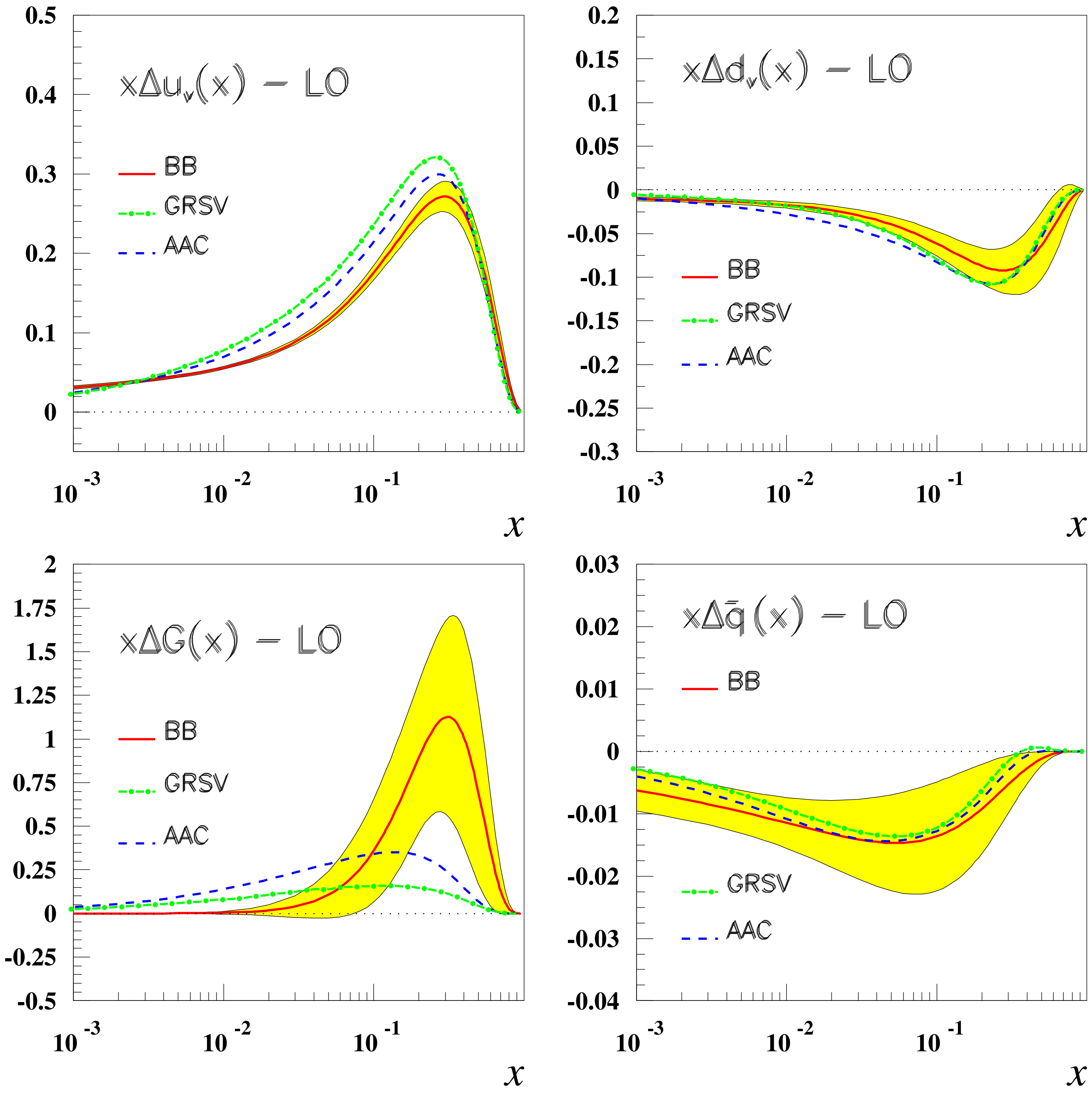
Covariance Matrices – 8 + 1 Parameter Fit

LO	$\Lambda^{(4)}_{QCD}$	a_{u_v}	b_{u_v}	a_{d_v}	b_{d_v}	$\eta_{\bar{q}}$	$a_{\bar{q}}$	η_G	a_G
$\Lambda^{(4)}_{QCD}$	3.17e-2								
a_{u_v}	-2.16e-4	6.42e-4							
b_{u_v}	-2.73e-2	3.59e-3	6.13e-2						
a_{d_v}	3.33e-3	-6.21e-4	-6.08e-3	4.11e-3					
b_{d_v}	2.18e-2	-1.05e-2	8.05e-3	4.57e-2	1.44e-0				
$\eta_{\bar{q}}$	1.37e-2	2.59e-3	1.03e-3	-3.64e-3	-3.39e-2	4.01e-2			
$a_{\bar{q}}$	3.73e-3	3.96e-3	1.84e-2	-7.63e-3	-9.39e-2	352e-2	4.12e-2		
η_G	-1.31e-1	-2.93e-3	9.91e-2	-6.21e-3	-1.11e-1	-1.17e-1	-5.74e-2	1.04e-0	
a_G	4.73e-2	1.69e-3	-1.58e-2	-6.13e-3	-1.83e-1	6.62e-2	5.60e-2	-3.91e-1	7.29e-1

NLO	$\Lambda^{(4)}_{QCD}$	a_{u_v}	b_{u_v}	a_{d_v}	b_{d_v}	$\eta_{\bar{q}}$	$a_{\bar{q}}$	η_G	a_G
$\Lambda^{(4)}_{QCD}$	3.17e-3								
a_{u_v}	4.34e-5	1.16e-3							
b_{u_v}	1.74e-4	4.95e-3	4.92e-2						
a_{d_v}	-1.15e-4	-3.38e-4	-1.46e-3	4.59e-3					
b_{d_v}	-1.97e-3	-9.68e-3	9.71e-2	5.34e-2	2.36e-0				
$\eta_{\bar{q}}$	-1.59e-4	-3.03e-3	-1.64e-2	5.07e-3	4.51e-2	1.58e-2			
$a_{\bar{q}}$	-7.55e-5	-1.13e-3	-6.89e-3	6.59e-4	9.21e-3	4.79e-3	3.33e-3		
η_G	1.15e-3	1.54e-2	7.44e-2	-1.62e-2	-9.43e-2	-6.35e-2	-1.98e-2	3.60e-1	
a_G	-3122e-4	-5.69e-3	-4.35e-2	3.86e-3	5.79e-2	8.75e-3	8.45e-3	-8.38e-2	1.81e-1

Pol. Parton Densities at $Q_0^2 = 4.0\,GeV^2$

- 8+1 Parameter Fit based on the Asymmetry Data:



$\Rightarrow$ Yellow error band: Fully correlated 1σ Gaussian error propagation at the input scale Q_0^2.

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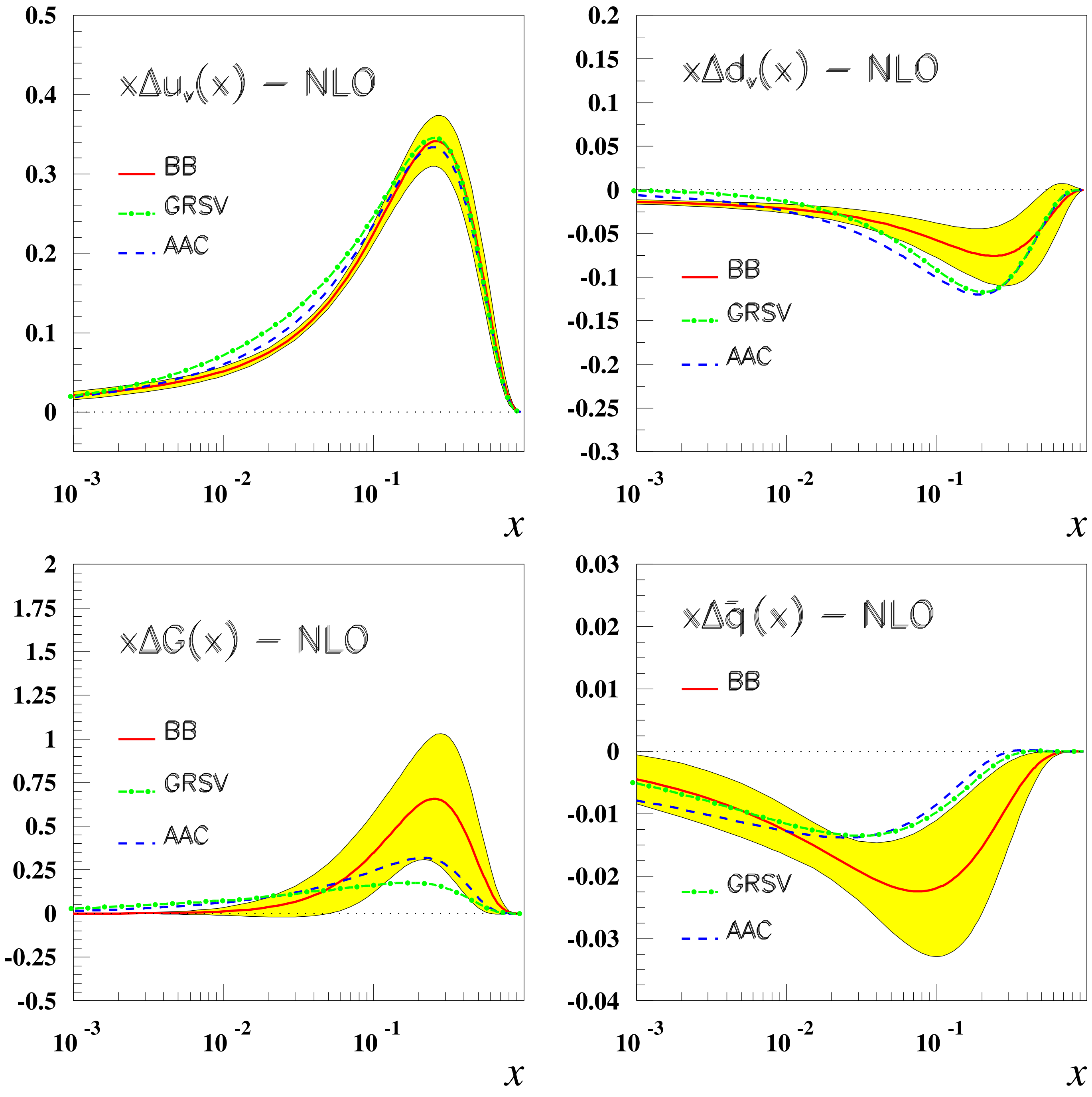

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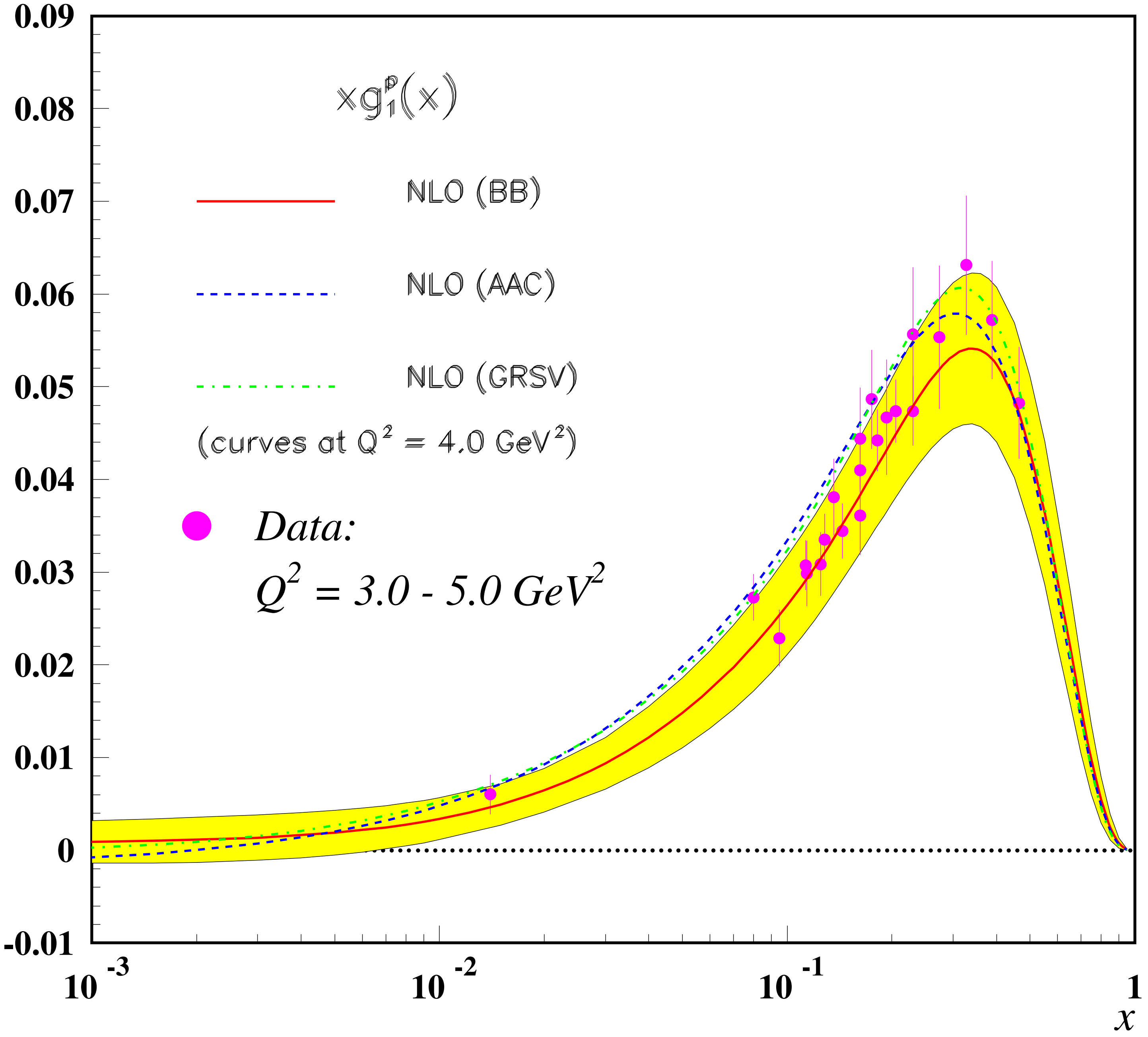
Comparison with measured $xg_1^p(x)$



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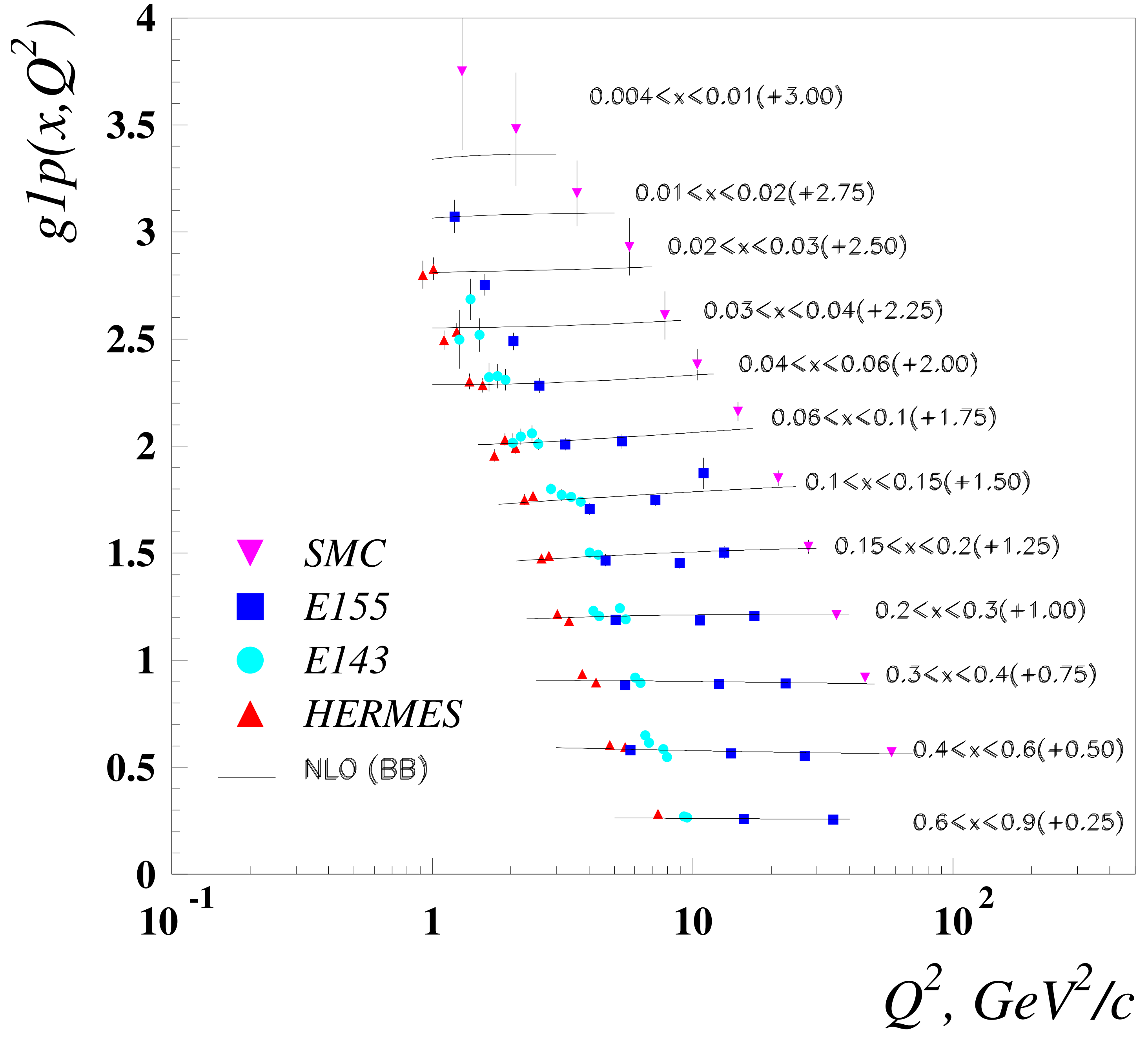
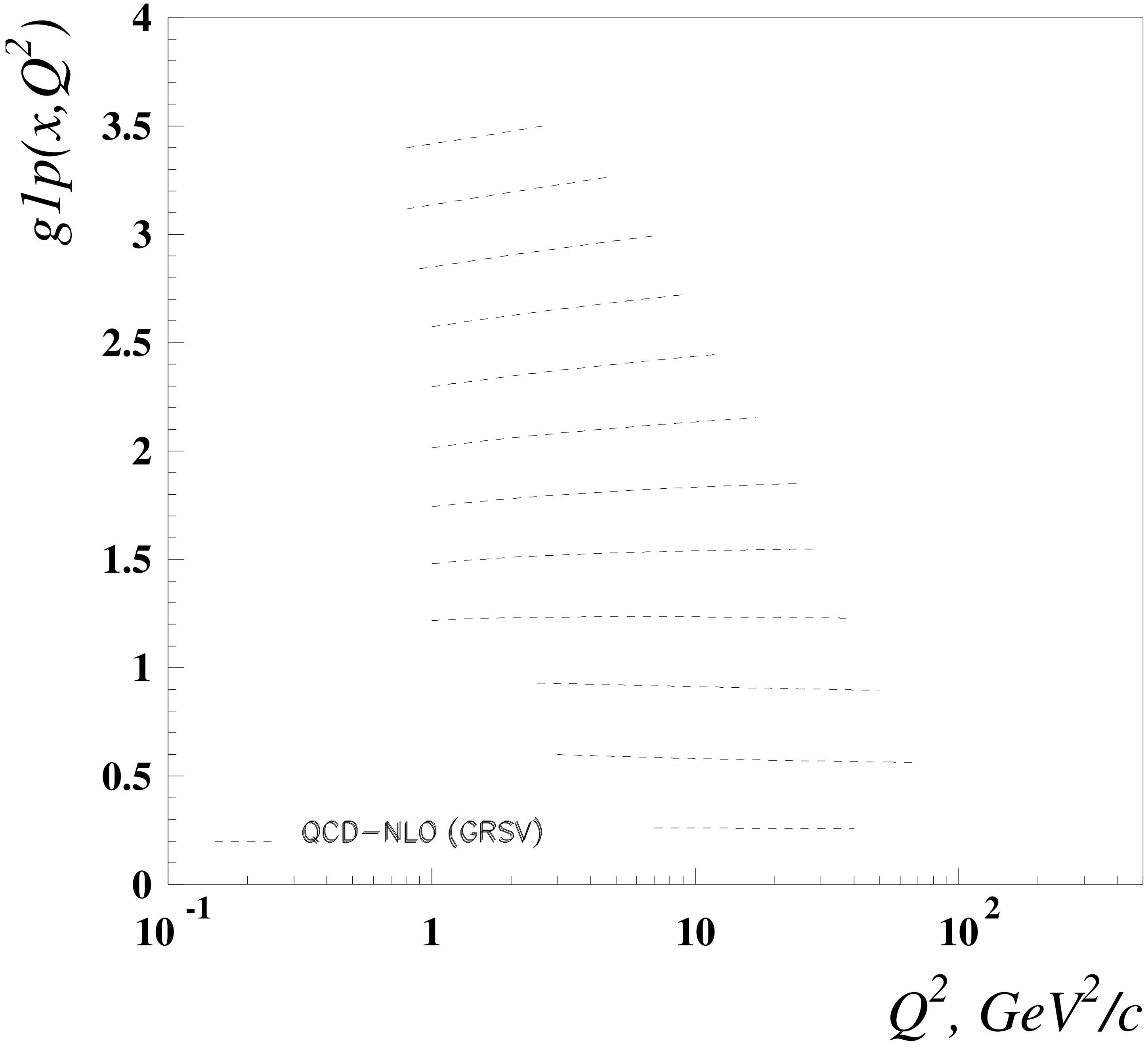
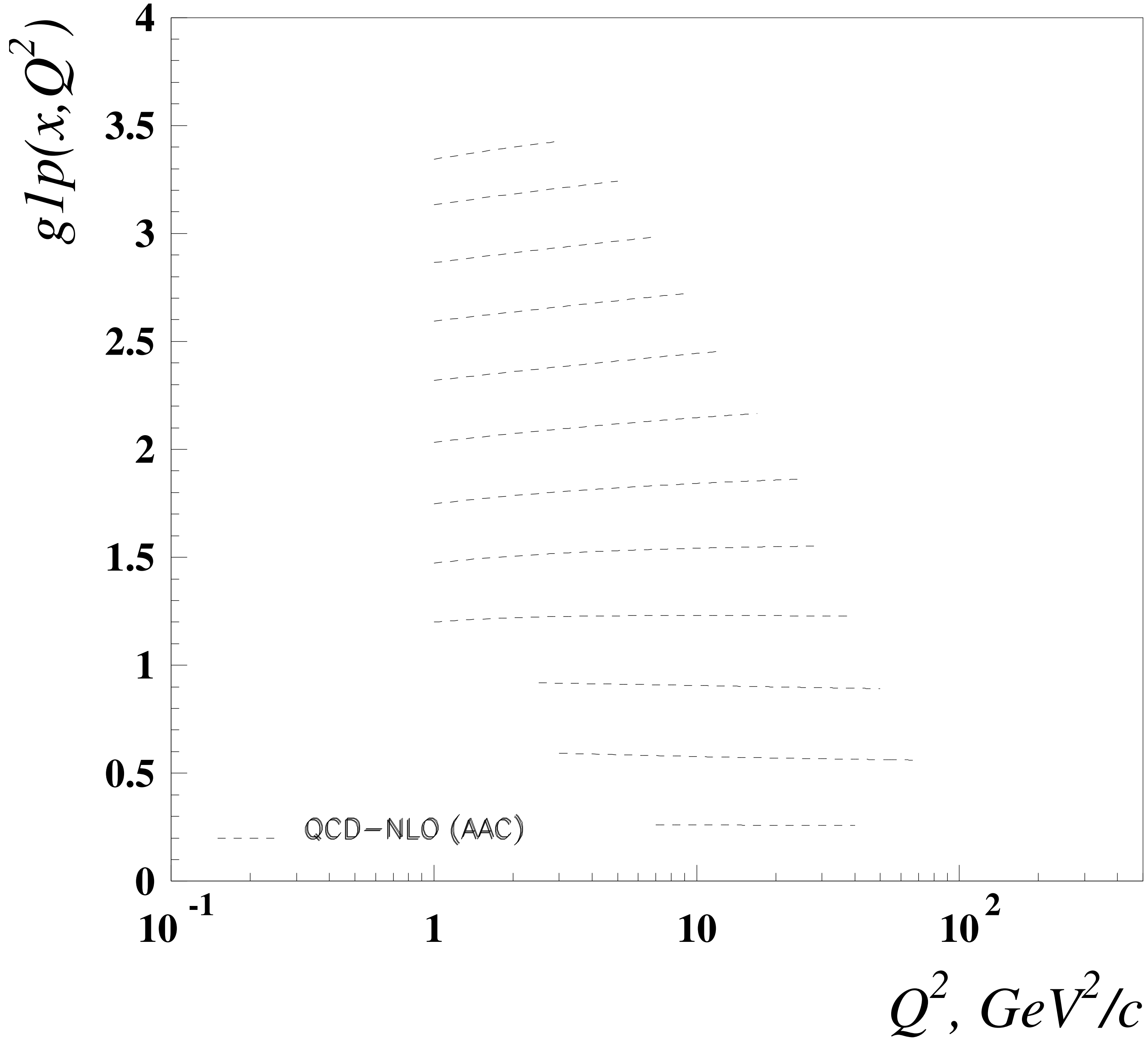
$g_1^p(x)$ versus Q^2



$g_1^p(x)$ **versus** Q^2

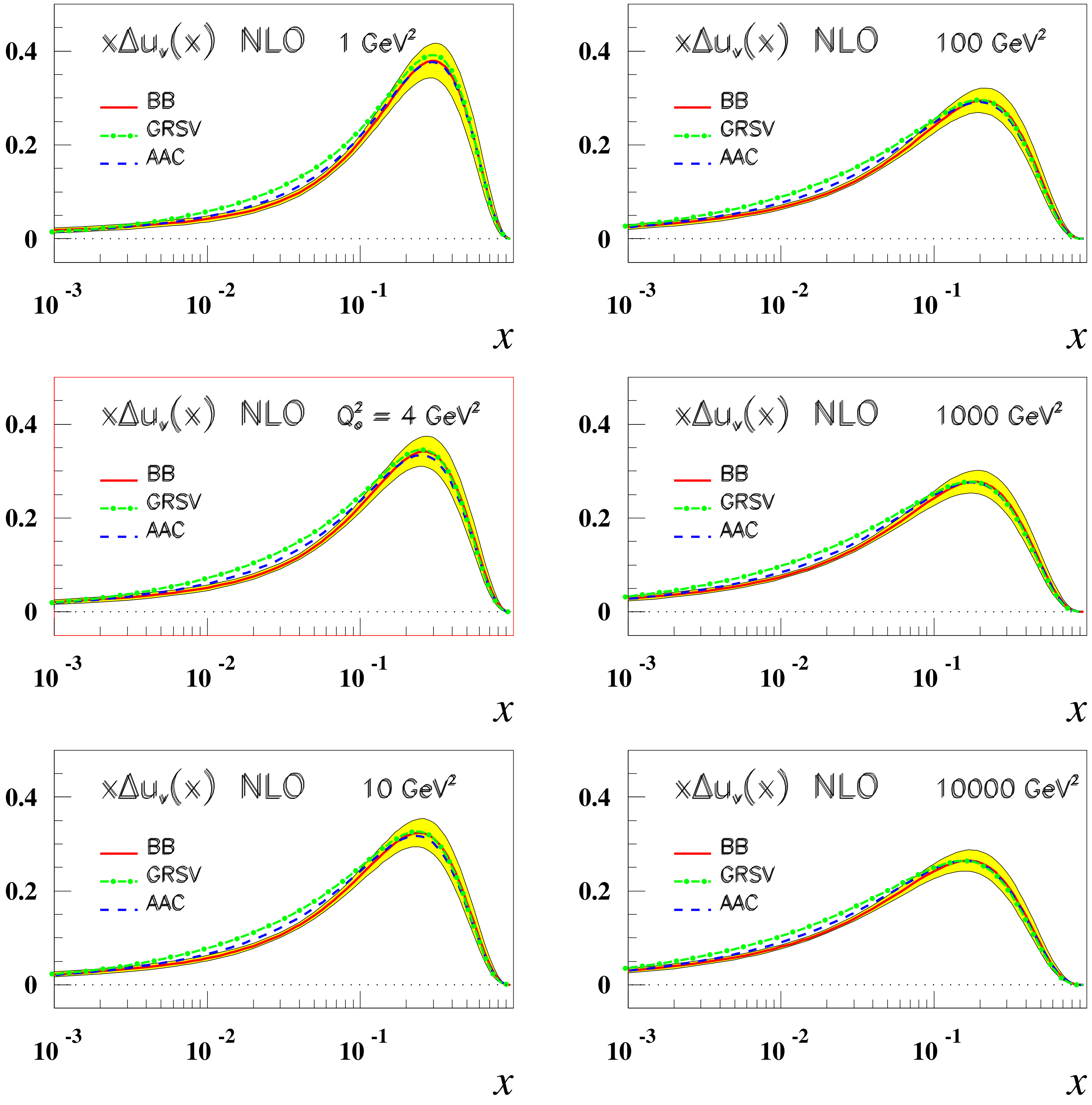


$g_1^p(x)$ versus Q^2



Evolution of Polarized Parton Densities

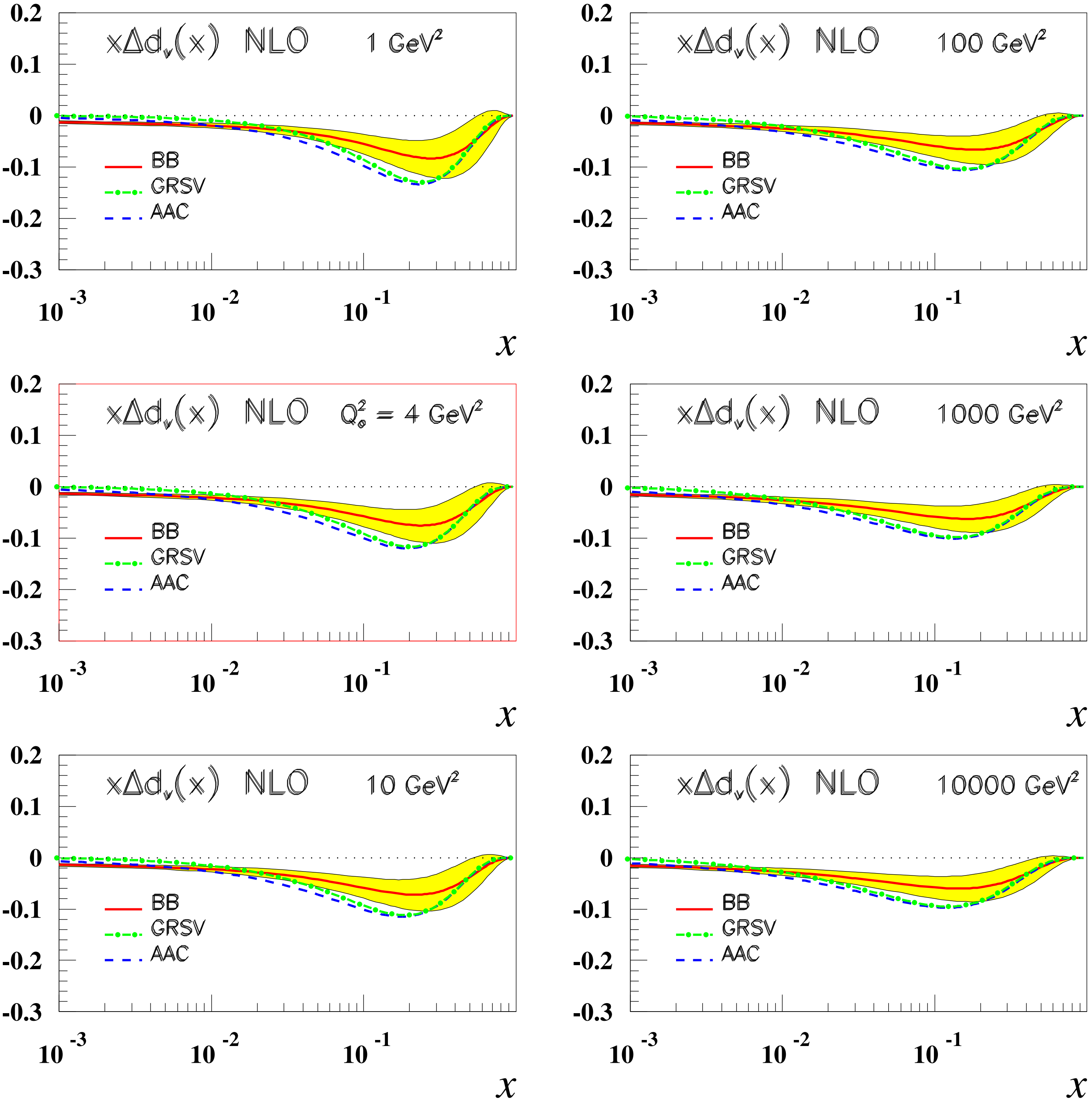
- 8+1 Parameter Fit based on the Asymmetry Data:



⇒ Yellow error band: Fully correlated 1σ Gaussian error propagation through the evolution equation.

Evolution of Polarized Parton Densities

- 8+1 Parameter Fit based on $A1(g1/F1)$ Data:



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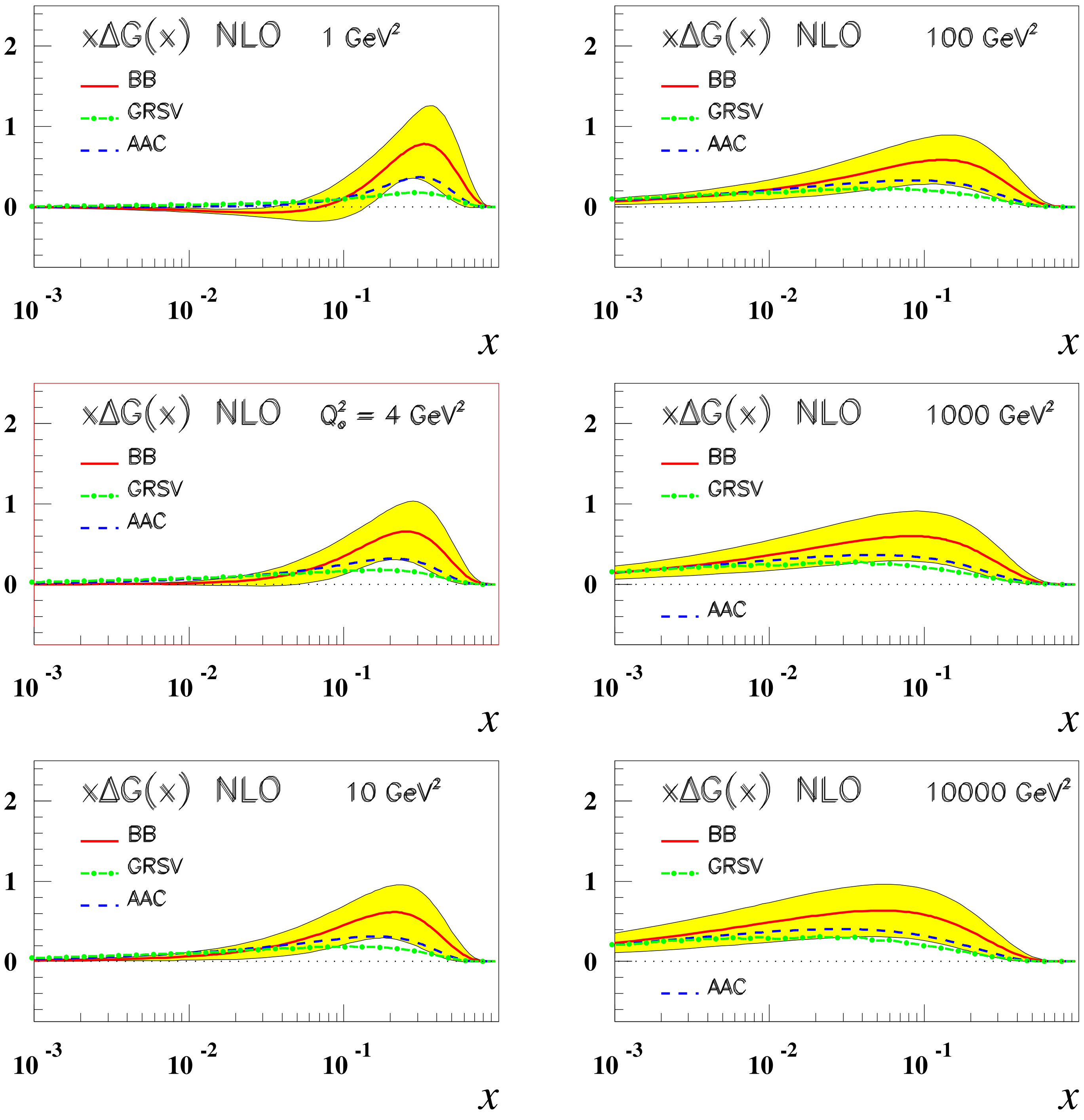

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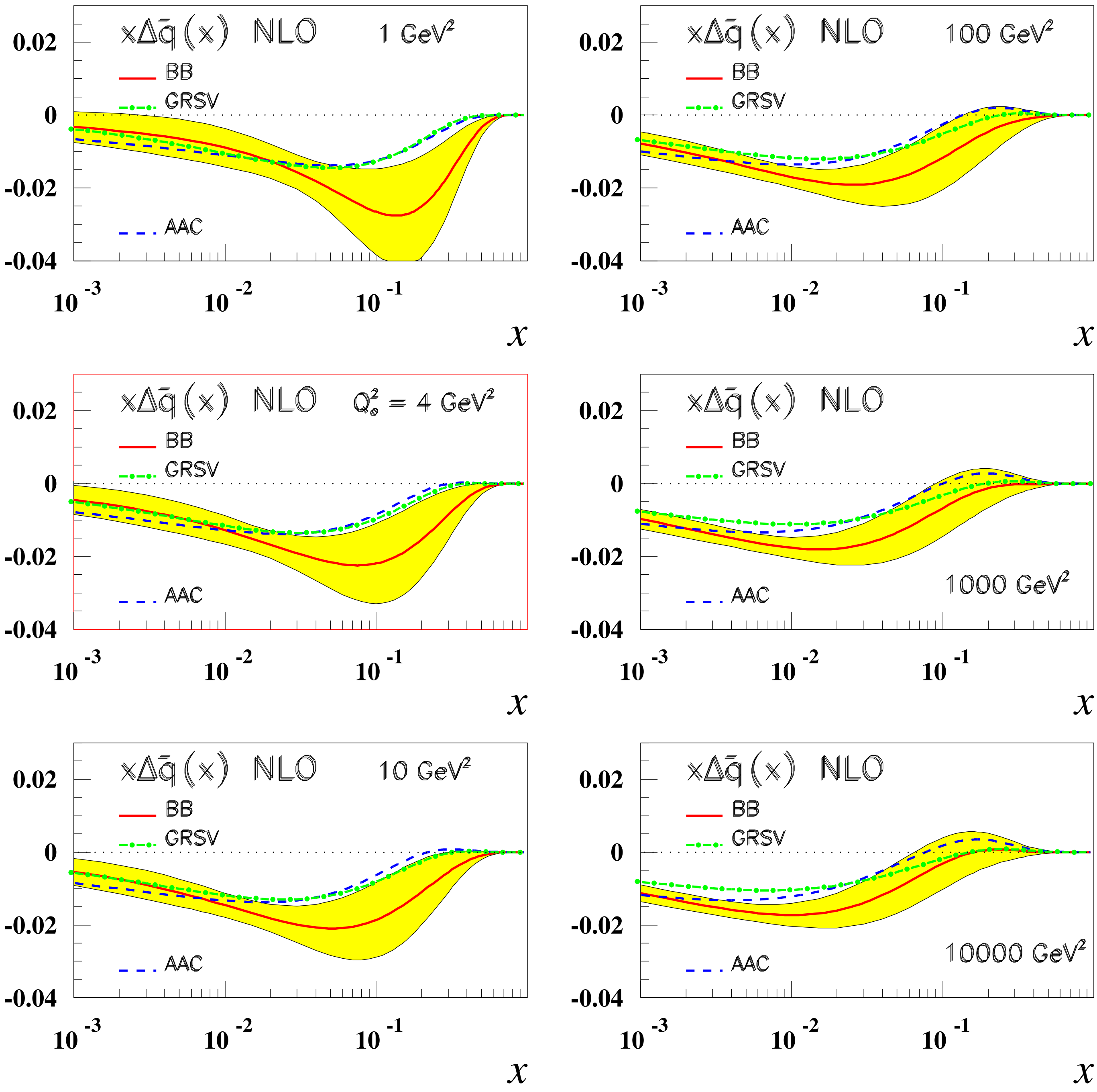


$\Rightarrow$ Yellow error band: Fully correlated 1σ Gaussian error propagation through the evolution equation.

8+1 parameter NLO fit: $\Lambda_{QCD}^{(4)} \Rightarrow \alpha_s(M_Z^2)$

$\Lambda_{QCD}^{(4)}$	$A1$		$g1$	
	value	error	value	error
FS/RS=1.0/1.0	0.241	± 0.056	0.245	± 0.067
FS/RS=0.5/1.0	0.192	– 0.049	0.195	– 0.050
FS/RS=2.0/1.0	0.304	+ 0.063	0.308	+ 0.063
FS/RS=1.0/0.5	0.345	+ 0.104	0.349	+ 0.104
FS/RS=1.0/2.0	0.181	– 0.060	0.184	– 0.061

- $A1$:

$$\alpha_s(M_Z^2) = 0.114 \begin{array}{c} +0.004 \\ -0.005 \end{array} \begin{array}{c} +0.004 \\ -0.004 \end{array} \begin{array}{c} +0.007 \\ -0.005 \end{array}$$

(fit) (fac) (ren)

- $g1$:

$$\alpha_s(M_Z^2) = 0.114 \begin{array}{c} +0.005 \\ -0.006 \end{array} \begin{array}{c} +0.004 \\ -0.004 \end{array} \begin{array}{c} +0.007 \\ -0.005 \end{array}$$

- SMC: $0.121 \pm 0.002(stat) \pm 0.006(syst + theor)$

E154: $0.108 - 0.116 \quad (bad\ for \geq 0.120)$

ABFR: $0.120 \begin{array}{c} +0.004 \\ -0.005 \end{array} (exp) \begin{array}{c} +0.009 \\ -0.006 \end{array} (theor)$

world average: 0.118 ± 0.002

Fac. Scheme Invariant Combinations

- Instead of Process-Independent Scheme-Dependent Evolution Equations for Partons one may think of Process-Dependent Scheme-Independent Evolution Equations for Observables, F_A, F_B.

$\Rightarrow$ The input densities are measured ! Control over the input directly.
$\Rightarrow$ No ΔG–Ansatz necessary.
$\Rightarrow$ A one parameter fit only – Λ_{QCD}.

Evolution Equations : [J. Blümlein, V. Ravindran, and W. L. van Neerven, Nucl. Phys **B586** (2000) 349.]

$$\frac{\partial}{\partial t}\left(\begin{array}{c} F_A^N \\ F_B^N \end{array}\right) = -\frac{1}{4}\left(\begin{array}{cc} K_{AA}^N & K_{AB}^N \\ K_{BA}^N & K_{BB}^N \end{array}\right)\left(\begin{array}{c} F_A^N \\ F_B^N \end{array}\right)$$

evolution variable :
$$t = -\frac{2}{\beta_0}\log\left(\frac{a_s(Q^2)}{a_s(Q_0^2)}\right)$$

$\Longrightarrow$ The evolution kernels K_{IJ}^N are also Physical Quantities! The Factorization Scheme Independence holds order by order.

$\Longrightarrow$ A possible choice: $F_A = g_1$ and $F_B = \partial g_1/\partial t$.

The renormalization scale dependence disappears only with more higher orders.

System : $g_1(x,Q^2), \partial g_1/\partial t(x,Q^2)$

Leading Order :

$$K_{22}^{N(0)} = 0$$

$$K_{2d}^{N(0)} = -4$$

$$K_{d2}^{N(0)} = \frac{1}{4}\left(\gamma_{qq}^{N(0)}\gamma_{gg}^{N(0)} - \gamma_{qg}^{N(0)}\gamma_{gq}^{N(0)}\right)$$

$$K_{dd}^{N(0)} = \gamma_{qq}^{N(0)} + \gamma_{gg}^{N(0)}$$

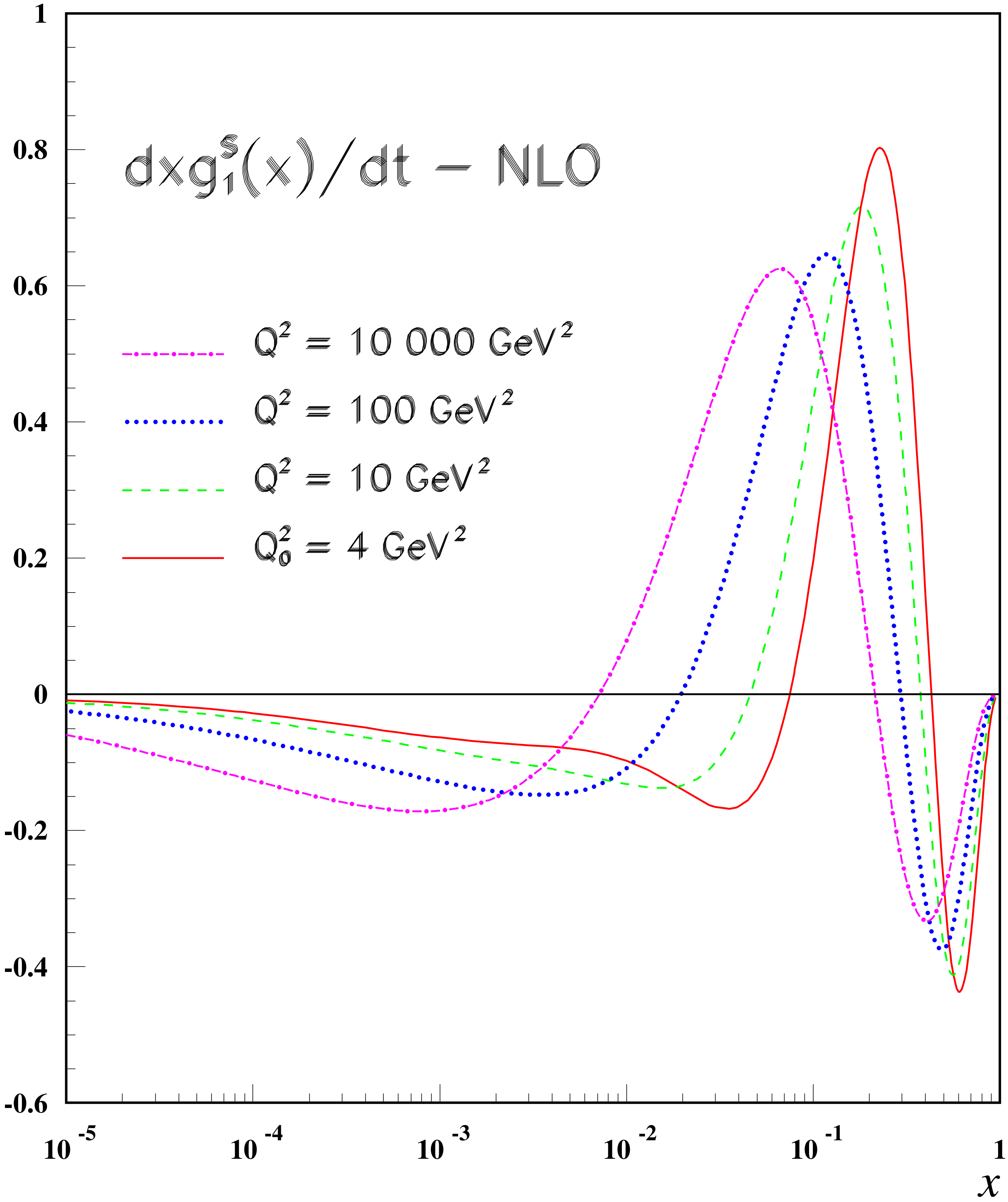
Next-to-Leading Order : [W. Furmanski and R. Petronzio, Z. Phys. **C 11** (1982) 293.]

$$K_{22}^{N(1)} = K_{2d}^{N(1)} = 0$$

$$\begin{aligned}
K_{d2}^{N(1)} = & \frac{1}{4}\left[\gamma_{gg}^{N(0)}\gamma_{qq}^{N(1)} + \gamma_{gg}^{N(1)}\gamma_{qq}^{N(0)} - \gamma_{qg}^{N(1)}\gamma_{gq}^{N(0)} - \gamma_{qg}^{N(0)}\gamma_{gq}^{N(1)}\right] \\
& -\frac{\beta_1}{2\beta_0}\left(\gamma_{qq}^{N(0)}\gamma_{gg}^{N(0)} - \gamma_{gq}^{N(0)}\gamma_{qg}^{N(0)}\right) \\
& +\frac{\beta_0}{2}C_{2,q}^{N(1)}\left(\gamma_{qq}^{N(0)} + \gamma_{gg}^{N(0)} - 2\beta_0\right) \\
& -\frac{\beta_0}{2}\frac{C_{2,g}^{N(1)}}{\gamma_{qg}^{N(0)}}\left[\gamma_{qq}^{N(0)^2} - \gamma_{qq}^{N(0)}\gamma_{gg}^{N(0)} + 2\gamma_{qg}^{N(0)}\gamma_{gq}^{N(0)} - 2\beta_0\gamma_{qq}^{N(0)}\right] \\
& -\frac{\beta_0}{2}\left(\gamma_{qq}^{N(1)} - \frac{\gamma_{qq}^{N(0)}\gamma_{qg}^{N(1)}}{\gamma_{qg}^{N(0)}}\right)
\end{aligned}$$

$$\begin{aligned}
K_{dd}^{N(1)} = & \gamma_{qq}^{N(1)} + \gamma_{gg}^{N(1)} - \frac{\beta_1}{\beta_0}\left(\gamma_{qq}^{N(0)} + \gamma_{gg}^{N(0)}\right) + 4\beta_0 C_{2,q}^{N(1)} - 2\beta_1 \\
& -\frac{2\beta_0}{\gamma_{qg}^{N(0)}}\left[C_{2,g}^{N(1)}\left(\gamma_{qq}^{N(0)} - \gamma_{gg}^{N(0)} - 2\beta_0\right) - \gamma_{qg}^{N(1)}\right]
\end{aligned}$$

$\partial x g_1^S/\partial t(x,Q^2)$ **and shift of** $\Lambda_{QCD}^{(4)}$



$$\Lambda_{QCD}^{(4)} : 0.241 \Longrightarrow 0.229 \quad , \quad \alpha_s(M_Z^2) : 0.114 \Longrightarrow 0.113$$

Conclusions

- An LO and NLO QCD Analysis of the current World-Data of Polarized Structure Functions was performed.

- New parametrizations of the parton densities including their Errors were derived. They are available via a fast `FORTRAN` program for the range:

$$1 < Q^2 < 10^6 \text{ GeV}^2 \text{ and } 10^{-4} < x < 1.$$

- The following results for $\Lambda_{\rm QCD}$ and $\alpha_s(M_Z^2)$ were obtained (8+1 parameter fit):

$$\Lambda_{\rm QCD}^{(4)} = 241 \pm 56 \ (\text{fit}) \ {}^{+63}_{-49} \ (\text{fac}) \ {}^{+104}_{-60} \ (\text{ren})$$

$$\alpha_s(M_Z^2) = 0.114 \ {}^{+0.004}_{-0.005} \ (\text{fit}) \ {}^{+0.004}_{-0.004} \ (\text{fac}) \ {}^{+0.007}_{-0.005} \ (\text{ren})$$

- First steps in a fac. scheme invariant QCD evolution based on the structure function $g_1(x,Q^2)$ and $\partial g_1(x,Q^2)/\partial \log Q^2$ were performed yielding similar results for $\alpha_s(M_Z^2)$.

- The latter analysis is a very promising way to proceed in the future, since it allows to extract $\Lambda_{\rm QCD}$ fixing all the input distributions by direct measurement.