Renormalization of Polyakov loops in different representations and the large-N limit

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Outline

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Preliminaries

- Lattice simulations of Yang-Mills theories with gauge group $\mathrm{SU}(N)$ at finite temperature
- The Lagrangian is characterized by *exact* center symmetry
- The Polyakov loop $L = \mathrm{tr} \prod_{t=1}^{N_T} U_4(t)$; order parameter for deconfinement
- The free energy associated with the *bare* Polyakov loop is divergent in the continuum: renormalization required **[Dotsenko and Vergeles, 1980]**

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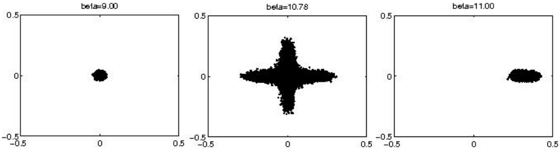

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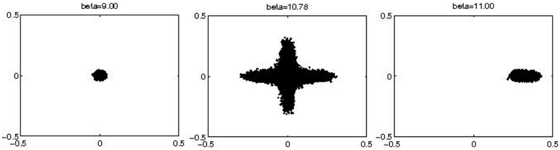

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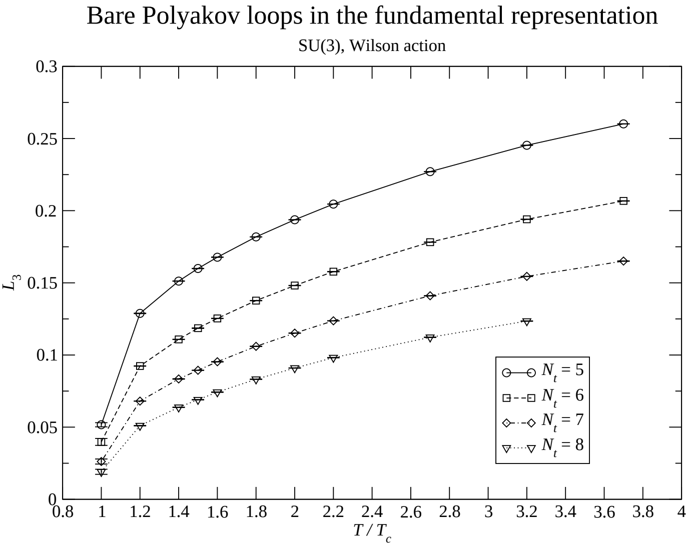
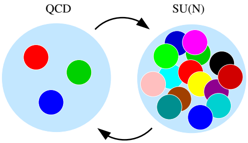
Bare Polyakov loops



Why large N?

- At fixed $\lambda = g^2 N$ and N_f, expansions in powers of $1/N$ give non-trivial insight onto some non-perturbative features of QCD **['t Hooft, 1974; Witten, 1979; Manohar, 1998]**



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- Formal connection to closed string theory; Topological expansions of amplitude $\leftrightarrow$ Loop expansion in Riemann surfaces **[Aharony, Gubser, Maldacena, Ooguri and Oz, 1999]**

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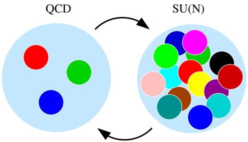

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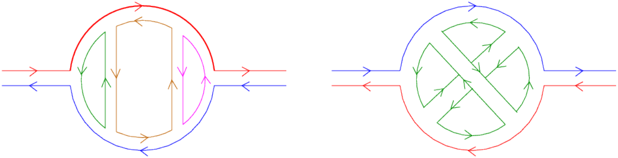

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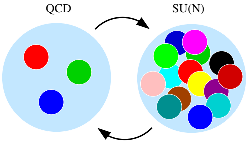

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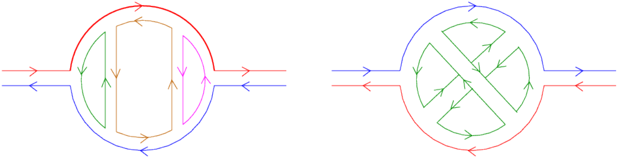

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- Volume reduction **[Eguchi and Kawai, 1982]**
- Implications for the phase diagram structure at large densities **[McLerran and Pisarski, 2007]**
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Why higher representations?

- Tests of Casimir scaling **[Döring *et al.*, 2007; Hübner and Pica, 2007; Del Debbio, Panagopoulos and Vicari, 2003]**
- Equivalence of different irreducible representations in the large-N limit
- Effective (matrix) models for the deconfinement region? **[Pisarski, 2002]**
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Polyakov loop renormalization methods

1. Using the $Q\bar{Q}$ potential at zero temperature **[Kaczmarek, Karsch, Petreczky and Zantow, 2002; Hübner and Pica, 2008]**

$$L_{ren} = Z^{N_t} L_{bare}, \qquad Z = \exp(V_0 a/2)$$

2. At fixed temperature T, remove the N_t-dependent contributions to the bare Polyakov loop free energy **[Dumitru *et al.*, 2003]**:

$$F^{\mathrm{bare}} = N_t F^{\mathrm{div}} + F^{\mathrm{ren}} + N_t^{-1} F^{\mathrm{lat}} + \dots$$

(however, note that g_0 is *not* fixed ...)

3. Iterative determination of the renormalization term, from simulations at two different bare couplings **[Gupta, Hübner and Kaczmarek, 2008; Creutz, 1981]**
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Simulation

- Simulations with the Wilson action **[Wilson, 1974]**:

$$S = \frac{2N}{g_0^2} \sum_x \sum_{\mu<\nu} \left\{ 1 - \frac{1}{N} \mathrm{Re}\ \mathrm{tr} U^{1,1}_{\mu,\nu}(x) \right\}$$

- . . . and with the tree-level improved action **[Curci, Menotti and Paffuti, 1983; Lüscher and Weisz, 1985]**:

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- Simulation algorithm based on a (standard) $1+3$ combination of heat-bath **[Creutz, 1980; Kennedy and Pendleton, 1985]** and overrelaxation **[Adler, 1981; Brown and Woch, 1987]** updates on $SU(2)$ subgroups **[Cabibbo and Marinari, 1982]**

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Setting the scale

- For the Wilson action: high-precision determinations available in the literature **[Necco and Sommer, 2001; Boyd *et al.*, 1996; Lucini, Teper and Wenger, 2004]**
- For the tree-level improved action: static potential at $T = 0$ from Wilson loops $W(r, L)$:

$$V(r) = \lim_{L \to \infty} \ln \frac{W(r, L-a)}{W(r, L)}, \qquad W(r, L) = e^{-L \cdot V(r)} + \dots$$

- Iteratively smeared spacelike links:

$$U_{\mu}^{(i+1)}(x) = U \in \mathrm{SU}(N) \quad \text{which maximizes} \quad \mathrm{Re}\,\mathrm{tr}(U^{\dagger} W)$$

with:

$$W = (1-k) U_{\mu}^{(i)}(x) + \frac{k}{4} \sum U_{staple}^{(i)}$$

- Fits to the Cornell potential to extract the string tension:

$$V(r) = \sigma r + V_0 + \frac{\gamma}{r}$$

- Comparison with a scale setting from the determination of the critical temperature **[Caselle, Panero and Piemonte, 2011]**

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Irreducible representations

- For $\mathrm{SU}(2)$, the recursive formula for obtaining characters of any irreducible representation:

$$\mathrm{tr}_{n+1} g = \mathrm{tr}_n g \, \mathrm{tr}_1 g - \mathrm{tr}_{n-1} g \quad \text{with: } \mathrm{tr}_0 g = 1$$

- For $\mathrm{SU}(3)$, the characters of higher representations are obtained using the Young calculus and the relation between the traces in the fundamental and anti-fundamental irreducible representation:

$$\frac{1}{2}[(\mathrm{tr}_f g)^2 - \mathrm{tr}_f(g^2)] = \mathrm{tr}_{\bar{f}} g = (\mathrm{tr}_f g)^\star$$

- For $\mathrm{SU}(N > 3)$ we combine the character relations derived from Young calculus with the Weyl formula **[Weyl, 1960; Itzykson and Nauenberg, 1966]**:

$$\mathrm{tr}_{\vec{\lambda}} g = \frac{\det F(\vec{\lambda})}{\det F(\vec{0})}$$

where $F_{kl}(\vec{\lambda}) = \exp\left[i\,(N-k)\,\alpha_l\right]$ and $e^{i\alpha_1}, e^{i\alpha_2}, \ldots\ e^{i\alpha_N}$ are the eigenvalues of g in the fundamental representation

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$$\mathrm{tr}_{n+1} g = \mathrm{tr}_n g \, \mathrm{tr}_1 g - \mathrm{tr}_{n-1} g \quad \text{with: } \mathrm{tr}_0 g = 1$$

- For $\mathrm{SU}(3)$, the characters of higher representations are obtained using the Young calculus and the relation between the traces in the fundamental and anti-fundamental irreducible representation:

$$\frac{1}{2}[(\mathrm{tr}_f g)^2 - \mathrm{tr}_f(g^2)] = \mathrm{tr}_{\bar{f}} g = (\mathrm{tr}_f g)^\star$$

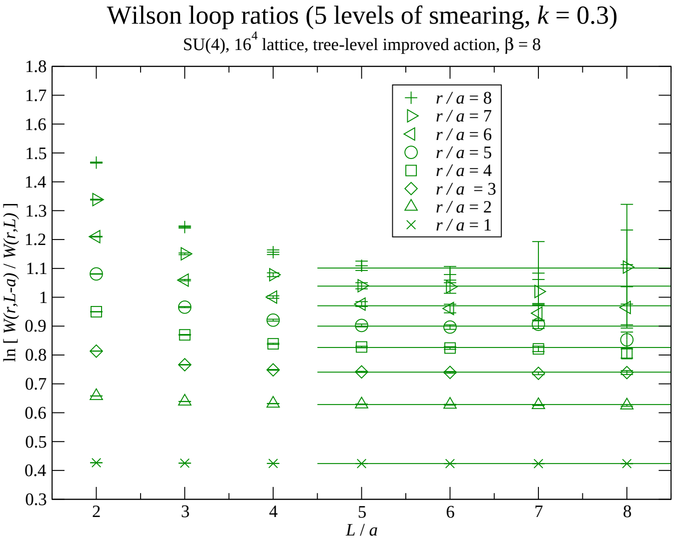
- For $\mathrm{SU}(N > 3)$ we combine the character relations derived from Young calculus with the Weyl formula **[Weyl, 1960; Itzykson and Nauenberg, 1966]**:

$$\mathrm{tr}_{\vec{\lambda}} g = \frac{\det F(\vec{\lambda})}{\det F(\vec{0})}$$

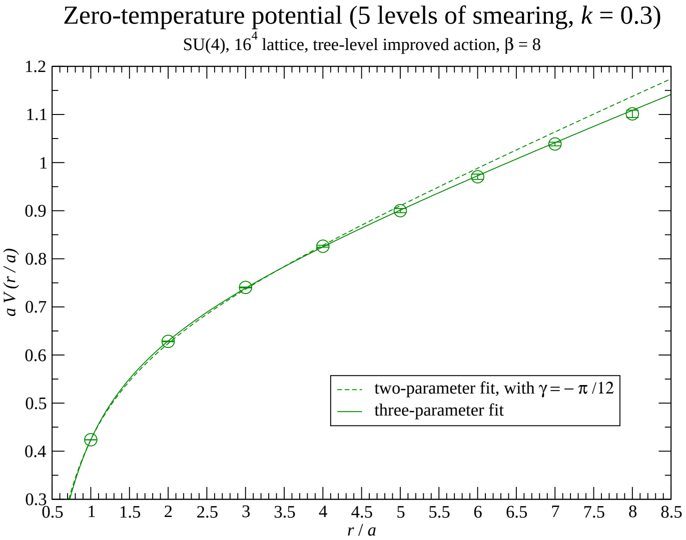
where $F_{kl}(\vec{\lambda}) = \exp\left[i\,(N-k)\,\alpha_l\right]$ and $e^{i\alpha_1}, e^{i\alpha_2}, \ldots \; e^{i\alpha_N}$ are the eigenvalues of g in the fundamental representation

Outline

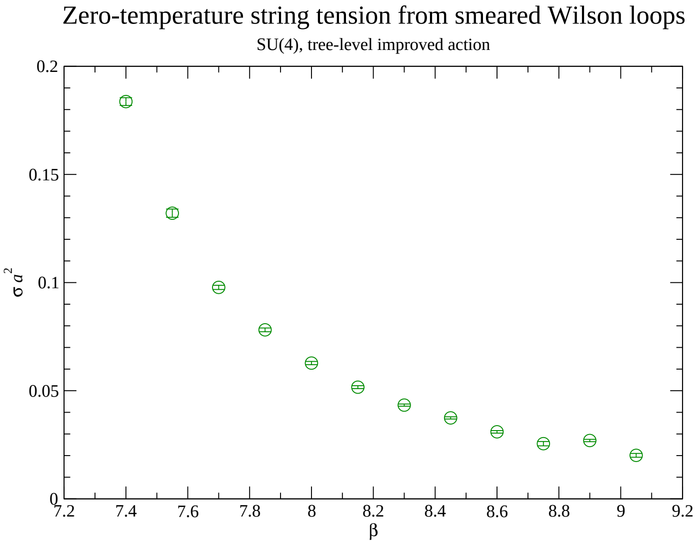
Scale determination from the zero-temperature potential



Scale determination from the zero-temperature potential



Scale determination from the zero-temperature potential



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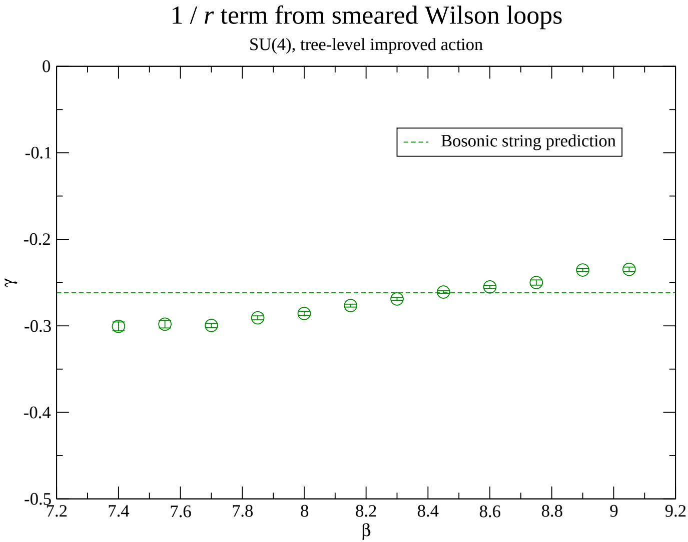

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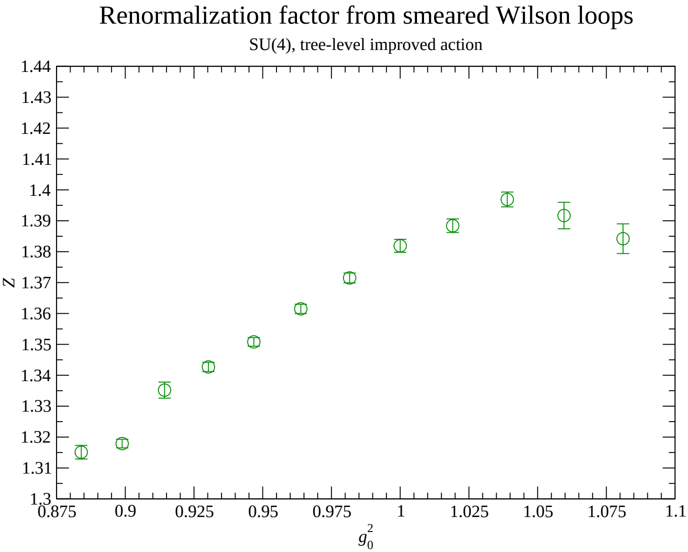

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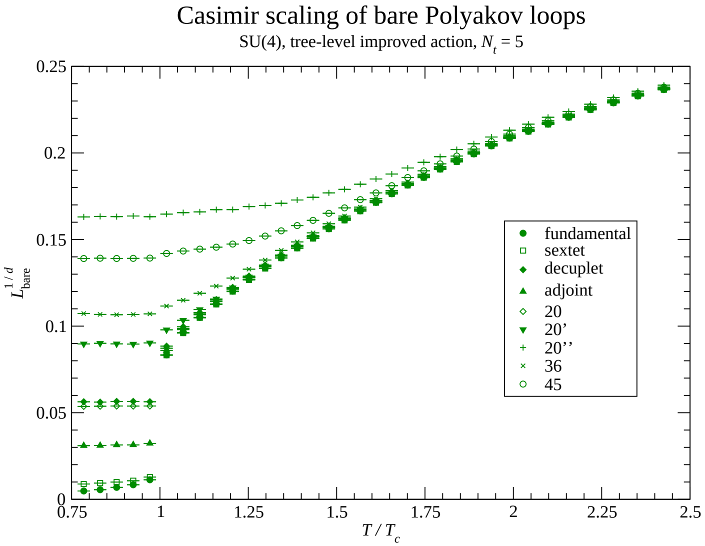

Scale determination from the zero-temperature potential