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Preface

The Wigner Symposia derive their name from Eugene P. Wigner (1902–95), Nobel laureate of 1963 and one of the greatest physicist of this century. To the majority of the physics community Wigner is known as the one who revealed the impact of symmetry principles in physics and showed how to use group theory to solve a vast number of problems, or at least to reduce their complexity substantially. But Wigner's work was not restricted to this activity only; his Collected Works, now being published in several volumes, clearly show that both his interest and his ability to make substantial contributions covered a much wider range: from chemical reactions to nuclear physics, from solid state theory to statistical mechanics, not to forget his papers on epistemological questions. As Nandor L. Balazs pointed out in his talk, it is this rare combination, the gift to see the underlying mathematical structure in a physical theory and the capacity to solve problems of practical interest, which is most characteristic of Wigner's work.

Therefore, a conference bearing his name should not be limited to the refinement, extension and generalization of mathematical methods introduced by Wigner. To live up to his memory it should also provide a forum where scientists with different professional backgrounds can discuss ideas which are relevant to more than one field, and propose solutions to yet unsolved fundamental problems.

Current trends and the policy of funding institutions favor conferences on specific topics, be it highly specialized workshops or general conferences which cover one field in all its aspects. But the preceding Wigner Symposia and the participation of nearly two hundred mathematicians, physicists and chemists in the Fifth Symposium prove that there exists also a vivid and steady interest in medium-size interdisciplinary meetings where emphasis is put on common methods and general ideas. If this spirit, which we associate with the name of Wigner, lives on, then many more stimulating Wigner Symposia should be expected in future.

March 1998 — The Editors

DURATION OF THE QUANTUM "ZENO" REGION FOR AN EXCITED STATE OF THE HYDROGEN ATOM

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The temporal behavior of 2P-1S transition of the hydrogen atom is analyzed and compared to the exponential decay law. The duration of the quantum Zeno region is found to be approximately $3.59 \cdot 10^{-15}$ s (the lifetime is $1.595 \cdot 10^{-9}$ s).

Unstable quantum mechanical systems decay according to an exponential law. However, although such a law has been experimentally verified with very high accuracy, the domain of its validity is limited:[1] a short-time quadratic behavior (leading to the so-called "quantum Zeno effect"[2]) and long-time power tails are unavoidable consequences of the mathematical properties of the Schrödinger equation. For a review, see Ref. [3].

We shall discuss this issue by considering an example: a two-level atom in interaction with the EM field, in the rotating wave approximation. The Hamiltonian reads ($\hbar = c = 1$)

$$H = \sum_{i=1}^{2} E_i |i\rangle\langle i| + \sum_\beta \int_0^\infty d\omega\, \omega\, a^\dagger_{\omega\beta} a_{\omega\beta} + \sum_\beta \int_0^\infty d\omega \left[\varphi_\beta(\omega)\, a^\dagger_{\omega\beta} |1\rangle\langle 2| + \text{h.c.}\right], \tag{1}$$

where the first term is the free Hamiltonian of the two-level atom, the second term is the Hamiltonian of the free EM field and the third term is the interaction Hamiltonian. We are using the energy-angular momentum basis for photons, with $\sum_\beta = \sum_{j=1}^{\infty} \sum_{m=-j}^{j} \sum_{\lambda=0}^{1}$, where λ defines the photon parity $P = (-1)^{j+1+\lambda}$, j is the total angular momentum (orbital + spin) of the photon, m its magnetic quantum number, ω its energy, $[a_{\omega\beta}, a^\dagger_{\omega'\beta'}] = \delta(\omega - \omega')\, \delta_{\beta\beta'}$ and $|i\rangle$, $i = 1, 2$, are the atomic states (of energy E_i). The quantities $\varphi_\beta(\omega)$ are the matrix elements of the interaction Hamiltonian between the states $|1; 1_{\omega\beta}\rangle$ and $|2; 0\rangle$ (the first number refers to the atom and the second to the photon). The details of the following calculation, as well as a discussion of the validity of the approximations involved, can be found in Ref. [4].

We concentrate on the 2P-1S transition of hydrogen and assume that the system is initially (at time $t = 0$) in the state $|2; 0\rangle$. By making use of an exact evaluation of the matrix elements,[5] one finds for the "Zeno" time (governing the dynamics for very short times, where the first deviations from exponential

behavior are expected)

$$\tau_Z = \langle 2,0| H^2_{\text{int}} |2,0\rangle^{-1/2} = \sqrt{\frac{6}{\chi}\frac{1}{\Lambda}} \simeq 3.593 \cdot 10^{-15}\,\text{s}, \tag{2}$$

where $\Lambda = \frac{3}{2}\alpha m_e \simeq 8.498 \cdot 10^{18}$ rad/s is the *natural* cutoff defining the atomic form factor and $\chi = \frac{2}{\pi}\left(\frac{2}{3}\right)^9 \alpha^3 \simeq 6.435 \cdot 10^{-9}$ (α is the fine structure constant and m_e the electron mass). This is, to our knowledge, the first estimate of the duration of the Zeno region for a truly unstable system.

It is also interesting to look at the temporal behavior of our system for longer times. There is previous related work [6] on this subject. A complete calculation [4] yields the Fermi "Golden Rule" (with the lifetime $\gamma^{-1} \simeq 1.595 \cdot 10^{-9}$ s), as well as the general expression (valid $\forall t \geq 0$) for the survival probability $P(t) \equiv \langle 2; 0| e^{-iHt} |2; 0\rangle$. The survival probability is displayed in Fig. 1, where, for convenience, arbitrary values of constants have been used.

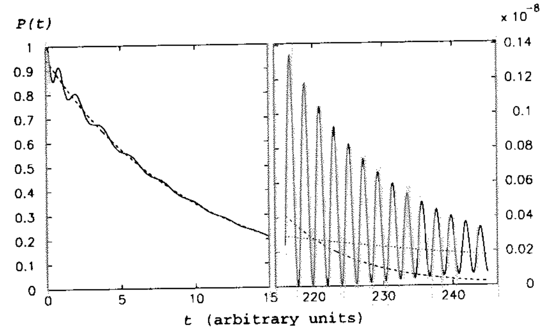

Figure 1: Behavior of the survival probability $P(t)$. For illustrative purposes, we set $\chi = 0.1, \gamma = 0.1\,\text{s}^{-1}$ and $\omega_0 = 3\,\text{s}^{-1}$. The frequency of the oscillations reaches its asymptotic value after a few oscillations. Notice that different scales have been used for the two graphs. The dashed line is the exponential and the dotted line the power law.

We only give here the expressions for short and long times:

$$P(t) \sim 1 - \frac{t^2}{\tau_Z^2} \qquad \text{for} \quad t \ll \tau_Z, \tag{3}$$

$$P(t) \sim \mathcal{Z}^2 e^{-\gamma t} + \chi^2 \frac{\mathcal{C}^2}{(\omega_0 t)^4} - 2\chi \frac{\mathcal{C}\mathcal{Z}}{(\omega_0 t)^2} e^{-\frac{\gamma}{2}t} \cos\left[(\omega_0 - \Delta E)t - \zeta\right], \qquad \text{for} \quad t \gg \Lambda^{-1}. \tag{4}$$

Here $\mathcal{Z}e^{i\zeta}$ and $\mathcal{C}$ are both $1 + O(\chi)$ and ΔE is a Lamb-shift-like contribution, negligible in comparison with $\omega_0 \equiv E_2 - E_1$. Notice that the "long-time" expansion is already valid for rather short times $\tau_Z > t \gg \Lambda^{-1} \simeq 10^{-19}$ s (well inside the Zeno region!) and observe the presence of an oscillatory term, which physically represents a quantum mechanical interference effect.[4] The above result clarifies an important point: in contrast with a widespread naive expectation, the short time behavior, yielding a vanishing decay rate, is nothing but the first of a series of oscillations, whose amplitude vanishes exponentially with time before being superseded by a power law. The frequency of the oscillations is essentially ω_0.

An interesting problem is to understand whether the initial quadratic behavior (3) is experimentally observable. This is an experimentally challenging task, that raises subtle theoretical and experimental questions about the problem of state preparation. This and related issues are discussed in Ref. [4].

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