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Understanding the quantum Zeno effect

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Abstract

The quantum Zeno effect consists in the hindrance of the evolution of a quantum system that is very frequently monitored and *found* to be in its initial state at *every* single measurement. On the basis of the correct formula for the survival probability, i.e. the probability of finding the system in its initial state at *every* single measurement, we critically analyze a recent proposal and experimental test that make use of an oscillating system.

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The seminal formulation of the quantum Zeno effect, due to Misra and Sudarshan [1], deals with the probability of observing an unstable system in its initial state *throughout* a time interval $\Delta = [0, t]$. The purpose of this note is to point out that the quantum Zeno effect has not been experimentally observed, yet, in its original formulation. Indeed, we shall argue that the interesting proposal by Cook [2], that makes use of a two-level system undergoing Rabi oscillations, as well as the beautiful experiment performed by Itano et al. [3], investigate the probability of finding the initial state *at* time *t*, *regardless* of the actual state of the system in the time interval Δ. As we shall see, in general, if the temporal behavior of the system is oscillatory, this probability includes the possibility that transitions of the type: initial state $\to$ other states $\to$ initial state, actually take place. Of course, this remark does not invalidate the soundness of the analysis in Ref. [2] and of the experiment in Ref. [3].

The temporal behavior of quantum mechanical systems is a long-standing issue of investigation [4] (for a review and a collection of recent developments, see Ref. [5]), and the curious features of the short-time behavior of the so-called "survival" probability of a quantum mechanical state, leading to what was to be named "quantum Zeno paradox" [1], were already known about 30 years ago [6]. However, renewed interest in the above topic was motivated by Cook's idea [2] and its subsequent experimental verification [3]. The experiment by Itano et al. provoked a lively debate [7–9], that has essentially focused on two aspects of the problem. First, it has been shown, and it is now becoming a widespread viewpoint, that the experimental results can be explained by making use of a unitary dynamics [10,8]. Notice that an analogous point was raised by Peres quite a few years ago [11]. Second, it has been argued that the so-called limit of continuous observation is in contradiction with Heisenberg's uncertainty principle and does not take into account unavoidable quantum mechanical losses, and is there-

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fore to be considered unphysical [12].

Nowadays most physicists tend to view this phenomenon as a purely dynamical process, in which von Neumann's projections can be substituted by spectral decompositions [13,14], so that the phase correlation among different branch waves is perfectly kept. For this reason, one often speaks of quantum Zeno *effect* (QZE) [7,8], rather than quantum Zeno *paradox* [1].

However, surprisingly, nobody seems to have realized that, strictly speaking, Cook's proposal and Itano et al.'s experiment are conceptually at variance with the original formulation of the QZE. Misra and Sudarshan, in their seminal paper [1], endeavoured to define "the probability $\mathcal{P}(0,T;\rho_0)$ that no decay is found *throughout the interval* $\Delta = [0,T]$ when the initial state of the system was known to be ρ_0". (Italics in the original. Some symbols have been changed.) The definition given in Ref. [1] is

$$\mathcal{P}(0,T;\rho_0) \equiv \lim_{N\to\infty} P^{(N)}(0,T;\rho_0), \tag{1}$$

where $P^{(N)}(0,T;\rho_0)$ is the probability of observing the initial state ρ_0 in a series of N observations, performed at times $t_n = nT/N$ $(n = 1,\ldots,N)$, in order to ascertain whether the system is still undecayed.

In order to facilitate comprehension of the following analysis, it is worth stressing that the above-mentioned "survival probability" of the initial state ρ_0 is the probability of finding the system under investigation in ρ_0 at *every* measurement, during the interval Δ. This is a subtle point, as we shall see.

For the sake of clarity, we shall first carefully analyze Itano et al.'s derivation of what they interpreted as a realization of the QZE, and then scrutinize Cook's formulae. Consider a three-level atomic system, on which an rf field of frequency ω provokes Rabi oscillations between levels 1 and 2. In the rotating wave approximation and in absence of detuning, the equations of motion for the density matrix ρ_{ij} $(i,j = 1,2)$ read

$$\dot\rho_{11} = \tfrac{1}{2}\mathrm{i}\omega(\rho_{21}-\rho_{12}), \quad \dot\rho_{12} = \tfrac{1}{2}\mathrm{i}\omega(\rho_{22}-\rho_{11}),$$
$$\dot\rho_{22} = \tfrac{1}{2}\mathrm{i}\omega(\rho_{12}-\rho_{21}), \tag{2}$$

where the dot denotes derivative with respect to time.

By applying a technique invented by Feynman, Vernon and Hellwarth [15], one can recast the above equations of motion in a very simple form, in which the use of rotating coordinates, introduced by Block [16] and Rabi, Ramsey and Schwinger [17], turns out to be particularly advantageous. Define

$$R_1 \equiv \rho_{21}+\rho_{12}, \quad R_2 \equiv \mathrm{i}(\rho_{12}-\rho_{21}),$$
$$R_3 \equiv \rho_{22}-\rho_{11} \equiv P_2 - P_1, \tag{3}$$

where $P_j \equiv \rho_{jj}$ is the probability that the atom is in level j $(j = 1,2)$. Since $P_1 + P_2 = 1$, one gets

$$P_2 = \tfrac{1}{2}(1+R_3). \tag{4}$$

In terms of the quantities $\boldsymbol{R} \equiv (R_1,R_2,R_3)$ and $\boldsymbol{\omega} \equiv (\omega,0,0)$, Eqs. (2) become

$$\dot{\boldsymbol{R}} = \boldsymbol{\omega}\times\boldsymbol{R}. \tag{5}$$

The solution of the above equation, with initial condition $\boldsymbol{R}(0) \equiv (0,0,-1)$ (only level 1 is initially populated) reads

$$\boldsymbol{R}(t) = (0,\sin\omega t,-\cos\omega t). \tag{6}$$

If the transition between the two levels is driven by an on-resonant π pulse, of duration $T = \pi/\omega$, one gets $\boldsymbol{R}(T) \equiv (0,0,1)$, so that $\rho_{22} = 1, \rho_{11} = 0$, and only level 2 is populated at time T.

The reasoning of Ref. [3] is the following. Assume you perform a measurement at time $\tau = \pi/N\omega = T/N$, by shining on the system a very short "measurement" pulse, that provokes transitions from level 1 to level 3, with subsequent spontaneous emission of a photon [1]. The measurement pulse "projects" the atom into level 1 or 2 ("naive wave function collapse"). Because a measurement "kills" the off-diagonal terms ρ_{12} and ρ_{21} of the density matrix, while leaving unaltered its diagonal terms ρ_{11} and ρ_{22}, one obtains

$$\boldsymbol{R}(\pi/N\omega) = [0,\sin(\pi/N),-\cos(\pi/N)]$$
$$\xrightarrow{\text{measurement}} [0,0,-\cos(\pi/N)] \equiv \boldsymbol{R}^{(1)}. \tag{7}$$

Then the evolution restarts, according to Eq. (5), but with the new initial condition $\boldsymbol{R}^{(1)}$. After N measurements, at time $T = N\tau = \pi/\omega$,

$$\boldsymbol{R}(T) = [0,0,-\cos^N(\pi/N)] \equiv \boldsymbol{R}^{(N)}. \tag{8}$$

[1] We are not addressing the (delicate) point that a measurement pulse, however short, must have a certain *finite* time duration. As a consequence, one must take into account the inevitable spread $\Delta\omega$ of the measurement pulse, and modify accordingly the following formulae. This problem is a very subtle one and will be properly addressed in a forthcoming paper [18].

The probabilities that the atom is in level 2 or 1 at time T, after the N measurements, are therefore given by (see Eq. (4))

$$P_2^{(N)}(T) = \tfrac{1}{2}(1 + R_3^{(N)}) = \tfrac{1}{2}[1 - \cos^N(\pi/N)], \tag{9}$$

$$P_1^{(N)}(T) = 1 - P_2^{(N)}(T) = \tfrac{1}{2}[1 + \cos^N(\pi/N)], \tag{10}$$

respectively. Since $P_2^{(N)}(T) \to 0$ and $P_1^{(N)}(T) \to 1$ as $N \to \infty$, this is interpreted as quantum Zeno effect [2]. The experimental result are in very good agreement with the above formulae. However, this is *not* the quantum Zeno effect à la Misra and Sudarshan: Eq. (9) ((10)) expresses only the probability that the atom is in level 2 (1) at time T, after N measurements, *independently* of its past history. In particular, Eqs. (9), (10) take into account the *possibility that one level gets repopulated after the atom has made transitions to the other level*. In order to shed light on this very important (and rather subtle) point, let us look explicitly at the first two measurements.

After the first measurement, $\boldsymbol{R}^{(1)}$ is given by Eq. (7) and

$$R_3^{(1)} = -\cos\frac{\pi}{N} = P_2^{(1)} - P_1^{(1)}, \tag{11}$$

where $P_j^{(1)}$ is the occupation probability of level j ($j = 1, 2$) at time $\tau = \pi/N\omega$, after the first measurement pulse:

$$P_2^{(1)} = \tfrac{1}{2}(1 + R_3^{(1)}) = \sin^2\frac{\pi}{2N}, \tag{12}$$

$$P_1^{(1)} = 1 - P_2^{(2)} = \cos^2\frac{\pi}{2N}. \tag{13}$$

After the second measurement, one obtains

$$R_3^{(2)} = -\cos^2\frac{\pi}{N} = P_2^{(2)} - P_1^{(2)}, \tag{14}$$

where the occupation probabilities at time $2\tau = 2\pi/N\omega$ read

$$P_2^{(2)} = \tfrac{1}{2}(1 + R_3^{(2)}) = 2\sin^2\frac{\pi}{2N}\cos^2\frac{\pi}{2N}, \tag{15}$$

[2] The $N \to \infty$ limit is in contradiction with the Heisenberg uncertainty principle, and is therefore unphysical. It is possible to set a physical limit on the maximum value that N can attain in a certain experimental situation [12,18].

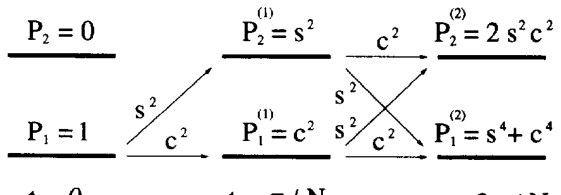

Fig. 1. Transition probabilities after the first two measurements ($s = \sin(\pi/2N)$ and $c = \cos(\pi/2N)$).

$$P_1^{(2)} = 1 - P_2^{(2)} = \cos^4\frac{\pi}{2N} + \sin^4\frac{\pi}{2N}. \tag{16}$$

It is then obvious that $P_1^{(2)}$, in Eq. (16), is *not the survival probability* of level 1, according to the seminal definition (1). It is just the probability that level 1 is populated at time $t = 2\pi/N\omega$, including the possibility that the transition $1 \to 2 \to 1$ took place, with probability $\sin^2(\pi/2N)\sin^2(\pi/2N) = \sin^4(\pi/2N)$. By contrast, the *survival* probability, namely the probability that the atom is found in level 1 *both* in the first and second measurements, is given by $P_1^{(1,2)} = \cos^2(\pi/2N)\cos^2(\pi/2N) = \cos^4(\pi/2N)$. Fig. 1 shows what happens during the first two measurements in the experiment in Ref. [3].

In the general case, after N measurements, the probability that level 1 is populated at time T, independently of its "history", is given by (10), and includes the possibility that transitions to level 2 took place. As a matter of fact, it is not difficult to realize that (9), (10) conceal a binomial distribution,

$$\begin{aligned}\sum_{n\ \mathrm{even}} \binom{N}{n} s^{2n} c^{2(N-n)} &= c^{2N} \sum_{n\ \mathrm{even}} \binom{N}{n} (s/c)^{2n} \\ &= \tfrac{1}{2} c^{2N} \Bigg[\sum_{n=0}^{N} \binom{N}{n} (s/c)^{2n} \\ &\quad + \sum_{n=0}^{N} \binom{N}{n} (-1)^n (s/c)^{2n} \Bigg] \\ &= \tfrac{1}{2} c^{2N} \{ [1 + (s/c)^2]^N + [1 - (s/c)^2]^N \} \\ &= \tfrac{1}{2}[1 + \cos^N(\pi/N)] \\ &= P_1^{(N)}(T) = 1 - P_2^{(N)}(T), \end{aligned} \tag{17}$$

where $\sum_{n\ \mathrm{even}}$ is a sum over all even values of n between 0 and N, $s = \sin(\pi/2N)$, $c = \cos(\pi/2N)$ [3].

[3] Mensky [9] first noticed the occurrence of a binomial dis-

Therefore

$$P_2^{(N)}(T) = 1 - \sum_{n\ \mathrm{even}} \binom{N}{n} \sin^{2n}\frac{\pi}{2N}\cos^{2(N-n)}\frac{\pi}{2N}, \tag{18}$$

$$P_1^{(N)}(T) = \sum_{n\ \mathrm{even}} \binom{N}{n} \sin^{2n}\frac{\pi}{2N}\cos^{2(N-n)}\frac{\pi}{2N}, \tag{19}$$

which clearly shows that Eqs. (9), (10) or (18), (19) include all possible transitions between levels 1 and 2, in such a way that at time T the system is, say, in level 1 after having made an even number ($n = 0, 2, \ldots$, etc.) of transitions between levels 1 and 2. It should be clear now that the result (10) is conceptually very different from Misra and Sudarshan's survival probability (1). The correct formula for the survival probability, in the present case, is obtained by considering *only* the $n = 0$ term in (19),

$$\mathcal{P}_1^{(N)}(T) = \cos^{2N}\frac{\pi}{2N}. \tag{20}$$

Eq. (20) is just the "survival probability", namely the probability that level 1 is populated at every measurement, at times $n\tau = nT/N$ ($n = 1, \ldots, N$) [4].

A comparison with the formulae of Ref. [3] is not straightforward, due to the fact that the authors analyzed their results in terms of the quantity $P_2^{(N)}(T)$, rather than $P_1^{(N)}(T)$. At any rate, Eq. (20) implies

$$\mathcal{P}_2^{(N)}(T) = 1 - \cos^{2N}\frac{\pi}{2N}. \tag{21}$$

Eq. (21) can be compared to (9): Even though they tend to the same limiting value 0 (in either case $\sin(\pi/2N) \to 0$ as $N \to \infty$), they give different results, in particular when N is small, as shown in Table 1.

It must be emphasized that we are not criticizing the soundness of the nice experiment in Ref. [3]. Indeed, the experimental results obtained by Itano et al. are in excellent agreement with Eqs. (9) or (18). We only claim that this experiment, although correctly performed, is conceptually at variance with the original idea of the QZE, as defined by Misra and Sudarshan, because the right expression for the survival probability, according to (1), is given by (20) and not by (19).

Let us now look at Cook's derivation of the QZE. For the sake of clarity, we shall present his analysis in a slightly simplified case. Starting from the set of Eqs. (2), Cook obtained the following rate equations,

$$\dot{P}_1 = k(P_2 - P_1), \tag{22}$$

$$\dot{P}_2 = k(P_1 - P_2), \tag{23}$$

where $k = \omega^2\tau/2$, τ being the time interval between measurement pulses. These equations yield, at time $T = \pi/\omega$,

$$P_2(T) = \tfrac{1}{2}[1 - \exp(-\pi^2/2N)]. \tag{24}$$

(A misprint in Ref. [2] has been corrected.) The above formula is interpreted as a quantum Zeno effect. Once again, this is not correct in a strict sense: The above equation expresses the occupation probability of level 2, independently of its history. Clearly, the rate equations (22), (23) take into account the possibility of transitions $1 \to 2 \to 1$, and so on, and therefore cannot be viewed as expressing "survival" probabilities, as in Eq. (1). It should be stressed that the conclusions drawn in this Letter hold true for all those situations in which the temporal behavior of the system under investigation is of the oscillatory type, and no precautions are taken in order to prevent repopulation of the initial state.

Finally, it is worth briefly commenting on the $N \to \infty$ limit (continuous observation). It was shown [12,18] that this limit is unphysical, for it is in contradiction with Heisenberg's uncertainty principle, and set a reasonable physical limit for the maximum value that N can attain in an experimental test of the QZE involving neutron spin. Venugopalan and Ghosh [19]

Table 1

N	1[a]	2	4	8	16	32	64
$P_2^{(N)}(T)$	1	0.5	0.3750	0.2346	0.1334	0.0716	0.0371
$\mathcal{P}_2^{(N)}(T)$	1	0.75	0.4692	0.2668	0.1431	0.0742	0.0378

[a] $N = 1$ means that only a final measurement is performed, at time T.

tribution in connection with the QZE for an oscillating system, without however pointing out the discrepancy with Misra and Sudarshan's definition of survival probability. The result (17) is, to our knowledge, new.

[4] Eq. (20) was first given in Section V of Ref. [10] (see in particular footnote 21).

criticized this result on the basis of an analysis whose starting point was Eq. (24). However, as we have seen, (24) is *not* related to the survival probability, according to the definition (1), so that the calculation of Ref. [19], although mathematically correct, is not physically relevant for our problem. Incidentally, in the light of our analysis, it is not surprising that the authors of Ref. [19], by applying the uncertainty principle, obtained the limiting value $P_2(T) \rightarrow 1/2$, in the large-N limit, from Eq. (24). Such a result is to be expected, on the basis of Cook's equations (22), (23), but refers to a physically different situation, not to the QZE.

In conclusion, we would like to put forward a few remarks. The real problem related with Cook's proposal and Itano et al.'s experiment is that the state of the atom is *not* observed at intermediate times. As a matter of fact, its observation would probably raise difficult technical problems, for one should be able to "select", after each measurement pulse, *which* atoms are in level 1 and discard those atoms that are in level 2.

The quantum theory of measurement [20,21] is still full of pitfalls and conceptual difficulties. One has to be extremely careful when applying von Neumann's projection postulate. A quantum measurement implies the occurrence of decoherence, but the inverse is not necessarily true, as we have seen: It may happen that the system is practically incoherent, but one still does not know, in practice, *which* state the atom is in.

Very promising candidates for an experimental observation of a genuine QZE seem to be those experiments involving neutron spin [12] or photon polarization [22]. There is certainly more to come, on this fascinating subject.

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