Macroscopic limit of a solvable dynamical model

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The interaction between an ultrarelativistic particle and a linear array made up of N two-level systems (AgBr molecules) is studied by making use of a modified version of the Coleman-Hepp Hamiltonian. Energy-exchange processes between the particle and the molecules are properly taken into account, and the evolution of the total system is calculated exactly both when the array is initially in the ground state and in a thermal state. In the weak-coupling, macroscopic ($N \to \infty$) limit, the system remains solvable and leads to interesting connections with the Jaynes-Cummings model, which describes the interaction of a particle with a maser. The visibility of the interference pattern produced by the two branch waves of the particle is computed, and the conditions under which the spin array behaves as a "detector" are investigated. The behavior of the visibility yields good insights into the issue of quantum measurements: It is found that, in the $N \to \infty$ limit, a superselection-rule space appears in the description of the (macroscopic) apparatus. In general, an initial thermal state of the "detector" provokes a more substantial loss of quantum coherence than an initial ground state. It is argued that a system increasingly loses coherence as the temperature of the detector increases. The problem of "imperfect measurements" is also briefly discussed.

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I. INTRODUCTION

Quantum mechanics is considered to be a fundamental theory of nature, due to its successful predictions in many practical applications. Nevertheless, we still lack a complete understanding of its interpretative postulates, in particular on the so-called quantum-measurement problem [1]. There is not even unanimous consensus about the very definition of the problem, and in fact there have been long discussions in order to understand whether a quantum-mechanical-measurement process can be analyzed within the quantum-mechanical formalism [2].

von Neumann's projection rules [1] are very useful computational tools, but the presence of an external "classical" measuring apparatus is required in order to provoke the "wave-function collapse." We feel that this is not satisfactory, because a measuring system is made up of elementary constituents that must be treated quantum mechanically, and is therefore a quantum-mechanical object itself. On the other hand, the "classical" nature of the macroscopic measuring system should be properly taken into account, because we know that the above-mentioned von Neumann's rules work well, in practical calculations.

In this paper we shall give a concrete example of interaction between an elementary quantum system Q and a model detector D. Notice that if we want to treat the $Q + D$ system quantum mechanically, we must consider the quantum-mechanical structure of both systems, and this is highly nontrivial if one of the two systems is made up of many elementary constituents, because we are forced to consider the interaction between the object particle Q and every single elementary constituent of D.

In order to study the interaction between an object particle and a detection system in the above-mentioned sense, solvable models are very helpful: Not only do they give good insights into physics, but they also provide us with a better understanding of the complicated phenomena involved. In this respect, a model Hamiltonian proposed by Hepp [3] is very well known: It describes the interaction between an ultrarelativistic particle and an ensemble of two-level systems, and is usually referred to as AgBr or the Coleman-Hepp model. Due to its relative simplicity, the model has received considerable attention in the past, and has played an important role in the literature on the measurement problem [4–8]. Another interesting solvable model which describes the interaction between a two-level system and the electromagnetic field in a cavity (maser) was proposed by Jaynes and Cummings some years ago [9].

Our purpose is twofold. First, by making use of a modified version recently proposed [10] for the AgBr Hamiltonian, we will study the interaction between the particle and the detector when the latter is initially in a thermal state. This situation is more realistic than the usual one, in which the detector is initially taken to be in the ground state, because D is macroscopic and cannot be completely isolated from its environment. We emphasize that it would be impossible to study the case of a thermal detector starting from the original Coleman-Hepp model, due to the absence of a free Hamiltonian for D. The introduction of the latter will also enable us to compute several physically relevant quantities, such as the energy "stored" in D as a result of the interaction, its fluctuation, and their ratio.

Second, we shall consider a weak-coupling, macroscopic limit for the AgBr model. We shall see that there is a connection between this limit and the Jaynes-Cummings model. The link can be seen only in our modified version, which is able to take into account energy-exchange processes between Q and D.

We will realize that the above-mentioned limit of a detection system is extremely important from the point of view of quantum measurements: Indeed, the visibility of the interference pattern can be exactly computed for the case of a finite number N of elementary constituents of D, and its behavior in the $N \to \infty$ limit is very interesting. It will be seen that a macroscopic system does not necessarily behave as a "detector," unless other important conditions are met.

This paper is organized as follows. We review the original Coleman-Hepp model in Sec. II, and introduce the modified version in Sec. III, where the case of a detector initially in a thermal state is also considered. In Sec. IV we compute the weak-coupling, $N \to \infty$ limit of some interesting physical quantities and of the scattering matrix. A slightly modified version of the Jaynes-Cummings model is displayed in Sec. V, and the relevant evolutions are considered. We will see that the latter model yields the same results obtained in the weak-coupling macroscopic limit from the AgBr case. In this limit and under certain conditions, the two Hamiltonians are shown to be identical in Sec. VI. The correspondence will be pushed further in Sec. VII, where the problem of quantum measurements will be considered, in particular in the light of the appearance of unitary-inequivalent representations in the many-Hilbert-space theory proposed by Machida and Namiki [11]. Section VIII contains additional considerations concerning this issue and briefly touches upon the concept of imperfect measurements.

II. REVIEW OF THE ORIGINAL AgBr HAMILTONIAN

Let us start off by introducing the Coleman-Hepp or AgBr Hamiltonian, and by reviewing the main results obtained by different authors in the past [3–8]. Even though the content of the present section is not original, light will be thrown on those results that are most important from the "macroscopic" point of view to be analyzed in the present paper.

The AgBr Hamiltonian describes the interaction between an ultrarelativistic particle Q and a one-dimensional N-spin array (D system). One can think, for instance, of a linear emulsion of AgBr molecules, the *down* state corresponding to the undivided molecule, and the *up* state corresponding to the dissociated molecule (Ag and Br atoms). The particle and each molecule interact via a spin-flipping local potential.

The total Hamiltonian for the $Q+D$ system is

$$H = H_Q + H'^{(0)}, \tag{2.1}$$

where H_Q is the free Hamiltonian of the particle and $H'^{(0)}$ the interaction Hamiltonian. These are explicitly written as

$$H_Q = c\hat{p},$$
$$H'^{(0)} = \sum_{n=1}^{N} V(\hat{x} - x_n)\sigma_1^{(n)} , \tag{2.2}$$

where $\hat{p}$ is the momentum of the particle, $\hat{x}$ its position, V is a real potential, x_n $(n = 1, ..., N)$ are the positions of the scatterers in the array, and $\sigma_1^{(n)}$ is the Pauli matrix acting on the nth site.

Notice that the Hamiltonian H is invariant under exchange of scatterers in the array. Therefore, if we call $\mathcal{P}_N$ the group of permutations on $\{1, \ldots, N\}$, we can restrict our attention to the $\mathcal{P}_N$-invariant sector $\mathcal{H}_N$ of the bigger Hilbert space $\mathcal{H}_{\{N\}}$ of the N scatterers. The former is generated by the symmetrized states $|j\rangle_N$, $j = 1, \ldots, N$, where j is the number of dissociated molecules, while the latter by the vectors $|\{j\}\rangle_N$, representing states in which j *particular* molecules are dissociated. The two types of vectors are related to each other via the formula

$$|j\rangle_N = \binom{N}{j}^{-1/2} \sum_{\{j\}} |\{j\}\rangle_N, \tag{2.3}$$

where the summation $\sum_{\{j\}}$ is over the permutations. Incidentally, observe that $\dim \mathcal{H}_{\{N\}} = 2^N$, while $\dim \mathcal{H}_N = N+1$. In the following, we shall concentrate our analysis on the *symmetrized* case, and give only a few comments for the other case. The symmetrization will become a delicate problem in the $N \to \infty$ limit, to be tackled in the following sections.

The above Hamiltonian is a nice model of a typical measurement process and can be solved exactly. Let us sketch rapidly the main results by making use of generalized coherent states [6]. A straightforward calculation yields the following S matrix:

$$S^{[N]} = \exp\left(-i\frac{V_0\delta}{\hbar c}\sum_{n=1}^{N}\sigma_1^{(n)}\right) = \prod_{n=1}^{N} S_{(n)} , \tag{2.4}$$

where

$$S_{(n)} = \exp\left(-i\frac{V_0\delta}{\hbar c}\sigma_1^{(n)}\right)$$
$$= \cos\left(\frac{V_0\delta}{\hbar c}\right) - i\sigma_1^{(n)} \sin\left(\frac{V_0\delta}{\hbar c}\right), \tag{2.5}$$

and $V_0\delta \equiv \int_{-\infty}^{\infty} V(x)dx$. This allows us to define the "spin-flip" probability, i.e., the probability of dissociating one AgBr molecule, as

$$q = \sin^2\left(\frac{V_0\delta}{\hbar c}\right). \tag{2.6}$$

The S matrix can alternatively be written as

$$S^{[N]} = \exp\left(-i\frac{V_0\delta}{\hbar c}N\Sigma_1^{(N)}\right), \tag{2.7}$$

where

$$\Sigma_j^{(N)} = \frac{1}{N}\sum_{n=1}^{N}\sigma_j^{(n)}, \qquad j = 1,2,3 \tag{2.8}$$

is the average spin. Observe that

$$\left[N\Sigma_i^{(N)}, N\Sigma_\ell^{(N)}\right] = 2iN\Sigma_k^{(N)}, \tag{2.9}$$

with i,ℓ,k any even permutation of 1, 2, 3, so that the operators $N\Sigma_j$ form a unitary representation of SU(2). Moreover, by defining

$$\Sigma_\pm^{(N)} = \tfrac{1}{2}\left(\Sigma_1^{(N)} \pm i\Sigma_2^{(N)}\right), \tag{2.10}$$

one gets the algebra

$$\begin{aligned} \left[N\Sigma_-^{(N)}, N\Sigma_+^{(N)}\right] &= -N\Sigma_3^{(N)}, \\ \left[N\Sigma_-^{(N)}, -N\Sigma_3^{(N)}\right] &= -2N\Sigma_-^{(N)}, \\ \left[N\Sigma_+^{(N)}, -N\Sigma_3^{(N)}\right] &= +2N\Sigma_+^{(N)}. \end{aligned} \tag{2.11}$$

The initial D state is taken to be the ground state $|0\rangle_N$ (N spins down), and we shall first consider the situation in which the initial Q state is a plane wave. The evolution is easily computed from Eq. (2.7) by observing that

$$\begin{aligned} N\Sigma_+^{(N)}|n\rangle_N &= \sqrt{(N-n)(n+1)}\;|n+1\rangle_N, \\ N\Sigma_-^{(N)}|n\rangle_N &= \sqrt{(N-n+1)n}\;|n-1\rangle_N, \\ N\Sigma_3^{(N)}|n\rangle_N &= (2n-N)\;|n\rangle_N, \end{aligned} \tag{2.12}$$

and by making use of the formula [12]

$$\begin{aligned} e^{-i\alpha N\Sigma_1^{(N)}} &= e^{\tanh(-i\alpha)N\Sigma_+^{(N)}}e^{-\ln[\cosh(-i\alpha)]N\Sigma_3^{(N)}} \\ &\quad\times e^{\tanh(-i\alpha)N\Sigma_-^{(N)}} \\ &= e^{-i\tan(\alpha)N\Sigma_+^{(N)}}e^{-\ln(\cos\alpha)N\Sigma_3^{(N)}} \\ &\quad\times e^{-i\tan(\alpha)N\Sigma_-^{(N)}}. \end{aligned} \tag{2.13}$$

The result is

$$\begin{aligned} S^{[N]}|p,0\rangle_N &= \sum_{j=0}^{N}\sum_{\{j\}}(-i\sqrt{q})^j\left(\sqrt{1-q}\right)^{N-j}|p,\{j\}\rangle_N \\ &= \sum_{j=0}^{N}\binom{N}{j}^{1/2}(-i\sqrt{q})^j\left(\sqrt{1-q}\right)^{N-j}|p,j\rangle_N, \end{aligned} \tag{2.14}$$

where we have used the notation $|p,\{j\}\rangle_N = |p\rangle|\{j\}\rangle_N$, $|p,j\rangle_N = |p\rangle|j\rangle_N$. The far right-hand side in Eq. (2.14) is a generalized coherent state [6].

In a typical interference experiment a divider splits an incoming wave function ψ into two branch waves ψ_1 and ψ_2, so that the initial state of the $Q+D$ system is

$$\Psi_I = (\psi_1 + \psi_2)\,|0\rangle_N\,, \tag{2.15}$$

where $|\psi_i\rangle = \int dp_i c(p_i)|p_i\rangle$ $(i = 1,2)$ are one-dimensional wave packets, normalized to unity. Assume that only ψ_2 interacts with D. The final state of the total system is

$$\Psi_F = |\psi_1\rangle|0\rangle_N + S^{[N]}|\psi_2\rangle|0\rangle_N\,, \tag{2.16}$$

and after recombination of the two branch waves the probability of observing the particle is

$$P = |\Psi_F|^2 = |\psi_1|^2 + |\psi_2|^2 + 2\mathrm{Re}\left[\psi_1^*\psi_2\;{}_N\langle 0|S^{[N]}|0\rangle_N\right]. \tag{2.17}$$

Interference is observed when a phase shifter is inserted in one of the two paths (neutron-interferometer type), or when the two branch waves originating from the slits are forwarded to a distant screen (Young-interferometer type). In both cases, the visibility of the interference pattern is readily calculated by Eqs. (2.14) and (2.17) as

$$\mathcal{V} = \frac{P_{\max} - P_{\min}}{P_{\max} + P_{\min}} = {}_N\langle 0|S^{[N]}|0\rangle_N = (1-q)^{N/2}. \tag{2.18}$$

Equations (2.7), (2.14), and (2.18) are the main results of the above analysis. Observe that the result is exact and holds true for every value of N. The $N \to \infty$ limit is a somewhat delicate problem, and will be one of the main objectives of the present study.

Notice that, as was to be expected, for finite $q \neq 0$, the interference pattern disappears in the N-infinity limit. This is essentially the case considered by Hepp [3] and Bell [4]. We shall instead consider the weak-coupling, macroscopic limit by letting $N \to \infty$ with $qN = \overline{n} =$ finite [7]. In this case, the visibility becomes

$$\mathcal{V} \xrightarrow[N\to\infty,\, qN=\text{finite}]{} e^{-qN/2} = e^{-\overline{n}/2}. \tag{2.19}$$

Note that $qN = \overline{n}$ represents the average number of excited molecules, so that interference gradually disappears as $\overline{n}$ increases. This is in contrast with the "sudden" disappearance of interference in the finite $q \neq 0$ case.

It remains to be stressed that the Hamiltonian H can be shown [8] to be equivalent to the one studied in Ref. [13], if we restrict our attention to the Hilbert space $\mathcal{H}_N$.

III. THE MODIFIED AgBr HAMILTONIAN

The previous results are very interesting, but we should remark that the above interaction Hamiltonian does not take into account the possibility of energy exchange between the particle and the spin system: Both systems never lose (or gain) energy as a consequence of the interaction. According to Ref. [14], a measuring apparatus that is not affected by the interaction simply acts as a "decomposer," i.e., a device that is only able to perform a spectral decomposition. In order to obtain a change of the apparatus state reflecting the state of the measured system one must, in general, modify the Hamiltonian of the total system. In the Coleman-Hepp case, even though the state of the spin array changes and the total energy of the $Q+D$ system is conserved, the energy levels of the spin system are completely neglected. This is not satisfactory, if we want to regard the spin system as a detecting device, because we are implicitly assuming to be able to distinguish *energetically* different states of the array: On the other hand, this can be made only via a free Hamiltonian of the spin system, which is absent in the above description.

This situation can be improved [10] by taking into account both the energy levels of the D system and the energy transfer between the Q and D systems: The free Hamiltonian of the spin array is added, and an appropriate operator is introduced into the interaction Hamiltonian. These modifications make the model more consistent and realistic. Remarkably, the model remains solvable if a "resonance condition" is met.

The total Hamiltonian for the $Q+D$ system becomes

$$H = H_0 + H', \qquad H_0 = H_Q + H_D, \tag{3.1}$$

where the free Hamiltonians of the particle and of the detector, H_Q and H_D, and the modified interaction Hamiltonian H' are written as

$$\begin{aligned} H_Q &= c\hat{p}, \\ H_D &= \tfrac{1}{2}\hbar\omega \sum_{n=1}^{N}\left(1+\sigma_3^{(n)}\right), \\ H' &= \sum_{n=1}^{N} V(\hat{x}-x_n)\sigma_1^{(n)} \exp\left(i\frac{\omega}{c}\sigma_3^{(n)}\hat{x}\right) \\ &= \sum_{n=1}^{N} V(\hat{x}-x_n)\left[\sigma_+^{(n)}\exp\left(-i\frac{\omega}{c}\hat{x}\right) + \sigma_-^{(n)}\exp\left(+i\frac{\omega}{c}\hat{x}\right)\right]. \end{aligned} \tag{3.2}$$

Notice that the energy difference between the two states of the molecule is $\hbar\omega$, and that the previous Hamiltonian [Eq. (2.2)] is reobtained in the $\omega \to 0$ limit.

Observe that, in contrast with every previous analysis [3–8], we are not neglecting the free energy of the scatterers, represented by H_D, and are taking into account the energy exchange between the Q particle and the spin system: This is accomplished by the above interaction Hamiltonian, whose action can be decomposed in the following way:

$$\begin{aligned} H'_{(n)}|p,\downarrow_{(n)}\rangle &= V(\hat{x}-x_n)\left|p-\frac{\hbar\omega}{c},\uparrow_{(n)}\right\rangle, \\ H'_{(n)}|p,\uparrow_{(n)}\rangle &= V(\hat{x}-x_n)\left|p+\frac{\hbar\omega}{c},\downarrow_{(n)}\right\rangle, \end{aligned} \tag{3.3}$$

where $H'_{(n)}$ is the H' term acting on the nth site, $|p,\downarrow_{(n)}\rangle$ represents a state in which the Q particle has momentum p and the nth molecule is undivided (spin down), and analogously for the other cases. We understand from Eq. (3.3) that the interaction Hamiltonian H' satisfies a "resonance condition," because the energy acquired or lost by the Q particle in every single interaction matches exactly the energy gap between the two spin states (i.e., the energy required to provoke one spin flip).

The analysis of the previous section is readily extended to the present case. The S matrix stems from the product of factors [10]

$$\begin{aligned} S_{(n)} &= \exp\left(-i\frac{V_0\delta}{\hbar c}\boldsymbol{\sigma}^{(n)}\cdot\mathbf{u}\right) \\ &= \cos\left(\frac{V_0\delta}{\hbar c}\right) - i\boldsymbol{\sigma}^{(n)}\cdot\mathbf{u}\sin\left(\frac{V_0\delta}{\hbar c}\right), \\ \mathbf{u} &= \left(\cos\left(\frac{\omega}{c}x\right), \sin\left(\frac{\omega}{c}x\right), 0\right), \end{aligned} \tag{3.4}$$

and is computed as

$$\begin{aligned} S^{[N]} = \prod_{n=1}^{N} S_{(n)} &= \exp\left(-i\frac{V_0\delta}{\hbar c}\sum_{n=1}^{N}\boldsymbol{\sigma}^{(n)}\cdot\mathbf{u}\right) \\ &= \exp\left(-i\frac{V_0\delta}{\hbar c}N\boldsymbol{\Sigma}^{(N)}\cdot\mathbf{u}\right). \end{aligned} \tag{3.5}$$

[Compare with Eqs. (2.4) and (2.7), and observe that the spin-flip probability q is the same.] We shall now compute the evolution of the total system in two interesting cases.

A. Initial ground state

If we take, as in the previous section, the ground state $|0\rangle_N$ as the initial D state, the evolution of the total system is easily calculated as

$$\begin{aligned} S^{[N]}|p,0\rangle_N &= \sum_{j=0}^{N}\sum_{\{j\}}(-i\sqrt{q})^j\left(\sqrt{1-q}\right)^{N-j}\left|p-j\frac{\hbar\omega}{c},\{j\}\right\rangle_N \\ &= \sum_{j=0}^{N}\binom{N}{j}^{1/2}(-i\sqrt{q})^j\left(\sqrt{1-q}\right)^{N-j}\left|p-j\frac{\hbar\omega}{c},j\right\rangle_N. \end{aligned} \tag{3.6}$$

Once again, we obtain the value

$$\mathcal{V} = \frac{P_{\max} - P_{\min}}{P_{\max} + P_{\min}} = {}_N\langle 0|S^{[N]}|0\rangle_N = (1-q)^{N/2} \quad (3.7)$$

for the visibility of the interference pattern. Notice that, for the sake of simplicity, we are suppressing the dependence on the "screen coordinate," in the second equality of Eq. (3.7) (see Appendix A). In the following, we shall always suppress the Q states unless confusion may arise.

It is interesting to calculate the energy "stored" in the array after the interaction with the particle. It is computed as

$$\langle H_D\rangle_F = {}_N\langle 0|S^{[N]\dagger} H_D S^{[N]}|0\rangle_N = qN\,\hbar\omega, \quad (3.8)$$

where F stands for final state and the Q-particle states are suppressed. The fluctuation around the average is

$$\langle \delta H_D\rangle_F = \sqrt{\langle\,(H_D - \langle H_D\rangle_F)^2\,\rangle_F} = \sqrt{pqN}\,\hbar\omega, \quad (3.9)$$

where $p = 1-q$, and their ratio is given by

$$\frac{\langle \delta H_D\rangle_F}{\langle H_D\rangle_F} = \sqrt{\frac{p}{qN}}. \quad (3.10)$$

We stress that the above results (3.8)–(3.10) could not be calculated starting from the original Coleman-Hepp Hamiltonian (2.2), due to the absence of the free Hamiltonian H_D.

The limit $N\to\infty$, $qN = \overline{n} < \infty$ is very interesting and will be discussed in the next section. We shall see that such a limit can be consistently taken only for the "modified" AgBr Hamiltonian introduced in this section.

B. Initial thermal state

In the previous subsection we have considered the interaction between a Q particle and a spin array D when the latter is initially in the ground state. This situation is not completely satisfactory, from the physical point of view: Indeed, our spin array is a caricature of a detector, and is therefore a macroscopic object. A more realistic description of D should therefore take into account such macroscopic quantities as volume, temperature, and so on. Let us now consider the case in which the detector is initially in a thermal state, characterized by the density matrix

$$\rho_{\rm th} = \frac{1}{Z}\exp\left[-\beta\frac{\hbar\omega}{2}\sum_{n=1}^{N}\left(1+\sigma_3^{(n)}\right)\right], \quad (3.11)$$

where Z is the partition function and $\beta = 1/k\Theta$, Θ being the temperature. As previously stated, we restrict ourselves to the symmetrized space $\mathcal{H}_N$, so that the identity is written as $\mathbf{1} = \sum_{j=0}^{N}|j\rangle_N\,{}_N\langle j|$, and

$$\rho_{\rm th} = \frac{1}{Z}\sum_{j=0}^{N}\exp\left[-j\beta\hbar\omega\right]|j\rangle_{NN}\langle j|. \quad (3.12)$$

The condition $\mathrm{Tr}\rho_{\rm th} = 1$ yields

$$Z = \frac{1-e^{-\beta\hbar\omega(N+1)}}{1-e^{-\beta\hbar\omega}}. \quad (3.13)$$

(Incidentally, notice that in the unsymmetrized space $\mathcal{H}_{\{N\}}$ there would appear different expressions for the above two quantities.)

We are implicitly assuming that the interaction between Q and D takes place when our detector is in contact with a thermal *reservoir*, at temperature Θ. Obviously, after the interaction, D will thermalize again, returning eventually to its initial state $\rho_{\rm th}$, so that no trace of the passage of the Q particle will be left. This situation is not very interesting, from our point of view, because we are just investigating under which conditions the spin array responds to the interaction with the Q particle, detecting its passage. Only in such a case can the D system be considered as a "detector." In the following analysis we shall assume that the coupling between D and the thermal *reservoir* is very weak compared to that between D and Q, so that the state of D immediately after the interaction with Q can be considered, to a very good approximation, as the final state. Alternatively, we can assume that the interaction between Q and D is much quicker than between D and the *reservoir*, so that the thermalization process of D after its interaction with Q requires a much longer time.

The initial D state is characterized by the quantities

$$\langle H_D\rangle_I^{\rm th} = \mathrm{Tr}\,(H_D\rho_{\rm th}) = \hbar\omega\left(\frac{e^{-\beta\hbar\omega}}{1-e^{-\beta\hbar\omega}} - \frac{(N+1)e^{-(N+1)\beta\hbar\omega}}{1-e^{-(N+1)\beta\hbar\omega}}\right),$$

$$\langle \delta H_D\rangle_I^{\rm th} = \hbar\omega\left(\frac{e^{-\beta\hbar\omega}}{(1-e^{-\beta\hbar\omega})^2} - \frac{(N+1)^2 e^{-(N+1)\beta\hbar\omega}}{\left(1-e^{-(N+1)\beta\hbar\omega}\right)^2}\right)^{1/2}, \quad (3.14)$$

where I stands for initial state.

The evolution of the $Q+D$ system can be computed explicitly, but the final state, expressed in terms of density matrices, does not have a simple expression due to the presence of the Q-particle states. We shall see, in Sec. VII, how it is possible to devise a formal expedient in order to get rid of the Q states. Here, we just calculate the value of the physically interesting quantities in the following way: From Eq. (3.4) we get

$$S_{(n)} = \exp\left[-i\varpi\boldsymbol{\sigma}^{(n)}\cdot\mathbf{u}\right] = \exp\left[-i\varpi\left(\sigma_+^{(n)}e^{-i\frac{\omega}{c}\hat{x}} + \sigma_-^{(n)}e^{i\frac{\omega}{c}\hat{x}}\right)\right], \quad (3.15)$$

where we have written $\varpi = V_0\delta/\hbar c$. [Notice that if we assume a small spin-flip probability q (of order N^{-1}), we get $q = \sin^2\varpi \simeq \varpi^2$.] It is then easy to compute

$$\begin{aligned} S_{(n)}\sigma_3^{(n)}S_{(n)}^\dagger &= \sigma_3^{(n)}\cos 2\varpi \\ &\quad + \left(\sigma_1^{(n)}\sin\frac{\omega}{c}\widehat{x} - \sigma_2^{(n)}\cos\frac{\omega}{c}\widehat{x}\right)\sin 2\varpi \\ &= \sigma_3^{(n)}\cos 2\varpi \\ &\quad + i\left(\sigma_+^{(n)}e^{-i\frac{\omega}{c}\widehat{x}} - \sigma_-^{(n)}e^{i\frac{\omega}{c}\widehat{x}}\right)\sin 2\varpi \\ &\equiv \sigma_3^{\star(n)}, \end{aligned} \tag{3.16}$$

so that

$$S^{[N]}H_D S^{[N]\dagger} = \frac{\hbar\omega}{2}\sum_{n=1}^{N}\left(1+\sigma_3^{\star(n)}\right). \tag{3.17}$$

It is then straightforward, if lenghty, to prove that

$$\begin{aligned} \langle H_D\rangle_F^{\rm th} &= {\rm Tr}\left(H_D\rho_{\rm th}^F\right) = {\rm Tr}\left(H_D S^{[N]}\rho_{\rm th}S^{[N]\dagger}\right) \\ &= {\rm Tr}\left(S^{[N]\dagger}H_D S^{[N]}\rho_{\rm th}\right) \\ &= \cos 2\varpi\langle H_D\rangle_I^{\rm th} + \frac{N}{2}\hbar\omega(1-\cos 2\varpi), \end{aligned} \tag{3.18}$$

where F stands for the final state.

Analogously, we get

$$\begin{aligned} \langle\delta H_D\rangle_F^{\rm th} = \Bigg[\left(\frac{\hbar\omega}{2}\right)^2\Big\{&\left(4\cos^2 2\varpi - 2\sin^2 2\varpi\right)\left[2\gamma_1^2+\gamma_1-(N+1)(N+1+2\gamma_1)\gamma_{N+1}\right] \\ &+\left[4N\cos 2\varpi(1-\cos 2\varpi)+2N\sin^2 2\varpi\right]\left[\gamma_1-(N+1)\gamma_{N+1}\right] \\ &+N^2(1-\cos 2\varpi)^2+N\sin^2 2\varpi\Big\} - \left(\langle H_D\rangle_F^{\rm th}\right)^2\Bigg]^{1/2}, \end{aligned} \tag{3.19}$$

where $\gamma_m = \exp(-m\beta\hbar\omega)/[1-\exp(-m\beta\hbar\omega)]$.

The calculation for the visibility is more involved and is explained in Appendix A. The final result is

$$\mathcal{V}^{\rm th} = {\rm Tr}\left(\rho_{\rm th}S^{[N]}\right) = \frac{e^{\beta\hbar\omega}\cos^{N+2}\varpi}{Z(t_+-t_-)}\left(\frac{1}{t_-^{N+1}}-\frac{1}{t_+^{N+1}}\right), \tag{3.20}$$

$$t_\pm = \tfrac{1}{2}\left[\cos^2\varpi\left(1+e^{\beta\hbar\omega}\right) \pm \sqrt{\cos^4\varpi\left(1+e^{\beta\hbar\omega}\right)^2 - 4\cos^2\varpi e^{\beta\hbar\omega}}\right].$$

Obviously, we recover the results of the previous subsection for $\Theta = 0$ ($\beta=\infty$).

Once again, we realize the advantage of keeping the free Hamiltonian H_D. If one started from the original Hamiltonian (2.2), one would not be able to discuss the temperature dependence of the physically interesting quantities [see Eqs. (3.14), (3.18), (3.19), and (3.20)]: Indeed, if there are no energy differences between different spin configurations, the D system, if it is to be represented by a mixture, is always described by the density matrix of a completely random ensemble [15], irrespectively of the temperature, and any discussion about the temperature dependence would be meaningless.

IV. THE WEAK COUPLING ($N \to \infty$) LIMIT

One of the main purposes of the present investigation is to study the thermodynamical limit of the modified AgBr model, introduced in the previous section. This will be done by keeping the quantity qN always finite. The physical meaning of this limit is appealing: It corresponds to admitting that the number of dissociated molecules $\overline{n} = qN$ is finite. Alternatively, one can say that the energy $\overline{n}\,\hbar\omega = qN\hbar\omega$ exchanged between the particle and the detector is kept finite, even though the number of elementary constituents of D becomes very large. It is also worth stressing the link with the $\lambda^2 t$ limit originally considered by Van Hove [16].

A. Interesting physical quantities

Let us now evaluate the physical quantities calculated in the previous section in the $N\to\infty$, $\overline{n} = qN =$ finite limit.

If the initial D state is the ground state $|0\rangle_N$ we obtain, from (3.8), (3.9), (3.10), and (3.7), respectively,

$$\begin{aligned}
\langle H_D\rangle_F &\to \overline{n}\,\hbar\omega,\\
\langle \delta H_D\rangle_F &\to \sqrt{\overline{n}}\,\hbar\omega,\\
\frac{\langle \delta H_D\rangle_F}{\langle H_D\rangle_F} &\to \frac{1}{\sqrt{\overline{n}}},\\
\mathcal{V} &\to e^{-\overline{n}/2}.
\end{aligned} \tag{4.1}$$

Notice that the $N\to\infty$ limit, with finite $q\neq 0$, yields only divergent or vanishing quantities.

On the other hand, if we start from the thermal D state we get, from Eq. (3.14),

$$\begin{aligned}
\langle H_D\rangle_I^{\rm th} &\to \hbar\omega n_{\rm th},\\
\langle \delta H_D\rangle_I^{\rm th} &\to \hbar\omega\, n_{\rm th} e^{\beta\hbar\omega/2},\\
\frac{\langle \delta H_D\rangle_I^{\rm th}}{\langle H_D\rangle_I^{\rm th}} &\to e^{\beta\hbar\omega/2},
\end{aligned} \tag{4.2}$$

where $n_{\rm th} = e^{-\beta\hbar\omega}/\left(1-e^{-\beta\hbar\omega}\right)$ is the number of excited (up) spins in the initial thermal state, and from Eqs. (3.18)–(3.20),

$$\begin{aligned}
\langle H_D\rangle_F^{\rm th} &\to \hbar\omega\left(n_{\rm th}+\overline{n}\right),\\
\langle \delta H_D\rangle_F^{\rm th} &\to \hbar\omega\, n_{\rm th}\sqrt{1+(2\overline{n}+1)\left(e^{\beta\hbar\omega}-1\right)+\overline{n}\left(e^{\beta\hbar\omega}-1\right)^2},\\
\frac{\langle \delta H_D\rangle_F^{\rm th}}{\langle H_D\rangle_F^{\rm th}} &\to \frac{\sqrt{1+(2\overline{n}+1)\left(e^{\beta\hbar\omega}-1\right)+\overline{n}\left(e^{\beta\hbar\omega}-1\right)^2}}{1+\overline{n}\left(e^{\beta\hbar\omega}-1\right)},\\
\mathcal{V}^{\rm th} &\to \exp\left[-\left(n_{\rm th}+\tfrac{1}{2}\right)\overline{n}\right].
\end{aligned} \tag{4.3}$$

Obviously, we recover the results of Eq. (4.1) for $\Theta=0$ ($\beta=\infty$). In particular,

$$\frac{\langle \delta H_D\rangle_F^{\rm th}}{\langle H_D\rangle_F^{\rm th}}\left\{\begin{array}{ll}\xrightarrow[\beta=0]{} & 1\\ \xrightarrow[\beta=\infty]{} & 1/\sqrt{\overline{n}}.\end{array}\right. \tag{4.4}$$

B. The scattering matrix

Let us now turn our attention to the $N\to\infty$ limit of the scattering matrix $S^{[N]}$ [Eq. (3.5)]. Observe that the S matrix can be rewritten as

$$\begin{aligned}
S^{[N]} &= \exp\left[-i\frac{V_0\delta}{\hbar c}\sum_{n=1}^{N}\boldsymbol{\sigma}^{(n)}\cdot\mathbf{u}\right]\\
&= \exp\left\{-i\frac{V_0\delta}{\hbar c}\sum_{n=1}^{N}\left(\sigma_+^{(n)}\exp\left[-i\frac{\omega}{c}\widehat{x}\right]+\sigma_-^{(n)}\exp\left[+i\frac{\omega}{c}\widehat{x}\right]\right)\right\}\\
&= \exp\left\{-i\frac{V_0\delta}{\hbar c}\sqrt{N}\left(\exp\left[-i\frac{\omega}{c}\widehat{x}\right]\frac{1}{\sqrt{N}}\sum_{n=1}^{N}\sigma_+^{(n)}+\exp\left[+i\frac{\omega}{c}\widehat{x}\right]\frac{1}{\sqrt{N}}\sum_{n=1}^{N}\sigma_-^{(n)}\right)\right\}.
\end{aligned} \tag{4.5}$$

Consider now that the condition $qN=$ finite, with $\sqrt{q}\simeq V_0\delta/\hbar c$, implies that the quantity $(V_0\delta/\hbar c)\sqrt{N}\equiv u_0\delta/\hbar c$ behaves "well" in the $N\to\infty$ limit, i.e., it does neither diverge, nor vanish. (We assume, for simplicity and without loss of generality, that δ is the same quantity used in Sec. II.) On the other hand, the operators $N^{-1/2}\sum_{n=1}^{N}\sigma_\pm^{(n)}$ and $(1/2)\sum_{n=1}^{N}\left(1+\sigma_3^{(n)}\right)$ [which is nothing but the free Hamiltonian of the detector in Eq. (3.2)] obey, in the $N\to\infty$ limit, the standard boson commutation relations for $a, a^\dagger$ and $\mathcal{N}=a^\dagger a$ [17]. This is shown in Appendix B for the reader's convenience. Summing up, we can identify

$$\begin{aligned}
\frac{1}{\sqrt{N}}\sum_{n=1}^{N}\sigma_+^{(n)} &= \sqrt{N}\Sigma_+^{(N)} \xrightarrow[N\to\infty]{} a^\dagger,\\
\frac{1}{\sqrt{N}}\sum_{n=1}^{N}\sigma_-^{(n)} &= \sqrt{N}\Sigma_-^{(N)} \xrightarrow[N\to\infty]{} a,\\
\frac{1}{2}\sum_{n=1}^{N}\left(1+\sigma_3^{(n)}\right) &= \frac{N}{2}\left(\mathbf{1}^{(N)}+\Sigma_3^{(N)}\right) \xrightarrow[N\to\infty]{} \mathcal{N}\equiv a^\dagger a,
\end{aligned} \tag{4.6}$$

so that the S matrix becomes

$$S^{[N]} \to S = \exp\left\{-i\frac{u_0\delta}{\hbar c}\left(a^\dagger \exp\left[-i\frac{\omega}{c}\widehat{x}\right] + a\exp\left[i\frac{\omega}{c}\widehat{x}\right]\right)\right\}. \quad (4.7)$$

The connection with a "maser" system is obvious, and will be made more precise in the next section.

V. THE MASER SYSTEM

Let us clarify the connection between the modified AgBr and the maser systems. First we consider the case in which the D system is an electromagnetic field in a cavity (maser). We keep the free Hamiltonian $H_Q = c\widehat{p}$ for the Q system, so that the total Hamiltonian is given by

$$H^{\mathrm{JC}} = H_0^{\mathrm{JC}} + H'^{\mathrm{JC}}, \quad H_0^{\mathrm{JC}} = H_Q + H_D^{\mathrm{JC}}, \quad (5.1)$$

where

$$H_Q = c\widehat{p},$$
$$H_D^{\mathrm{JC}} = \hbar\omega\mathcal{N} = \hbar\omega a^\dagger a,$$
$$H'^{\mathrm{JC}} = u(\widehat{x})\left[a^\dagger \exp\left(-i\frac{\omega}{c}\widehat{x}\right) + a\exp\left(i\frac{\omega}{c}\widehat{x}\right)\right]. \quad (5.2)$$

Here, we wrote JC in order to stress the resemblance with the so-called Jaynes-Cummings [9] Hamiltonian that describes the interaction between a two-level system and the electromagnetic field in a cavity. The JC Hamiltonian differs from the present one only because it contains terms of the type $\tau_\pm$, instead of $\exp\left(\pm i\frac{\omega}{c}\widehat{x}\right)$, $\tau_\pm$ being the raising or lowering operator for a two-level system. In the case we are considering, the Q particle has a continuous spectrum, and can exchange an arbitrary number of quanta of energy $\hbar\omega$. Clearly, this difference is not important for our analysis, one of the purposes of which is to understand the behavior of the spin array in the $N \to \infty$ limit.

We solve the interaction between the Q particle and the D system (maser). The S matrix is

$$S = \exp\left\{-i\frac{u_0\delta}{\hbar c}\left(a^\dagger \exp\left[-i\frac{\omega}{c}\widehat{x}\right] + a\exp\left[i\frac{\omega}{c}\widehat{x}\right]\right)\right\}, \quad (5.3)$$

where $\int u(x)dx = u_0\delta$, δ being the same quantity used in Eq. (4.5). Notice that the S-matrix obtained here is exactly the same as that derived in the $N \to \infty$ limit for the modified AgBr model (4.7). We will see below that the results obtained in Secs. III A and III B can be extended to the "JC" case in a fully consistent way. The analogy between the two cases will be pushed much further in Sec. VII. We first calculate the physically interesting quantities for the JC case in this section, and then we put forward a correspondence between the modified AgBr and JC Hamiltonians in Sec. VI.

A. Initial ground state

Let us first assume the initial state to be $|p,0\rangle = |p\rangle|0\rangle$, where $|0\rangle$ is the ground state of the maser cavity. The evolution is

$$S|p,0\rangle = e^{-\overline{\kappa}/2}\sum_{j=0}^{\infty}\frac{(-i\sqrt{\overline{\kappa}})^j}{\sqrt{j!}}\left|p - j\frac{\hbar\omega}{c}, j\right\rangle, \quad (5.4)$$

$$\overline{\kappa} = \left(\frac{u_0\delta}{\hbar c}\right)^2,$$

with $|p_j, j\rangle = |p_j\rangle|j\rangle$, $|j\rangle$ being the number state of the cavity. By observing that

$$e^{i\alpha(a^\dagger + a)}ae^{-i\alpha(a^\dagger+a)} = a - i\alpha, \quad (5.5)$$

we easily obtain

$$\langle H_D^{\mathrm{JC}}\rangle_F = \overline{\kappa}\,\hbar\omega,$$
$$\langle \delta H_D^{\mathrm{JC}}\rangle_F = \sqrt{\overline{\kappa}}\,\hbar\omega,$$
$$\frac{\langle \delta H_D^{\mathrm{JC}}\rangle_F}{\langle H_D^{\mathrm{JC}}\rangle_F} = \frac{1}{\sqrt{\overline{\kappa}}}, \quad (5.6)$$
$$\mathcal{V}^{\mathrm{JC}} = e^{-\overline{\kappa}/2},$$

where F denotes the final state and the matrix elements of the Q-particle states are trivially computed. As was to be expected, the above equations allow us to interpret $\overline{\kappa}$ as the average number of boson excitations in the cavity. We can see the perfect correspondence between Eqs. (4.1) and (5.6), if we identify

$$\overline{n} \Leftrightarrow \overline{\kappa}. \quad (5.7)$$

Incidentally, notice that, if we neglect altogether the Q-particle states, the generalized coherent state of Eq. (3.6) becomes, in the $N \to \infty$, $qN < \infty$ limit, the Glauber coherent state of Eq. (5.4). We shall come back to this point in Sec. VII.

The analogy between the two cases, i.e., between the $N \to \infty$ limit of the N-spin system and the maser system, has thus been estabilished when the ground state is chosen as the initial D state. What happens if we choose a thermal state as the initial state? We shall analyze this case in the next subsection.

B. Initial thermal state

The thermal state of the cavity can be written

$$\rho_{\mathrm{th}}^{\mathrm{JC}} = \frac{1}{\mathcal{Z}}\exp\left[-\beta\hbar\omega a^\dagger a\right], \quad (5.8)$$

where $\mathcal{Z}$ is the partition function and $\beta = 1/k\Theta$, Θ being the temperature. In the Fock space $\mathcal{H}$ the identity is $\mathbf{1} = \sum_{j=0}^{\infty}|j\rangle\langle j|$, so that

$$\rho_{\mathrm{th}}^{\mathrm{JC}} = \frac{1}{\mathcal{Z}}\sum_{j=0}^{\infty}\exp\left[-j\beta\hbar\omega\right]|j\rangle\langle j|, \quad (5.9)$$

and the condition $\mathrm{Tr}\rho_{\rm th}^{\rm JC} = 1$ yields

$$\mathcal{Z} = \frac{1}{1 - e^{-\beta\hbar\omega}}. \tag{5.10}$$

Notice the correspondence with the $N \to \infty$ limit of Eqs. (3.11)–(3.13), and remember the importance of choosing the symmetrized space $\mathcal{H}_N$ for the spin case: As already stressed, the unsymmetrized space $\mathcal{H}_{\{N\}}$ would have given different expressions in Eqs. (3.12) and (3.13).

The initial maser state is characterized by the quantities

$$\begin{aligned} \langle H_D^{\rm JC}\rangle_I^{\rm th} &= \hbar\omega\, \kappa_{\rm th}, \\ \langle \delta H_D^{\rm JC}\rangle_I^{\rm th} &= \hbar\omega\, \kappa_{\rm th} e^{\beta\hbar\omega/2}, \\ \frac{\langle \delta H_D^{\rm JC}\rangle_I^{\rm th}}{\langle H_D^{\rm JC}\rangle_I^{\rm th}} &= e^{\beta\hbar\omega/2}, \end{aligned} \tag{5.11}$$

where $\kappa_{\rm th} = e^{-\beta\hbar\omega}/\left(1 - e^{-\beta\hbar\omega}\right)$ is the number of boson excitations in the initial thermal state. This is identical to Eq. (4.2). It is not difficult to prove that

$$\begin{aligned} &\langle H_D^{\rm JC}\rangle_F^{\rm th} = \hbar\omega\left(\kappa_{\rm th} + \overline{\kappa}\right), \\ &\langle \delta H_D^{\rm JC}\rangle_F^{\rm th} \\ &\quad = \hbar\omega\, \kappa_{\rm th}\sqrt{1 + (2\overline{\kappa}+1)\left(e^{\beta\hbar\omega}-1\right) + \overline{\kappa}\left(e^{\beta\hbar\omega}-1\right)^2}\,, \\ &\frac{\langle \delta H_D^{\rm JC}\rangle_F^{\rm th}}{\langle H_D^{\rm JC}\rangle_F^{\rm th}} = \frac{\sqrt{1 + (2\overline{\kappa}+1)\left(e^{\beta\hbar\omega}-1\right) + \overline{\kappa}\left(e^{\beta\hbar\omega}-1\right)^2}}{1 + \overline{\kappa}\left(e^{\beta\hbar\omega}-1\right)}. \end{aligned} \tag{5.12}$$

Once again, the correspondence with Eq. (4.3) is perfect. Of course, the values of Eq. (5.6) are recovered for $\Theta = 0$ ($\beta = \infty$).

Observe also that, in agreement with Eq. (4.4),

$$\frac{\langle \delta H_D^{\rm JC}\rangle_F^{\rm th}}{\langle H_D^{\rm JC}\rangle_F^{\rm th}} \begin{cases} \xrightarrow[\beta=0]{} & 1 \\ \xrightarrow[\beta=\infty]{} & 1/\sqrt{\overline{\kappa}}. \end{cases} \tag{5.13}$$

The calculation for the visibility of the interference pattern is somewhat more involved, and is given in Appendix C. The result is

$$\mathcal{V}_{\rm th}^{\rm JC} = \exp\left[-\left(\kappa_{\rm th} + \tfrac{1}{2}\right)\overline{\kappa}\right], \tag{5.14}$$

and is identical to the value given in Eq. (4.3).

VI. IDENTIFYING THE HAMILTONIANS

From the complete correspondence between the physically interesting quantities calculated in the JC case and in the weak-coupling ($N \to \infty$) limit of the modified AgBr case, we may expect that there exists a weak-coupling macroscopic limit of the modified AgBr Hamiltonian (3.2), which reproduces the JC Hamiltonian (5.2). We have already seen that as far as the S matrix is concerned, the detailed structure of the potential V does not play any role: Only the integrated quantity $\int_{-\infty}^{\infty} V(x)dx = V_0\delta$ has relevance. Notice also that we have restricted our attention to the $\mathcal{P}_N$-invariant sector $\mathcal{H}_N$ of the total space $\mathcal{H}_{\{N\}}$, so that we are mainly interested in "global" quantities like the total number of spin flips, and information like which spins are flipped and which are not is of no importance, in particular in the macroscopic limit to be considered. Therefore, we can try to neglect the x_n dependence of the potential $V(\widehat{x} - x_n)$ from the beginning, in order to establish a link between the two Hamiltonians (3.2) and (5.2).

Here we shall consider two of the possible limiting procedures for the modified AgBr Hamiltonian. Since we have already established the $N \to \infty$ limit for the free Hamiltonian H_D [see Eq. (4.6)], let us concentrate our attention on the interaction Hamiltonian

$$H' = \sum_{n=1}^{N} V(\widehat{x} - x_n)\sigma_1^{(n)} \exp\left(i\frac{\omega}{c}\sigma_3^{(n)}\widehat{x}\right). \tag{6.1}$$

One possibility to take the $N \to \infty$ limit is to consider the case in which the spins are all placed at the same position, say

$$x_n \equiv x_0 = 0, \qquad \forall n = 1, \ldots, N. \tag{6.2}$$

Another possibility is to consider a kind of average potential over the positions of the scatterers x_n, and replace $V(\widehat{x} - x_n)$ with its average [say $\overline{V}(\widehat{x})$]. In the latter case, we are implicitly assuming that all spins are distributed in a rather small region [8].

In either case we obtain (writing V for $\overline{V}$ in the latter case)

$$\begin{aligned} H' &= V(\widehat{x}) \sum_{n=1}^{N} \sigma_1^{(n)} \exp\left(i\frac{\omega}{c}\sigma_3^{(n)}\widehat{x}\right) \\ &= V(\widehat{x}) \sum_{n=1}^{N} \left[\sigma_+^{(n)} \exp\left(-i\frac{\omega}{c}\widehat{x}\right) + \sigma_-^{(n)} \exp\left(+i\frac{\omega}{c}\widehat{x}\right)\right] \\ &= V(\widehat{x})\sqrt{N}\left[\exp\left(-i\frac{\omega}{c}\widehat{x}\right)\frac{1}{\sqrt{N}}\sum_{n=1}^{N}\sigma_+^{(n)} \right. \\ &\qquad \left. + \exp\left(+i\frac{\omega}{c}\widehat{x}\right)\frac{1}{\sqrt{N}}\sum_{n=1}^{N}\sigma_-^{(n)}\right]. \end{aligned} \tag{6.3}$$

Once again, the condition qN = finite, with $\sqrt{q} \simeq V_0\delta/\hbar c = \int V(y)dy/\hbar c$, implies that the quantity $V(\widehat{x})\sqrt{N} = u(\widehat{x})$ behaves well in the $N \to \infty$ limit. Therefore, in this limit, we can see that the modified AgBr Hamiltonian (3.2) is transformed into the JC Hamiltonian (5.2). In conclusion, the N-spin system behaves, in the $N \to \infty$ limit, as a "cavity," in which boson-like excitations (collective modes) can be created, as a consequence of the interaction with the Q particle.

We close this section with a remark: The AgBr and JC Hamiltonians have been identified when the detailed internal structure of the particle-spin interaction and/or the spin locations are neglected [see, for instance, Eq. (6.2)]. However, this assumption is *not* fundamental because, as we have seen in Sec. V, all the physically interesting quantities, such as energy, energy fluctuation, visibility of the interference pattern, and so on, can be calculated by making use *only* of the S matrix, whose limit can be computed in full generality, as seen in Sec. IV B.

VII. AN INTERPRETATION

In the previous sections we have seen that there is a nice correspondence between the weak-coupling, $N \to \infty$ limit of the AgBr model and the JC model. We have proven the correspondence of the S matrices and the Hamiltonians in Secs. IV B and VI, respectively, and have shown the identity of the final results when the D system is initially in the ground state and in a thermal state in Secs. V A and V B, respectively.

In the present section, we wish to push further the correspondence between the two cases. In order to do this, we shall introduce a suitable notation to denote generalized and Glauber coherent states, and shall explicitly compute the relevant evolutions. It turns out to be convenient, in the following, to suppress the Q-particle states. Needless to say, these could be explicitly taken into account, but at the price of making the notation cumbersome and the formulas more involved. Therefore, in this section, we shall exclusively consider the D states. This can be accomplished via the following expedient: Let us consider the extreme situation in which the energy of the Q system is so large that the loss of energy due to the interaction with the spin system can safely be neglected. That is, we assume that $cp \gg \hbar\omega$. Notice that we still keep $H_D \neq 0$.

Under the above-mentioned condition, if we take an initial ground state, the evolution of the total system may be written as

$$S^{[N]}|p,0\rangle_N \simeq |p\rangle \sum_{j=0}^{N} \binom{N}{j}^{1/2} (-i\sqrt{q})^j \left(\sqrt{1-q}\right)^{N-j} |j\rangle_N$$
$$\equiv |p\rangle |-i\sqrt{q}\rangle_N, \tag{7.1}$$

because the state $|p - j\frac{\hbar\omega}{c}, j\rangle_N \simeq |p,j\rangle_N$ for small j, and the probability of losing a large amount of energy (for large j) is very small ($\simeq q^N$) for small $q \simeq O(N^{-1})$. Here the state $|-i\sqrt{q}\rangle_N$ is a generalized coherent state [6], and the Q-particle state, being factorized, can be neglected. Incidentally, we stress the correspondence between this case and the original Coleman-Hepp model reviewed in Sec. II. We can write

$$S^{[N]}|0\rangle_N \simeq |-i\sqrt{q}\rangle_N \equiv |0^\star\rangle_N, \tag{7.2}$$

where $|0^\star\rangle_N$ is the usual outgoing state of scattering theory, and represents here a new "vacuum," in a sense to be clarified later. Analogously, for the JC case, from Eq. (5.4) we get

$$S|0\rangle \simeq e^{-\overline{\kappa}/2} \sum_{j=0}^{\infty} \frac{(-i\sqrt{\overline{\kappa}})^j}{\sqrt{j!}} |j\rangle \equiv |-i\sqrt{\overline{\kappa}}\rangle \equiv |0^\star\rangle, \tag{7.3}$$

where the coherent state $|-i\sqrt{\overline{\kappa}}\rangle$ has been written in the z representation ($a|z\rangle = z|z\rangle$). Obviously, the $N \to \infty$, $qN = \overline{n} = \overline{\kappa}$ limit of the right-hand side of Eq. (7.1) yields the coherent state of Eq. (7.3). Once again, the action of the S matrix on the vacuum $|0\rangle$ has the effect of generating a new vacuum. In the same spirit, by making use of Eq. (2.12), we define

$$S^{[N]}|m\rangle_N = S^{[N]} \frac{\left(\sqrt{N}\Sigma_+^{(N)}\right)^m}{\sqrt{m!}\prod_{k=0}^{m-1}\sqrt{1-\frac{k}{N}}} |0\rangle_N$$
$$= \frac{\left(\sqrt{N}\Sigma_+^{\star(N)}\right)^m}{\sqrt{m!}\prod_{k=0}^{m-1}\sqrt{1-\frac{k}{N}}} |0^\star\rangle_N \equiv |m^\star\rangle_N, \tag{7.4}$$
$$\sqrt{N}\Sigma_\pm^{\star(N)} \equiv S^{[N]}\sqrt{N}\Sigma_\pm^{(N)} S^{[N]\dagger},$$

and, in the JC case,

$$S|m\rangle = S\frac{(a^\dagger)^m}{\sqrt{m!}}|0\rangle$$
$$= \frac{(a^{\star\dagger})^m}{\sqrt{m!}}|0^\star\rangle \equiv |m^\star\rangle, \tag{7.5}$$
$$a^\star \equiv SaS^\dagger = a + i\sqrt{\overline{\kappa}}.$$

In the thermal case, the evolution is given by

$$\rho_{\rm th}^F = S^{[N]}\rho_{\rm th}S^{[N]\dagger}$$
$$\simeq \frac{1}{Z}\exp\left[-\beta\frac{\hbar\omega}{2}S^{[N]}\sum_{n=1}^{N}\left(1+\sigma_3^{(n)}\right)S^{[N]\dagger}\right],$$
$$= \frac{1}{Z}\exp\left[-\beta\frac{\hbar\omega}{2}\sum_{n=1}^{N}\left(1+\sigma_3^{\star(n)}\right)\right], \tag{7.6}$$

where the notation is the same as in Eq. (3.16). Analogously, one gets

$$S\rho_{\rm th}^{\rm JC}S^\dagger \simeq \frac{1}{\mathcal{Z}}\exp\left[-\beta\hbar\omega a^{\star\dagger}a^\star\right]. \tag{7.7}$$

It is easy to prove that this is the same quantity obtained in the $N \to \infty$ limit from Eq. (7.6).

Summarizing, if the energy change of Q is neglected, we understand the following correspondence in the $N \to \infty, qN = \overline{n} < \infty$ limit:

$$\begin{aligned}
\sqrt{N}\Sigma_+^{(N)} &\longrightarrow a^\dagger,\\
\sqrt{N}\Sigma_-^{(N)} &\longrightarrow a,\\
\frac{N}{2}\left(\mathbf{1}^{(N)}+\Sigma_3^{(N)}\right) &\longrightarrow \mathcal{N} = a^\dagger a,\\
S^{[N]} &\longrightarrow S,\\
|0\rangle_N &\longrightarrow |0\rangle,\\
|m\rangle_N &\longrightarrow |m\rangle,\\
\rho_{\rm th} &\longrightarrow \rho_{\rm th}^{\rm JC},
\end{aligned} \tag{7.8}$$

and analogously for the * states and operators. We are now ready to put forward an interpretation of the results obtained in the previous sections. First, notice that in this paper we have not considered "decoherence" (loss of coherence) effects [8,11], namely we have not tried to understand why and how the density matrix of the $Q+D$ systems evolves from a pure to a mixed state: We have simply introduced a solvable dynamical model describing the interaction between a particle and a "detector," without fully addressing the problem of the loss of quantum coherence. From the measurement-theoretical point of view, the interest of the present model lies in the appearance of a superselection-rule space in the $qN=\overline{n}=\overline{\kappa}\to\infty$ limit. The phenomenon is well known in the many-Hilbert-space theory [11], where the macroscopic apparatus (detector) is described by means of a unitary inequivalent representation.

In order to understand the above-mentioned point, start from the general expression given in Eq. (3.7) and Appendix A, and observe that the visibility of the interference pattern (4.1) and (5.6) disappears in the $\overline{n}=\overline{\kappa}\to\infty$ limit as the two "vacua" become orthogonal:

$$\mathcal{V}=\langle 0|S|0\rangle \simeq \langle 0|0^\star\rangle = e^{-\overline{\kappa}/2} \underset{\overline{\kappa}\to\infty}{\longrightarrow} 0. \tag{7.9}$$

Most of the analyses previously performed on the AgBr Hamiltonian have dealt with this situation.

Moreover, it is very interesting to rewrite the visibility in the thermal case (4.3) and (5.14) as

$$\begin{aligned}\mathcal{V}^{\rm th} &= {\rm Tr}(\rho_{\rm th}S)\\ &\simeq \sum_j \frac{e^{-j\beta\hbar\omega}}{\mathcal{Z}}\langle j|e^{-i\sqrt{\overline{\kappa}}(a^\dagger+a)}|j\rangle\\ &= \sum_j \frac{e^{-j\beta\hbar\omega}}{\mathcal{Z}}\langle 0|\frac{a^j}{\sqrt{j!}}\frac{\left(a^\dagger-i\sqrt{\overline{\kappa}}\right)^j}{\sqrt{j!}}e^{-i\sqrt{\overline{\kappa}}(a^\dagger+a)}|0\rangle\\ &= \sum_j \frac{e^{-j\beta\hbar\omega}}{\mathcal{Z}}\langle 0|\frac{a^j\left(a^{\star\dagger}\right)^j}{j!}|0^\star\rangle\\ &= \sum_j \frac{e^{-j\beta\hbar\omega}}{\mathcal{Z}}\langle j|j^\star\rangle = e^{-\left(\kappa_{\rm th}+\frac{1}{2}\right)\overline{\kappa}} \underset{\overline{\kappa}\to\infty}{\longrightarrow} 0.\end{aligned} \tag{7.10}$$

One clearly sees that interference disappears as the two basis $\{|j\rangle\}$ and $\{|j^\star\rangle\}$ become *orthogonal* to each other. In this sense, we may say that the $N\to\infty$, $qN=\overline{n}=\overline{\kappa}=$ finite limit considered in this paper "foreruns" the appearance of a *superselection-rule* space.

The visibility is often considered as a physical quantity able to characterize the loss of coherence between the two interfering branch waves ("collapse of the wave function"). This is not always correct: Indeed, even though a loss of quantum coherence implies a loss of interference, the opposite is not necessarily true, because the interference pattern may vanish even though the total $Q+D$ system is still in a pure state [18]. In the case described in the present paper, all evolutions are described by S matrices and are therefore strictly *unitary*. If the initial state is a pure state, the final state remains pure, and in this sense quantum coherence is always preserved.

Nevertheless, one may safely regard the above result as a first step towards the loss of quantum coherence ("collapse"), because the D system, being macroscopic, undergoes internal motions that tend to destroy the delicate coherence between its elementary constituents. In the AgBr model considered, for example, we have neglected the presence of interactions between the molecules, as well as their positions (the x_n's play the role of simple parameters, and not of dynamical variables). All these additional effects, if taken into account, would have randomized the process and provoked decoherence, so that statistically, after many repetitions of the "experiment," phase-correlation effects would have been washed out. In this statistical sense one can state that if all additional randomization processes had been taken into account the "collapse of the wave function" would have occurred.

VIII. ADDITIONAL REMARKS

We have studied the interaction between an ultrarelativistic particle Q and a "detector" D, schematized as a linear array of two-level systems ("AgBr molecules"), that can be excited (dissociated) as a consequence of the interaction. We have seen that if the original AgBr model is suitably modified, it is possible to take into account energy-exchange processes between Q and D: This is physically appealing, because the state of a detector should show trace of the passage of the particle, also from an energetic viewpoint. We have computed the weak-coupling, macroscopic limit of this system and stressed a correspondence with the Jaynes-Cummings model.

As mentioned in the previous section, the examples considered in this paper are particularly interesting from the point of view of quantum measurements. We have seen that the visibility has a remarkable behavior in all the cases considered, and in particular in the $N\to\infty$ limit. Notice, that while $\overline{n}=\overline{\kappa}$ represents the strength of the interaction between Q and D, $n_{\rm th}$ and $\kappa_{\rm th}$ express the presence of (thermal) noise. Obviously, from Eqs. (4.1) and (4.3) [or alternatively, from Eqs. (5.6) and (5.14)], the visibility disappears in both cases as $\overline{n}=\overline{\kappa}\to\infty$: This means, in a certain sense, that the macrosystem "works better" as a "measuring system," as the strength of the interaction between Q and D increases. On the other hand, as was to be expected, as soon as the Q and D systems are dynamically coupled ($\overline{n}=\overline{\kappa}\neq 0$), the visibility tends to vanish more quickly if the detector is initially in a thermal state rather than in the ground state.

Notice also that the visibility vanishes quickly (exponentially) as the temperature increases. If we consider, within the limits stressed at the end of the last section, the visibility as a physical quantity able to characterize the loss of coherence between the two interfering branch waves of the object system ("collapse of the wave function"), we realize that the Q system *decoheres* more as the temperature of the D system increases. In the above-mentioned sense, one could speak of *imperfect measurements*: The visibility plays the role of a parameter that controls how "effective" a measurement of the Q-particle trajectory is. The value $\mathcal{V}=1$ ($\overline{n}=\overline{\kappa}=0$) signifies absence of interaction between Q and D: The Q system is not affected by the detector and behaves as a "wave." Interference between the two branch waves is complete.

On the other hand, the value $\mathcal{V} = 0$ represents a "particle" behavior, and a total loss of interference. Notice that the latter situation can be achieved if $\overline{n} = \overline{\kappa} \to \infty$ or, alternatively, when D is initially in a thermal state, if $\overline{n} = \overline{\kappa} \neq 0$ and $n_{\text{th}} = \kappa_{\text{th}} \to \infty$: The latter case simply means a nonvanishing interaction between Q and a D system that is initially at infinite temperature. The intermediate cases $0 < \mathcal{V} < 1$ represent imperfect measurements, after which the branch waves of the Q system are still able to interfere, at least to a partial extent.

We stress that the problem of decoherence and imperfect measurements is certainly much more delicate than implied by the above discussion. In particular, notice that the off-diagonal terms (with respect to Q) of the total $(Q + D)$ density matrix have not been shown to vanish, in the cases considered in the present paper, so that, strictly speaking, the problem of decoherence has not been addressed in its full generality. More careful investigation is required on this point.

It is also interesting to comment on a remark put forward by Busch, Lahti, and Mittelstaedt [19] about the occurrence of nonseparable Hilbert spaces when (continuous) superselection rules appear in the description of macroscopic apparatuses. It seems to us that there are cases (and the model discussed in this paper provides an example) in which physics itself "suggests" which limit and space are more suitable to describe the situation investigated: In the AgBr system, one could have considered other possible situations, such as, for instance, the space $\mathcal{H}_{\{N\}}$ or the $N \to \infty$ limit without keeping the quantity qN finite. We have already observed that $\dim \mathcal{H}_{\{N\}} = 2^N$, so that in the $N \to \infty$ limit the space $\mathcal{H}_{\{N\}}$ is nonseparable. On the other hand, $\dim \mathcal{H}_N = N + 1$, and $\mathcal{H}_N$ tends to a separable Hilbert space: In fact, the qN-finite case investigated in this paper turns out to be equivalent to a maser system, that is describable in Fock space. It seems therefore that the requirement of physically reasonable conditions (such as *finite* energy exchange between Q and D and restriction to *symmetrized* D states) lead, in the weak-coupling macroscopic limit, to a *separable* Hilbert space, and therefore the emergence of nonseparability is not necessarily a problem that must be faced.

The model discussed in this paper has proven to be a very fertile example for discussions on quantum measurements. Even though the argument remains open, in particular on the problem of decoherence, we hope to have convincingly shown that a quantum-mechanical measurement process can be treated within quantum mechanics, and one need not postulate a "classical" behavior for the measuring apparatus.

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APPENDIX A

Let us first derive a general formula for the visibility $\mathcal{V}$. We consider a typical Young-type experiment displaying the interference pattern (the treatment of a neutron-interferometric-type experiment is analogous). Let $|\psi_1\rangle$ and $|\psi_2\rangle$ be the two branch waves of the initial Q system and $|j\rangle$ a complete orthonormal set of the D system and assume that the evolution of the total system is described by the S matrix whose action is given by

$$S|\psi_1\rangle|j\rangle = |\psi_1\rangle|j\rangle,$$
$$S|\psi_2\rangle|j\rangle = \sum_k C_{jk}|\psi'_{2k}\rangle|k\rangle. \tag{A1}$$

We assumed that only $|\psi_2\rangle$ interacts with the detector D.

Let, without loss of generality, the D system be initially in one of the above $|j\rangle$ states, say $|n\rangle$. The final state of the total system is described by the density matrix

$$\begin{aligned}\rho^F_{\text{tot}} &= S\rho^I_{\text{tot}}S^\dagger \\ &= S\Big[(|\psi_1\rangle\langle\psi_1| + |\psi_2\rangle\langle\psi_2| + |\psi_1\rangle\langle\psi_2| + \text{H.c.}) \otimes |n\rangle\langle n|\Big]S^\dagger \\ &= |\psi_1\rangle\langle\psi_1| \otimes |n\rangle\langle n| + \sum_{j,k} C_{nj}C^*_{nk}|\psi'_{2j}\rangle\langle\psi'_{2k}| \otimes |j\rangle\langle k| + \sum_j C^*_{nj}|\psi_1\rangle\langle\psi'_{2j}| \otimes |n\rangle\langle j| + \text{H.c.}\end{aligned} \tag{A2}$$

Therefore the probability of observing the particle after the interaction, say at $\mathbf{y}_0$, is given by

$$P(\mathbf{y}_0) = \mathrm{Tr}\Big[|\mathbf{y}_0\rangle\langle\mathbf{y}_0|\rho^F_{\rm tot}\Big] = |\psi_1(\mathbf{y}_0)|^2 + |\psi_2(\mathbf{y}_0)|^2 + 2\,\mathrm{Re}\Big[C^*_{nn}\psi_1(\mathbf{y}_0)\psi'^*_{2n}(\mathbf{y}_0)\Big], \tag{A3}$$

where the trace is taken over both the Q and D states, $\psi_1(\mathbf{y}_0) = \langle\mathbf{y}_0|\psi_1\rangle$ is the branch-wave function of the particle, and so on. (Notice that the S matrix, which is responsible only for the interaction between the Q and D systems, does not contain $\mathbf{y}_0$ as a dynamical variable.) The position $\mathbf{y}_0$ corresponds to the location of the particle's spot on the screen. To simplify the discussion, we assume that the wave functions after the interaction with the D system can be well approximated by plane waves. Then we understand that the first two terms in (A3) no longer depend on $\mathbf{y}_0$ and the interference pattern is produced by the last term. By assuming $|\psi_1|^2 = |\psi_2|^2 = |\psi'_{2n}|^2$, the visibility $\mathcal{V}$ is given by

$$\mathcal{V} = \frac{P_{\max} - P_{\min}}{P_{\max} + P_{\min}} = |C_{nn}|, \tag{A4}$$

where the indices max and min are relative to the screen coordinate $\mathbf{y}_0$. Notice that C_{nn} can also be written as

$$C_{nn} = \mathrm{Tr}\Big[|\mathbf{y}_0\rangle\langle\mathbf{y}_0| \otimes |n\rangle\langle n|\, S\Big], \tag{A5}$$

or, if we suppress the Q states,

$$C_{nn} = \mathrm{Tr}\Big[|n\rangle\langle n|\, S\Big] = \langle n|S|n\rangle. \tag{A6}$$

This is the formula used in Eqs. (2.18), (3.7), and (5.6).

Similarly, if the initial D system is described by a density matrix

$$\rho^I_D = \sum_n p_n |n\rangle\langle n|, \tag{A7}$$

the probability of finding the particle at $\mathbf{y}_0$ is calculated as

$$P(\mathbf{y}_0) = \mathrm{Tr}\Big[|\mathbf{y}_0\rangle\langle\mathbf{y}_0|\rho^F_{\rm tot}\Big] = |\psi_1(\mathbf{y}_0)|^2 + |\psi_2(\mathbf{y}_0)|^2 + 2\mathrm{Re}\Big[\sum_n p_n C^*_{nn}\,\psi_1(\mathbf{y}_0)\psi'^*_{2n}(\mathbf{y}_0)\Big], \tag{A8}$$

which yields

$$\mathcal{V} = \sum_n p_n |C_{nn}|. \tag{A9}$$

Thus we find that the visibility $\mathcal{V}$ is simply given by

$$\mathrm{Tr}\Big[|\mathbf{y}_0\rangle\langle\mathbf{y}_0| \otimes \rho^I_D\, S\Big] \tag{A10}$$

or, if we suppress the Q state, by

$$\mathrm{Tr}\Big[\rho^I_D\, S\Big]. \tag{A11}$$

This is the formula used in Eqs. (3.20) and (5.14).

Let us proceed to the explicit calculation of $\mathcal{V}^{\rm th}$ when the initial D state ρ^I_D and the S matrix are given by $\rho_{\rm th}$ (3.12) and $S^{[N]}$ (3.5), respectively.

First observe that the S matrix in Eq. (3.5) is expressed as

$$S^{[N]} = \exp\Big\{-i\alpha\Big[N\Sigma^{(N)}_+(\omega) + N\Sigma^{(N)}_-(\omega)\Big]\Big\}, \tag{A12}$$

$$\Sigma^{(N)}_\pm(\omega) \equiv \frac{1}{N}\sum_{n=1}^N \sigma^{(n)}_\pm \exp\left(\mp i\frac{\omega}{c}\widehat{x}\right),$$

where $\alpha = V_0\delta/\hbar c$. Moreover, $N\Sigma^{(N)}_\pm(\omega)$ and $N\Sigma^{(N)}_3$, which is defined in Eq. (2.8), satisfy the algebra (2.11):

$$\Big[N\Sigma^{(N)}_-(\omega), N\Sigma^{(N)}_+(\omega)\Big] = -N\Sigma^{(N)}_3,$$
$$\Big[N\Sigma^{(N)}_\pm(\omega), -N\Sigma^{(N)}_3\Big] = \pm 2N\Sigma^{(N)}_\pm(\omega). \tag{A13}$$

This allows us to rewrite $S^{[N]}$ as [12]

$$S^{[N]} = e^{-i\tan(\alpha)N\Sigma^{(N)}_+(\omega)} e^{-\ln[\cos(\alpha)]N\Sigma^{(N)}_3} e^{-i\tan(\alpha)N\Sigma^{(N)}_-(\omega)}. \tag{A14}$$

It is straightforward to obtain [8,6]

$$N\Sigma^{(N)}_+(\omega)|p,n\rangle_N = \sqrt{(N-n)(n+1)}\left|p - \frac{\hbar\omega}{c}, n+1\right\rangle_N,$$

$$N\Sigma^{(N)}_-(\omega)|p,n\rangle_N = \sqrt{(N-n+1)n}\left|p + \frac{\hbar\omega}{c}, n-1\right\rangle_N, \tag{A15}$$

$$N\Sigma^{(N)}_3|p,n\rangle_N = (2n-N)\;|p,n\rangle_N,$$

so that

$$e^{-i\tan(\alpha)N\Sigma_-^{(N)}(\omega)}|p,n\rangle_N = \sum_{k=0}^{\infty}\frac{(-i\tan\alpha)^k}{k!}\left[N\Sigma_-^{(N)}(\omega)\right]^k|p,n\rangle_N$$
$$= \sum_{k=0}^{n}\frac{(-i\tan\alpha)^k}{k!}\sqrt{\frac{(N-n+k)!n!}{(N-n)!(n-k)!}}\left|p+k\frac{\hbar\omega}{c},n-k\right\rangle_N, \tag{A16}$$

and similarly

$${}_N\langle p,n|e^{-i\tan(\alpha)N\Sigma_+^{(N)}(\omega)} = \sum_{k=0}^{n}\frac{(-i\tan\alpha)^k}{k!}\sqrt{\frac{(N-n+k)!n!}{(N-n)!(n-k)!}}\;{}_N\left\langle p+k\frac{\hbar\omega}{c},n-k\right|. \tag{A17}$$

Therefore

$${}_N\langle p,n|S^{[N]}|p,n\rangle_N = \sum_{k=0}^{n}(-1)^k\tan^{2k}\alpha\binom{N-n+k}{k}\binom{n}{k}(\cos\alpha)^{N-2(n-k)}$$
$$= \cos^N\alpha\left(\frac{1}{\cos^2\alpha}\right)^n\frac{1}{(N-n)!}\frac{d^{N-n}}{dx^{N-n}}x^{N-n}(1+x)^n\Big|_{x=-\sin^2\alpha}. \tag{A18}$$

Since p_n, in Eq. (A9), is given by $\exp(-n\beta\hbar\omega)/Z$, the problem is reduced to evaluating $[g\equiv\exp(-\beta\hbar\omega)/\cos^2\alpha]$

$$\sum_{n=0}^{N}\frac{g^n}{(N-n)!}\frac{d^{N-n}}{dx^{N-n}}x^{N-n}(1+x)^n, \tag{A19}$$

which can be expressed as a complex z integration around $z=x$

$$\sum_{n=0}^{N}\frac{g^n}{2\pi i}\oint\frac{z^{N-n}(1+z)^n}{(z-x)^{N-n+1}}dz = \frac{1}{2\pi i}\oint\frac{z^N}{(z-x)^{N+1}}\sum_{n=0}^{N}\left(\frac{(1+z)(z-x)g}{z}\right)^n dz$$
$$= \frac{1}{2\pi i}\oint\frac{z^N}{(z-x)^{N+1}}\frac{1}{1-\frac{(1+z)(z-x)g}{z}}dz. \tag{A20}$$

In the last equality, an analytic term around $z=x$ has been omitted because its contribution vanishes. Changing the integration variable from z to $t=(z-x)/z$, the last expression becomes

$$\frac{1}{2\pi i}\oint\frac{dt}{t^{N+1}}f(t), \tag{A21}$$

where

$$f(t) = \frac{1}{1-t(1+(1+x)g)+gt^2}, \tag{A22}$$

and the integration contour includes $t=0$. By writing $f(t)$ as

$$f(t) = \frac{1}{g(t_+-t_-)}\left(\frac{1}{t-t_+}-\frac{1}{t-t_-}\right), \tag{A23}$$

where $t_\pm$ are the two roots of the denominator of $f(t)$

$$t_\pm = \frac{1+(1+x)g\pm\sqrt{[1+(1+x)g]^2-4g}}{2g}, \tag{A24}$$

the integral in Eq. (A21) is readily evaluated to yield

$$-\frac{1}{g(t_+-t_-)}\left(\frac{1}{t_+^{N+1}}-\frac{1}{t_-^{N+1}}\right). \tag{A25}$$

By taking into account the proper normalization factor, given by the inverse of Eq. (3.13), and evaluating Eq. (A25) at $x=-\sin^2\alpha$, we finally arrive at the desired result Eq. (3.20).

APPENDIX B

We shall prove, following Ref. [17], that the operators $N^{-1/2}\sum_{n=1}^{N}\sigma_\pm^{(n)} = N^{1/2}\Sigma_\pm^{(N)}$ and $(1/2)\sum_{n=1}^{N}(1+\sigma_3^{(n)}) = (N/2)(\mathbf{1}^{(N)}+\Sigma_3^{(N)})$ obey, in the $N\to\infty$ limit, the commutation relations for $a, a^\dagger$ and $\mathcal{N}=a^\dagger a$.

We start from the generators of SU(2) given in Eq. (2.11), and perform the following change of basis:

$$\begin{pmatrix}h_+\\h_-\\h_3\\1\end{pmatrix} = \begin{pmatrix}N^{-1/2}&&&\\&N^{-1/2}&&\\&&1&N/2\\&&&1\end{pmatrix}\begin{pmatrix}N\Sigma_+^{(N)}\\N\Sigma_-^{(N)}\\N\Sigma_3^{(N)}/2\\\mathbf{1}^{(N)}\end{pmatrix}. \tag{B1}$$

The commutation properties for $\mathbf{h}, 1$ are

$$\begin{aligned}[h_3, h_\pm] &= \pm h_\pm, \\ [h_-; h_+] &= 1 - \frac{2}{N} h_3, \\ [\mathbf{h}, 1] &= 0, \end{aligned} \tag{B2}$$

and yield, in the $N \to \infty$ limit, the standard boson commutation relations. In conclusion,

$$\begin{aligned} h_+ &= \frac{1}{\sqrt{N}} \sum_{n=1}^{N} \sigma_+^{(n)} = \sqrt{N} \Sigma_+^{(N)} \underset{N\to\infty}{\longrightarrow} a^\dagger, \\ h_- &= \frac{1}{\sqrt{N}} \sum_{n=1}^{N} \sigma_-^{(n)} = \sqrt{N} \Sigma_-^{(N)} \underset{N\to\infty}{\longrightarrow} a, \\ h_3 &= \frac{1}{2} \sum_{n=1}^{N} \left(1 + \sigma_3^{(n)}\right) \\ &= \frac{N}{2} \left(\mathbf{1}^{(N)} + \Sigma_3^{(N)}\right) \underset{N\to\infty}{\longrightarrow} \mathcal{N} \equiv a^\dagger a. \end{aligned} \tag{B3}$$

APPENDIX C

Here we compute the visibility of the interference pattern when the cavity is initially in a thermal state.

The S matrix for the maser case is given by Eq. (5.3) and can be rewritten as

$$\begin{aligned} S = \exp(-\overline{\kappa}/2) \exp\left[-i\sqrt{\overline{\kappa}} a^\dagger \exp\left(-i\frac{\omega}{c}\widehat{x}\right)\right] \\ \times \exp\left[-i\sqrt{\overline{\kappa}} a \exp\left(i\frac{\omega}{c}\widehat{x}\right)\right], \end{aligned} \tag{C1}$$

where $\overline{\kappa}$ was defined in Eq. (5.4). The initial thermal state $\rho_{\rm th}^{\rm JC}$ [Eq. (5.8)] has the following P representation [20] in terms of coherent states

$$\rho_{\rm th}^{\rm JC} = \int \frac{d^2\beta}{\pi \kappa_{\rm th}} |\beta\rangle e^{-|\beta|^2/\kappa_{\rm th}} \langle\beta|, \tag{C2}$$

where $a|\beta\rangle = \beta|\beta\rangle$ and $\kappa_{\rm th}$ was defined after Eq. (5.11). By the same reasoning explained at the beginning of Appendix A, we understand that the visibility is given by

$$\mathcal{V}_{\rm th}^{\rm JC} = {\rm Tr}_D \langle p|\rho_{\rm th}^{\rm JC} S|p\rangle, \tag{C3}$$

and is explicitly computed as

$$\begin{aligned} \mathcal{V}_{\rm th}^{\rm JC} &= \int \frac{d^2\beta}{\pi\kappa_{\rm th}} \langle p, \beta|S|p, \beta\rangle e^{-|\beta|^2/\kappa_{\rm th}} \\ &= \exp(-\overline{\kappa}/2) \int \frac{d^2\beta}{\pi\kappa_{\rm th}} \langle p| \exp\left[-i\sqrt{\overline{\kappa}}\beta^* \exp\left(-i\frac{\omega}{c}\widehat{x}\right)\right] \\ &\qquad \times \exp\left[-i\sqrt{\overline{\kappa}}\beta \exp\left(i\frac{\omega}{c}\widehat{x}\right)\right] |p\rangle e^{-|\beta|^2/\kappa_{\rm th}}. \end{aligned} \tag{C4}$$

Observe that

$$\begin{aligned} &\exp\left[-i\sqrt{\overline{\kappa}}\beta \exp\left(i\frac{\omega}{c}\widehat{x}\right)\right] |p\rangle \\ &\quad = \sum_{n=0}^{\infty} \frac{(-i\sqrt{\overline{\kappa}}\beta)^n}{n!} e^{in\omega\widehat{x}/c} |p\rangle \\ &\quad = \sum_{n=0}^{\infty} \frac{(-i\sqrt{\overline{\kappa}}\beta)^n}{n!} \left|p + \frac{n\hbar\omega}{c}\right\rangle, \end{aligned} \tag{C5}$$

and similarly

$$\begin{aligned} &\langle p| \exp\left[-i\sqrt{\overline{\kappa}}\beta^* \exp\left(-i\frac{\omega}{c}\widehat{x}\right)\right] \\ &\quad = \sum_{n=0}^{\infty} \frac{(-i\sqrt{\overline{\kappa}}\beta^*)^n}{n!} \left\langle p + \frac{n\hbar\omega}{c}\right|. \end{aligned} \tag{C6}$$

Therefore Eq. (C4) becomes

$$\mathcal{V}_{\rm th}^{\rm JC} = \exp(-\overline{\kappa}/2) \int \frac{d^2\beta}{\pi\kappa_{\rm th}} \sum_{n=0}^{\infty} \frac{(\overline{\kappa}|\beta|^2)^n}{n!} e^{-|\beta|^2/\kappa_{\rm th}}. \tag{C7}$$

The integration over β is easily performed in two-dimensional polar coordinates, and yields

$$\begin{aligned} \mathcal{V}_{\rm th}^{\rm JC} &= \exp(-\overline{\kappa}/2) \int_0^\infty \frac{d|\beta|^2}{\kappa_{\rm th}} \sum_{n=0}^{\infty} \frac{(-\overline{\kappa}|\beta|^2)^n}{n!} e^{-|\beta|^2/\kappa_{\rm th}} \\ &= \exp(-\overline{\kappa}/2) \sum_{n=0}^{\infty} \frac{1}{\kappa_{\rm th}} \frac{(-\overline{\kappa})^n}{n!} \kappa_{\rm th}^{n+1} \Gamma(n+1) \\ &= \exp(-\overline{\kappa}/2) \sum_{n=0}^{\infty} \frac{(-\overline{\kappa}\kappa_{\rm th})^n}{n!} \\ &= \exp\left[-\left(\kappa_{\rm th} + \tfrac{1}{2}\right)\overline{\kappa}\right]. \end{aligned} \tag{C8}$$

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