EXPERIMENTAL TESTS OF BELL INEQUALITIES. ARE ALL LOCAL MODELS REALLY EXCLUDED?

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Bell inequality tests performed by atomic cascade experiments always make use of a coincidence counting electronics. It is shown that such electronics can act as a "source of nonlocality" in the correlation function, even if every phenomenon involved is perfectly local. A simple example illustrating this situation is given.

The aim of the present paper is to show that experimental tests of Bell inequalities do not actually check all local theories as they are thought to do [1,2]. The reason for this is that they rely upon the assumption up to now *always implicitly made*, that the coincidence counting electronics is a perfectly local device. This is not the case, as we hope to convincingly show, because a "source of nonlocality" is in fact introduced by the electronics, which can and may be responsible for the violations of the inequalities tested. A simple counterexample will be shown, which emphasizes these nonlocal features of the electronics, and some conclusions will be inferred.

We shall deal in the following with Clauser and Horne's (CH) version [1–3] of the Bell inequality [4], and we shall of course refer to the atomic cascade experiments performed up to this date [5].

The derivation of the CH inequality rests on the following premises [2]: let Λ be the space of the possible states λ describing a physical system. Let ρ_λ be the distribution function of these states λ on the space Λ. Of course we require the normalization condition $\int_\Lambda d\rho_\lambda = 1$. Let $p_1(\lambda, a)$ ($p_2(\lambda, b)$) be the probability that the first (second) photomultiplier counts a photon, given the orientation a (b) of the relative polarizer. Then, according to Bell's definition of locality [4], we assume the probability to detect both photons (a coincidence), given the state λ, to be:

$$p_{12}(\lambda, a, b) = p_1(\lambda, a) p_2(\lambda, b). \tag{1}$$

Defining the average probability of a coincidence count as:

$$p_{12}(a, b) = \int_\Lambda p_{12}(\lambda, a, b)\, d\rho_\lambda = \int_\Lambda p_1(\lambda, a) p_2(\lambda, b)\, d\rho_\lambda, \tag{2}$$

we obtain the inequality

$$p_{12}(\infty, \infty) \leqslant p_{12}(a, b) - p_{12}(a, b') + p_{12}(a', b) + p_{12}(a', b') - p_{12}(a', \infty) - p_{12}(\infty, b) \leqslant 0, \tag{3}$$

where ∞ denotes the experimental situations in which the polarizer is absent [‡1]. The experimentally tested inequality

$$|R(\pi/8) - R(3\pi/8)|/R_0 < 1/4 \tag{3'}$$

follows directly from (3), once rotational invariance and proportionality between coincidence count rates

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[‡1] The so-called no-enhancement hypothesis is also necessary to get (3); we shall not consider, in the following, the problems deriving from such an assumption (see ref. [6]).

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$R(\phi)$ and probabilities $p_{12}(\phi)$ have been assumed ($\phi = |a-b|$ and R_0 is proportional to $p_{12}(\infty,\infty)$).

Our criticism concerns the supposed locality of eqs. (1) and (2): as we will show, it is possible to think of local models which satisfy (1) and (2), but show nonlocal properties because of the coincidence counting electronics used in the experiments.

Let us start off with some necessary observations. We can write (2) assuming λ to be any kind of (hidden) variable describing the observed system and the source, as well as the two polarizers and photomultipliers. The locality assumption implies that (hidden) variables describing the first measuring device (photomultiplier + polarizer) at the time t cannot influence the result of a measurement carried out on the second measuring device at the same time and vice versa. We can quite well outline this situation as follows: let us write the Λ-space as

$$\Lambda = \mathrm{A} \cup \mathrm{S} \cup \mathrm{B}, \tag{4}$$

where A (B) is the space of the possible states α (β) describing the first (second) measuring apparatus (namely polarizer and photomultiplier) and S is the space of the possible states σ describing the "source" (namely the state of the emitter atom, the states of the atoms close to this one, which can give rise to cooperative effects in the emission, the density of the atomic beam, the presence of some external field, etc.). It is a very simple matter to show that, with this hypothesis, (2) becomes

$$p_{12}(a,b) = \int_{\mathrm{A}\cup\mathrm{S}\cup\mathrm{B}} p_1(a,\alpha,\sigma) p_2(b,\beta,\sigma)\, \mathrm{d}\rho_\alpha\, \mathrm{d}\rho_\beta\, \mathrm{d}\rho_\sigma$$
$$= \int_{\mathrm{S}} p_1'(a,\sigma) p_2'(b,\sigma)\, \mathrm{d}\rho_\sigma, \tag{5}$$

where $p_1'(a,\sigma) = \int_{\mathrm{A}} p_1(a,\alpha,\sigma)\, \mathrm{d}\rho_\alpha$ (ρ_α is the distribution function of the α-states in A) and a similar relation holds for $p_2'(b,\sigma)$. In (5) we have simply factored out the dependence of the correlation function $p_{12}(a,b)$ on the (hidden) variables of the two measuring devices: the final expression of $p_{12}(a,b)$ is an integral over the possible states of what is globally defined as the "source". Note that (5) is formally identical to (2), so that (3) can still be derived.

The question is: is it always possible to obtain a perfectly local expression such as (5) for the correlation function? To show that this is not the case, we start with an example of local hidden variable model for which the factorization (5) *cannot be carried out*. Let one of the hidden variables describing the behaviour of the photon be the position of the particle in the wave packet, according to the old-fashioned Einstein–de Broglie hypothesis. Let us assume that a wave packet has a "leading edge" in the emission direction and let x be the distance separating the particle from this leading edge (we shall consider, for the sake of simplicity, the unidimensional model; the following reasoning holds of course also in the more general three-dimensional case). Now let us assume that *there exists a dependence of the probability that the photon (wave + particle) crosses the polarizer and is detected by the photomultiplier, on the position x of the particle in the wave*. This means that:

$$p_1 = p_1(a,\alpha,\sigma,x_1), \quad p_2 = p_2(b,\beta,\sigma,x_2), \tag{6}$$

where the dependence of p_i on x_i ($i = 1, 2$) has been stressed [‡2]. Therefore:

$$p_{12}(a,b) = \int_{\mathrm{S}} \mathrm{d}\rho_\sigma \int_{\mathrm{A}\cup\mathrm{X}_1} \mathrm{d}\rho_\alpha\, \mathrm{d}\rho_{x_1}\, p_1(a,\alpha,\sigma,x_1)$$
$$\times \int_{\mathrm{B}\cup\mathrm{X}_2} \mathrm{d}\rho_\beta\, \mathrm{d}\rho_{x_2}\, p_2(b,\beta,\sigma,x_2), \tag{7}$$

where X_i and ρ_{x_i} ($i = 1, 2$) are the x_i-domain and the corresponding distribution function. Formulae (6) and (7) contain a "concealed" danger: to make it come to light we have to parametrize our situation in terms of a different hidden variable. Let t be the time elapsed between the arrival of the leading edge of the wave packet and the arrival of the particle in the same place. Of course the descriptions in terms of t and x are *perfectly equivalent*. We get

$$p_1 = p_1(a,\alpha,\sigma,t_1), \quad p_2 = p_2(b,\beta,\sigma,t_2), \tag{6'}$$

and

[‡2] One could think that the possibility that $p_1 = p_1(a,\alpha,\sigma,x_1,x_2)$ and $p_2 = p_2(b,\beta,\sigma,x_1,x_2)$ cannot be ruled out, because, since the two particles are emitted by the same atom, each particle might have information about the position of the other one in the other wave packet, and the probability that the count is triggered in the measuring device could as well depend on this (previously and locally obtained) information. As we shall see below this cannot modify any conclusion which is going to be drawn.

$$p_{12}(a,b) = \int_S \mathrm{d}\rho_\sigma \int_{A \cup T_1} \mathrm{d}\rho_\alpha \, \mathrm{d}\rho_{t_1} \, p_1(a,\alpha,\sigma,t_1)$$

$$\times \int_{B \cup T_2} \mathrm{d}\rho_\beta \, \mathrm{d}\rho_{t_2} \, p_2(b,\beta,\sigma,t_2) \qquad (7')$$

(the meaning of the new variables is obvious).

Let us consider now the experimental situation involved: it is obvious to assume, in this model for the photon, that the photoelectric effect takes place when the particle impinges on the photomultiplier photocathode. Then, according to the definition, t_i ($i = 1$, 2) has to be regarded as the time interval elapsed between the arrival time of the ith wave packet leading edge on the ith photocathode, and the extraction of the photoelectron. If τ_i is defined as the time needed by the ith photomultiplier to turn the photoelectron into an electrical impulse (by means of the well-known amplification process) plus the time necessary to "shape" this pulse (by the discriminator), then the time interval t_i^* separating the arrival of the leading edge of the wave packet on the photocathode from the appearance of a shaped electrical pulse in the measuring device is

$$t_i^* = t_i + \tau_i \quad (i = 1, 2). \qquad (8)$$

Now, the *coincidence counting electronics imposes that*

$$t_2^* + T \leqslant t_1^* + \omega, \qquad (9)$$

where T is the lifetime of the excited atom which is going to emit the second photon of the cascade (in our model T can be interpreted as the time interval between the emissions of the two wave packets leading edges), and ω is the so-called coincidence window. Note that ω is a priori fixed in every coincidence experiments: it is practically the maximum time separation allowed between two signals in order for them to be considered a coincidence.

From (8) and (9) we obtain:

$$t_2 \leqslant t_1 + \tau_1 - \tau_2 + \omega - T, \qquad (10)$$

so that, according to the very definition:

$$0 \leqslant t_2 \leqslant t_1 + \tau_1 + \tau_2 + \omega - T. \qquad (10')$$

Eq. (10′) is quite an amazing result because it is equivalent to admitting that in formula (7′)

$$T_2 = T_2(t_1, \tau_1, \tau_2, \omega, T). \qquad (11)$$

In other words the domain T_2 of a hidden variable (t_2) is a function of "properties" of the measuring apparatus! This feature of T_2 is *manifestly nonlocal*, the source of nonlocality being only the electronics of the experimental arrangement.

We would like to prevent an objection which could be put forward: namely the one outlined in footnote 2, according to which the second particle could "have information" about the variable x_1 (and t_1). We stress that, in a local philosophy, the second particle of a correlated couple

(a) has information about τ_2,

(b) may have information about t_1 and T (see footnote 2),

(c) might have information (*but this does not seem very likely*) about ω (ω being fixed in every experiment),

(d) *absolutely cannot* have any information about τ_1, because τ_1 is a function of the actual experimental situation of the first polarizer and first photomultiplier, which is a space-like distance apart from the second photon polarization measurement.

How easy it is to violate inequality (3) when the hidden variable domain exhibits nonlocal features, as in (11), can easily be guessed; such an example is given for instance, in ref. [7]. We emphasize that (11) just accomplishes what is explicitly forbidden by CH in note 13 of ref. [1], i.e. the dependence of the distribution function ρ_λ on variables describing the two analyzers: in fact (11) is perfectly equivalent to dealing with a new distribution function

$$\rho'_{t_2} = \rho_{t_2}, \quad \text{if} \quad 0 \leqslant t_2 \leqslant t_1 + \tau_1 - \tau_2 + \omega - T,$$

$$= 0, \quad \text{if} \quad t_2 > t_1 + \tau_1 - \tau_2 + \omega - T,$$

that is to say, a distribution function for the variable t_2 (or x_2) depending, among other things, on τ_1. Attention should be paid to the very likely eventuality that the time elapsed between the arrival of the wave packet leading edge on the polarizer and the creation of an electrical pulse beyond the distriminator could quite well be a function of the polarizer axis $\boldsymbol{n}$; it is possible, for instance, that $\boldsymbol{n}$-polarized photons would need less time to cross the polarizer than differently polarized ones. Moreover, it should be noticed that, if $T_2 = T_2(\tau_1)$ (we retain only this dependence because it is impossible to rule out by "local" reasoning such as (a), (b), (c), listed above), then (7′) gives:

$$p_{12}(a,b) = \int_S \mathrm{d}\rho_\sigma \int_A \mathrm{d}\rho_\alpha\, p_1(a,\alpha,\sigma) p_2'(b,\sigma,\tau_1), \quad (12)$$

where the variable t_1 has been included in the σ's and

$$p_2'(b,\sigma,\tau_1) = \int_{B \cup T_2(\tau_1)} \mathrm{d}\rho_\beta\, \mathrm{d}\rho_{t_2}\, p_2(b,\beta,\sigma,t_2).$$

It is evidently impossible to recover formula (5) from (12) because $\tau_1 = \tau_1(\alpha)$, α being the variables describing the *first* measuring device.

In conclusion, we stress that the alleged locality of eq. (2) is ruled out when the coincidence counting electronics is taken into account. It is the very definition of coincidence which, overwhelmingly entering formula (5), makes us face the troubles coming from formulae (7′) and (11). We think that these problems cannot be overlooked, unless some additional assumption is made, so as to avoid the situation outlined in formula (11). This will be the subject of future work. We emphasize, anyway, that even if it were possible, by means of some supplementary hypothesis, to obtain a constraint like (3) valid for any local theory satisfying this assumption, a "loophole" would always exist: models for which (11) holds will generally be able to violate (3) and (3′). It would be very difficult to comment on the physical soundness of such models. But the concepts at stake (locality and realism) are too important to leave even the smallest loophole unexplored. We hope to have convincingly shown, in this paper, that another loophole does exist to escape from the supposedly universal validity of eqs. (3), (3′).

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