Physica B 276–278 (2000) 970–972

PHYSICA B

www.elsevier.com/locate/physb

Decoherence in neutron interferometry

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Abstract

The coherence properties of a neutron are analyzed by making use of the Wigner quasi-distribution function. We discuss, in particular, highly non-classical, Schrödinger-cat-like neutron states that can be obtained in an interferometer or in a magnetic field. The dephasing and decoherence effects are quantitatively defined by introducing a "decoherence parameter", that enables one to emphasize some peculiar aspects of irreversibility and decoherence in neutron scattering. © 2000 Elsevier Science B.V. All rights reserved.

PACS: 03.65.Bz; 42.50.Wm; 42.50.Hz; 42.50.Vk

Keywords: Polarized neutrons; Quantum effects; Non-equilibrium systems

The notion of decoherence [1,2] is attracting increasing attention, disclosing unexpected applications as well as innovative technology. Neutron physics (neutron optics in particular) has played an important role in this context, both on theoretical and experimental grounds. Highly non-classical states can be obtained in neutron interferometry, e.g. by superposing wave packets [3] or different spin states in a magnetic field [4]. We analyze here the coherence properties of cold and thermal neutrons, by making use of the Wigner function [5–7]. The main motivation of this work is to use the coherence properties of the wave function in order to "probe" the "degree of disorder" of an atmosphere. Some of the results to be discussed below are counterintuitive and are at variance with expectation.

We assume that the wave function is approximated by a Gaussian wave packet of initial average position, momentum and spread $x_0, p_0 = \hbar k_0$ and δ, respectively (we work in one dimension for simplicity), and consider two physical situations. In the first one, a polarized neutron crosses a magnetic field parallel to its spin of intensity B, contained in a region of length L. Its total energy is conserved and its kinetic energy changes by $\Delta E = \mu B$, where μ is the neutron magnetic moment. This entails a change in average momentum $\Delta k = m\mu B/\hbar^2 k_0$ and an additional shift of the neutron phase proportional to $\Delta \equiv L\Delta k/k_0$. In the second situation, a polarized neutron crosses a magnetic field perpendicular to its spin. Its total energy is again conserved, but due to Zeeman splitting, the two neutron spin states in the direction of the B field have different kinetic energies and travel with different speeds. After the neutron has crossed the B field only its initial spin component is observed. This situation is physically most interesting, for it yields Schrödinger-cat-like states [4], whose coherence properties are of great interest. We only give here our main results: more details can be found in Refs. [6,7].

Let the field fluctuate, so that the shifts Δ are statistically distributed according to a distribution law $w(\Delta)$. Their collective "degree of disorder" can be quantitatively evaluated in terms of the entropy

$$S = -\int \mathrm{d}\Delta\, w(\Delta)\, \log(w(\Delta)). \tag{1}$$

Let W_{m} be the Wigner function averaged over the distribution $w(\Delta)$. The coherence properties of the neutron

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PII: S0921-4526(99)01275-2

ensemble can be analyzed in terms of a *decoherence parameter* [6,7]

$$\varepsilon = 1 - \frac{\mathrm{Tr}\,\rho^2}{(\mathrm{Tr}\,\rho)^2} = 1 - \frac{2\pi\int \mathrm{d}x\,\mathrm{d}k\,W_{\mathrm{m}}(x,k)^2}{(\int \mathrm{d}x\,\mathrm{d}k\,W_{\mathrm{m}}(x,k))^2}. \qquad (2)$$

This quantity measures the degree of "purity" of a quantum state: it is maximum when the state is maximally mixed ($\mathrm{Tr}\,\rho^2 < \mathrm{Tr}\,\rho$) and vanishes when the state is pure ($\mathrm{Tr}\,\rho^2 = \mathrm{Tr}\,\rho$). One might think that the two quantities S and ε should agree at least qualitatively: in other words, the loss of quantum mechanical coherence is larger when the neutron interacts with a fluctuating field of larger entropy. Such a naive expectation turns out to be incorrect.

We assume first that the shifts fluctuate according to a Gaussian law with standard deviation σ. The entropy is an increasing function with σ. If the B field is parallel to the neutron spin, the decoherence parameter (2) is shown in Fig. 1(a) and is in qualitative agreement with the behavior of the entropy. Consider now the "Schrödinger-cat" state obtained when the neutron interacts with a B field perpendicular to its spin. The behavior of ε is shown in Fig. 1(b): observe that for $\delta \gtrsim 3\,\text{Å}$ it is *not* a monotonic function of σ. This is at variance with the behavior of the entropy and with the naive expectation mentioned after Eq. (2).

We consider now a different example. Let the shifts change according to the law

$$\Delta(t) = \Delta_0 + \Delta_1[\sin(\Omega t) + \sin(r_j \Omega t)], \qquad (3)$$

where t is the time and Ω a frequency much smaller than v_0/L, the inverse time of flight of the neutron in the field region. We choose "increasingly less rational" frequencies $r_j = f_j/f_{j+1}$, where f_j are the Fibonacci numbers, so that the sequence r_j tends to the golden mean (the "most irrational" number [8]). Our results for the entropy and the decoherence parameter are summarized in Fig. 2. Note that although S is a monotonically increasing function of the Fibonacci number in the sequence, ε reaches a maximum for $r_j = \frac{3}{5}$ ($j = 3$). It is remarkable that the maximum is obtained for the same Fibonacci ratio in both cases (Gaussian wavepackets and "cats").

Decoherence is a very useful and prolific concept. However, it has some counterintuitive aspects: in particular, it is not correct to think that a quantum system, interacting with an increasingly entropic atmosphere will suffer an increasing loss of quantum coherence.

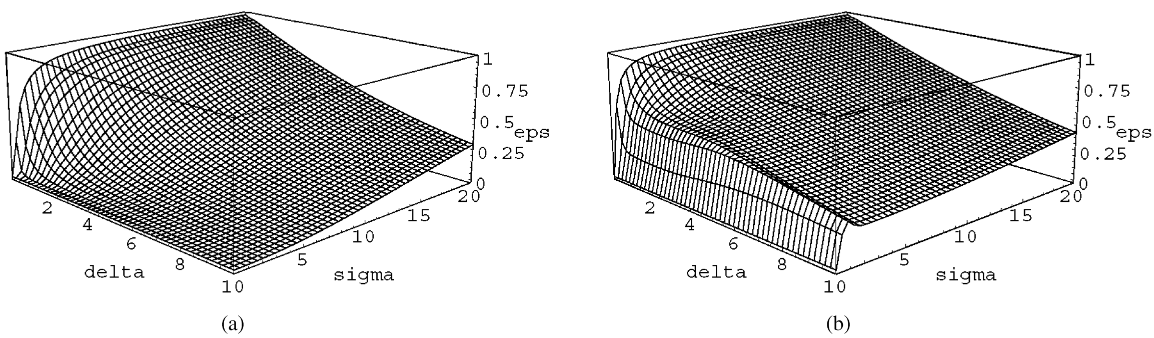

Fig. 1. Decoherence parameter versus coherence length of the wave packet δ(Å) and standard deviation of the fluctuation σ(Å). (a) Gaussian wave packet. (b) Schrödinger "cats" with $\Delta_0 = 16.1$ Å. In both cases $k_0 = 1.7$ Å^{-1} [4].

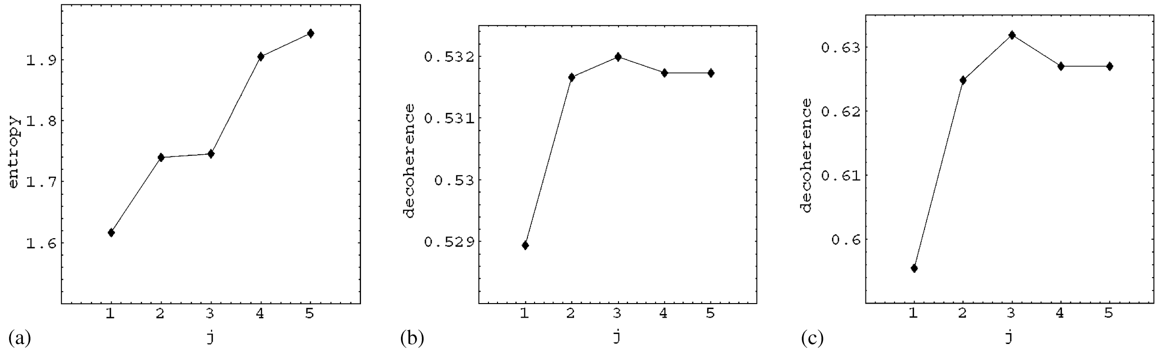

Fig. 2. (a) Entropy versus j (index in the Fibonacci sequence). (b) Decoherence parameter versus j: Gaussian. (c) Decoherence parameter versus j: "cats".

We thank G. Badurek, H. Rauch and M. Suda for useful comments. The numerical computation was performed on the "Condor" pool of the Italian Istituto Nazionale di Fisica Nucleare (INFN). This project was realized within the framework of the TMR European Network on "Perfect Crystal Neutron Optics" (ERB-FMRX-CT96-0057).

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