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Infinitely frequent measurements and quantum Zeno effect

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Abstract

The limit of infinitely many measurements is critically analyzed within the quantum mechanical framework, in connection with the quantum Zeno effect. It is shown that such a limit is unphysical and that quantum losses are unavoidable. A specific example involving neutron spin is considered. © 1998 Elsevier Science B.V.

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The quantum Zeno effect is the hindrance of the transition to a state different from the initial one when a large number of measurements in rapid succession are performed on a quantum system in order to check whether it is still in its initial state. The main features of the quantum mechanical dynamics that lead to the quantum Zeno effect (QZE) were already known to von Neumann [1], but were discussed in detail only a few decades later [2–6]: In a few words, the inhibition of the quantum evolution is due to the quadratic behavior of the transition probability at short times. Notice that deviations from a purely exponential behavior were expected on the basis of early work on the temporal behavior of quantum mechanical systems [7,8]. The temporal behavior of a quantum mechanical system is reviewed in Ref. [9].

There is no direct test of this effect on a truly unstable system, as the seminal proposals suggested. However, in 1988 Cook [10] proposed to test the QZE by using optical transitions in two-level atoms. This proposal inspired an interesting experiment, performed by Itano et al. [11], that provoked a very lively debate [12,13]. It is now almost unanimously accepted that the QZE is susceptible to a purely dynamical explanation [12,13] and it has also been proposed that the above-mentioned experiment, although correctly performed, is not a direct test of the QZE and should be correctly reinterpreted [14].

In the most paradoxical version of the QZE, one demands that an *infinite* number of measurements be performed in a finite elapse of time, in order to com-

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pletely "freeze" the quantum mechanical evolution and leave the system in its initial state with unit probability. Such a limit is usually called "continuous observation" in the literature, and has been very critically analyzed in the past [15–18]. Open questions arise about the behavior at high repetition rates of the measurements, even being aware that the limit of infinite number of measurements is simply unphysical: Not only because it is physically meaningless to conceive of the possibility of performing an *infinite* number of measurements in a finite elapse of time, but also because of (i) the unavoidable losses that are always present in any physical system and (ii) a considerable limitation imposed by the uncertainty principle.

The purpose of this Letter is simply to reanalyze critically the limit of continuous observation, without entering questions about the completeness of quantum mechanics. We shall consider a specific example involving neutron spin [13] and shall also counter a criticism recently put forward by Pan et al. [19].

Let Q be a quantum system, undergoing an evolution governed by the unitary operator $U(t) = \exp(-\mathrm{i}Ht)$, where H is the Hamiltonian. Q is prepared in a given initial state and N observations are performed at times $T/N, 2T/N, \ldots, (N-1)T/N, T$, in order to check whether Q is still in its initial state. We shall say that there is a quantum Zeno effect if the probability $P^{(N)}(T)$ that the system is found in its initial state after N measurements is such that

$$P^{(N)}(T) > P^{(N')}(T) \quad \text{for } N > N'. \tag{1}$$

The above condition holds true for a wide class of quantum systems, under very general conditions, when N is sufficiently large. Notice that the above-mentioned measurements are supposed to be *instantaneous* and the total duration T of the experiment is finite.

We emphasize that the idea that an observation (measurement) can be performed *instantaneously* is useful for computational purposes, but it is also physically misleading, because any known physical process takes place in a finite elapse of time, which can sometimes be very short on a macroscopic scale, but is usually long on a microscopic scale. This is even more true for a quantum mechanical measurement which involves the interaction with a macroscopic device [20]. We should also note that the idea of instantaneous measurement does not take into account the finite response time of the apparatus [15,13].

The example introduced in Ref. [13] involves neutron spin and is similar to early proposals [6,21]. A polarized neutron, whose speed is v and whose spin is up along a certain direction (say z), crosses N identical regions of longitudinal size ℓ in which there is a static magnetic field B, that provokes a rotation of the neutron spin around its axis (say x). The interaction time with each B region is $t = \ell/v$ and the total interaction time with the magnetic field is $T = Nt$. By choosing $T = \pi/\omega$, where $\omega = \mu B/\hbar$ (μ being the neutron magnetic moment), the final neutron spin is down with probability $P_{\downarrow}^{(N)}(T) = 1$. The spin turn can be accomplished by d.c. Mezei flippers, where a magnetic field perpendicular to the initial direction of the polarization causes Larmor rotation around the axis of the field (Fig. 1) [22].

A quantum Zeno effect is obtained by "monitoring" the neutron spin at every step: This is easily accomplished by selecting and detecting the spin-down component, for instance by placing a polarized He-3 filter into the beam which transmits the spin-up components and also absorbs and measures the spin-down component [23]. The probability that the neutron spin remains up at time T reads [13]

$$P_{\uparrow}^{(N)}(T) = \left(\cos^2\frac{\omega t}{2}\right)^N = \left(\cos^2\frac{\pi}{2N}\right)^N, \tag{2}$$

which obeys Eq. (1). This is an ideal result, valid as far as N is not too large and all losses are neglected and when it is assumed that the different stages act independently. In particular,

$$P_{\uparrow}^{(N)}(T) \stackrel{N\to\infty}{\longrightarrow} 1, \tag{3}$$

which signifies that the initial (up) neutron spin is completely "frozen" as N tends to infinity. This is what we consider paradoxical: We argued in Ref. [16] that the limit (3) is unphysical, not only because it is unattainable for practical reasons, but also because it is in contradiction with Heisenberg's uncertainty principle. In a somehow more general treatment one has to consider not only the situation in one parameter space (spin) but also in other parameter spaces, because it may happen that the spin state remains conserved but the intensity or momentum distribution changes drastically. The aim of this Letter is threefold: We shall

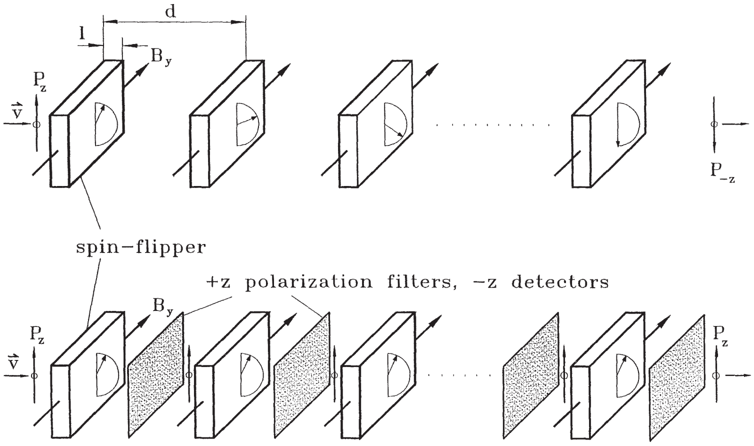

Fig. 1. Multistage d.c. spin flipper (above) and Zeno effect measurement arrangement when the polarization behind each is filtered in the z-direction.

first clarify the above issue by sharpening our previous argument, then counter the criticism by Pan et al. [19] and finally present a general discussion, still based on the uncertainty principle.

We start by noticing that the quantity

$$\phi \equiv \omega t = \frac{\mu B \ell}{\hbar v} = \frac{\pi}{2N}, \tag{4}$$

although of order $O(N^{-1})$, cannot vanish if we assume that the size ℓ of the interaction region is larger than the longitudinal spread Δx of the neutron wave packet: Indeed, there are unavoidable uncertainties in the neutron speed Δv and position Δx so that

$$\phi \sim \phi_0 \equiv \frac{\mu B \ell}{\hbar v_0} > \frac{\mu B \Delta x}{\hbar v_0} > \frac{\mu B}{2 m v_0 \Delta v} = \frac{1}{4} \frac{\Delta E_{\rm m}}{\Delta E_{\rm k}}, \tag{5}$$

where v_0 is the mean velocity of the neutron beam and we defined the magnetic energy gap $\Delta E_{\rm m} = 2\mu B$ and the kinetic energy spread of the neutron beam $\Delta E_{\rm k} = m v_0 \Delta v$. In this case Eq. (2) gives a completely different result when $N \to \infty$, namely zero. Pan et al. do not question the above derivation. Thus, the discussion concerns the opposite situation, namely the case $\ell < \Delta x$ [19]. They claim that the "interaction" and "passage" times of the neutron in each B region should be distinguished from the "evolution" time, which tends to zero if $\ell < \Delta x$. As a consequence, the argument made above does not hold anymore. This point is well taken. However, it is possible to modify our analysis and counter their criticism.

Let us therefore carefully analyze the case $\ell < \Delta x$. First of all notice that, as the size ℓ of the B regions and their relative distance d is reduced, there are cases in which the different stages may *not* act independently on the quantum system (see, e.g., Ref. [24]). This occurs when $\ell \simeq n\lambda$ or $d = n\lambda$, where n is integer and λ is the neutron wavelength. In this case, the various rotation stages can act together coherently, like a periodic potential; as a consequence, the reflectivity of such a coherent lattice-like structure behaves like N^2. For large n values ($n \gg \lambda/\Delta\lambda$), wavelengths within a Gaussian packet become resonant and strongly attenuated, but most of the neutrons of the whole packet are still transmitted. When n becomes small ($n \ll \lambda/\Delta\lambda$), typical Bragg diffraction occurs. This can be treated by dynamical diffraction and causes a total back-reflection over a rather wide bandwidth which is well known from multilayer reflection [25]. This observation sheds light on the difficulties one has to face when taking the $N \to \infty$ limit: As N is increased and ℓ reduced, one may encounter coherent effects that tend to wash out any realization of the QZE. When a random distribution of a characteristic length is chosen, the coherent resonant effects of reflection disappear, but a diffusive component appears yielding the same integral contribution as the coherent diffraction effect.

When one reduces ℓ even further, one reaches the dimension of an atomic diameter which defines a further limit on how thin magnetic fields can be made in practice. The neutron effectively interacts with only some of the atoms constituting the support of the B field, say, those within the neutron wave packet. Let us first rewrite Eq. (4) in terms of the *relative* speed $v_{\rm rel}$ between the neutron and these atoms or field region,

$$\phi = \frac{\mu B \ell}{\hbar v_{\rm rel}}, \tag{6}$$

and then think of the following situation (relativistically equivalent to that considered after Eq. (4)): The incident neutron does not move and the bunch of atoms considered move backwards (towards the neutron) with speed $V = v_{\rm rel}$. Since the atoms are to be viewed as fully quantum mechanical objects, they are also subject to the uncertainty principle, so that

$$\phi \sim \phi_0' \equiv \frac{\mu B \ell}{\hbar V} > \frac{\mu B \Delta X}{\hbar V} > \frac{\mu B}{2 M V \Delta V} = \frac{1}{4}\frac{\Delta E_{\rm m}}{\Delta E_{\rm k}'}, \tag{7}$$

where ΔX and ΔV are the uncertainties in the position and speed of the bunch of atoms, M their mass and $\Delta E_{\rm k}'$ are their kinetic energy spreads. Once again, ϕ is bounded from below and cannot vanish in the $N \to \infty$ limit. This result is conceptually similar to the one obtained in the opposite case ($\ell > \Delta x$, Eq. (5)) preventing the limit of Eq. (3) from being physically sensible.

This result can also be obtained by comparing the Zeeman energy change with the energy content of the magnetic sheet within the coherence area of the neutron beam. One should also notice that the result (7) does not depend on whether one regards Eq. (6) as due to the "passage" or "interaction" times [19].

Both Eqs. (5) and (7) depend on quantities that are characteristic of the situation considered, like masses, magnetic fields and so on. In this sense, they cannot provide a general estimate of how large N can be, in order that Eq. (2) makes sense, from a physical point of view. For this reason, it seems convenient to look for a bound on N that, although related to the experimental setup considered (say, thermal neutrons in a B field) are rather independent on those quantities that may vary from one experiment to another. This estimate is all the more important in view of the possible experimental realization.

The following analysis is based on the time–energy uncertainty principle,

$$\Delta E \Delta t > \hbar, \tag{8}$$

and is very general, within the framework of the experiment considered in this Letter. Notice that a previous argument, also based on the time–energy uncertainty principle, was given by Ghirardi et al. [15].

Consider a neutron undergoing a spin rotation in a B region, whose longitudinal size is ℓ. The volume $\mathcal{V}$ of the interaction region is roughly given by $\mathcal{V} \simeq \ell \Sigma_{\rm c}$, where $\Sigma_{\rm c} \simeq \Delta y \Delta z \simeq 100\ \text{Å}^2$ is the transverse coherence area of the neutron wave packet (whose spreads perpendicular to the direction of motion x are Δy and Δz). The energy of the B field in the interaction volume is of order

$$\Delta E \simeq \frac{B^2}{2\mu_0}\mathcal{V}. \tag{9}$$

(This is a *conservative* estimate: a smaller value of ΔE would lead to a more stringent limit on N.) On the other hand, the interaction time is of order

$$\Delta t \simeq \ell / v. \tag{10}$$

Combining Eqs. (8), (9) and (10), we obtain

$$B\ell > \sqrt{\frac{2\mu_0 \hbar v}{\Sigma_c}}, \tag{11}$$

which yields, via Eq. (4), the bounds

$$\phi = \frac{\mu B \ell}{\hbar v} > \sqrt{\frac{2\mu^2 \mu_0}{\hbar v \Sigma_c}} \simeq 10^{-6} \Longleftrightarrow N < 10^6 \tag{12}$$

for a thermal neutron ($v \simeq 2000$ m/s). This is the maximum value that N can take in Eq. (2); it is not very different from the value obtained in Ref. [16], but it is derived from more general premises. For larger values of N, Eq. (2) makes no sense anymore. In particular, the limit (3) is physically incorrect; this is a general conclusion for the experiment considered in Refs. [13,16]. It is obviously also true for related theoretical proposals [6] and experiments [21] and casts doubts about the possibility of performing truly "interaction-free" measurements. Analogous arguments are valid for the case of Refs. [10,11], which have been discussed in Refs. [17,18].

Finally, it is interesting to observe that all the bounds derived in this Letter arise when one compares the energy involved in the interaction between neutron and magnetic field ($H_{int} = \mu B$) with the spread of the other relevant energies. For example, Eq. (5) stems from the comparison of uncertainty of the kinetic energy of the incident neutron, Eq. (7), with the uncertainty of the kinetic energy of the atoms supporting the magnetic field, and Eq. (12) from comparison with the energy of the magnetic field in the relevant physical region. This seems to suggest the presence of a very fundamental mechanism preventing the limit (3) from being physical.

The analysis in this Letter shows that an extrapolation of a single stage behavior in a Zeno type experiment to an infinite number of stages is unphysical. Collective effects of a periodic structure have to be taken into account and uncertainty effects and coupling in phase space have to be considered as well.

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