

## Quantum Zeno Phenomena: Pulsed versus Continuous Measurement

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### Abstract

The time evolution of an unstable quantum mechanical system coupled with an external measuring agent is investigated. According to the features of the interaction Hamiltonian, a quantum Zeno effect (hindered decay) or an inverse quantum Zeno effect (accelerated decay) can take place, depending on the response time of the apparatus. The transition between the two regimes is analyzed for both pulsed and continuous measurements.

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### 1. Quantum Zeno Effect: Fundamentals

Let  $H$  be the total Hamiltonian of a quantum system. The survival probability of the system in state  $|a\rangle$  is

$$P(t) = |\mathcal{A}(t)|^2 = |\langle a | e^{-iHt} | a \rangle|^2. \quad (1)$$

An elementary expansion yields a quadratic behavior at short times

$$P(t) \sim 1 - t^2/\tau_Z^2, \quad \tau_Z^{-2} \equiv \langle a | H^2 | a \rangle - \langle a | H | a \rangle^2, \quad (2)$$

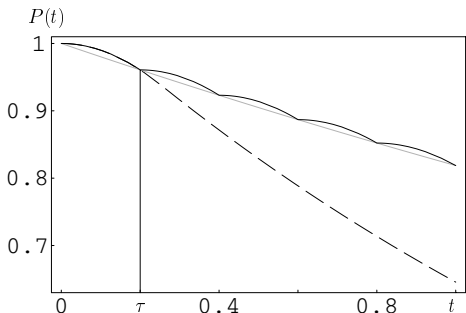
where  $\tau_Z$  is called Zeno time. Observe that if one divides the Hamiltonian into a free and an interaction part  $H = H_0 + H_1$ , with  $H_0 |a\rangle = \omega_a |a\rangle$  and  $\langle a | H_1 | a \rangle = 0$ , the Zeno time reads  $\tau_Z^{-2} = \langle a | H_1^2 | a \rangle$  and depends only on the off-diagonal part of the Hamiltonian.

We first consider “pulsed” measurements, as in the seminal approach [1]. The complementary notion of “continuous measurement” will be discussed in Sec. 4. Perform  $N$  (instantaneous) measurements at time intervals  $\tau = t/N$  (pulsed observation), in order to check whether the system is still in its initial state  $|a\rangle$ . The survival probability after the measurements reads

$$P^{(N)}(t) = P(\tau)^N = P(t/N)^N \sim \exp(-t^2/\tau_Z^2 N) \xrightarrow{N \rightarrow \infty} 1. \quad (3)$$

The (mathematical) limit is the quantum Zeno paradox: “A watched pot never boils”. For large (but finite)  $N$  the evolution is slowed down (quantum Zeno effect). Indeed, the survival probability after  $N$  pulsed measurements ( $t = N\tau$ ) is interpolated by an exponential law [2]

$$P^{(N)}(t) = P(\tau)^N = \exp(N \log P(\tau)) = \exp(-\gamma_{\text{eff}}(\tau) t), \quad (4)$$



**Fig. 1.** Evolution with frequent “pulsed” measurements: quantum Zeno effect. The dashed (full) line is the survival probability without (with) measurements. The gray line is the interpolating exponential (4)

with an effective decay rate

$$\gamma_{\text{eff}}(\tau) \equiv -\frac{1}{\tau} \log P(\tau) \geq 0. \tag{5}$$

For  $\tau \rightarrow 0$  one gets  $P(\tau) \sim \exp(-\tau^2/\tau_Z^2)$ , whence

$$\gamma_{\text{eff}}(\tau) \sim \tau/\tau_Z^2 : \quad (\tau \rightarrow 0) \tag{6}$$

increasingly frequent measurements hinder the evolution and tend to “freeze” it. The Zeno evolution is represented in Fig. 1.

## 2. Unstable Systems

Consider the spontaneous decay of state  $|a\rangle$  into state  $|b\rangle$  described by the Hamiltonian

$$H = H_0 + H_1 = \omega_a |a\rangle \langle a| + \sum_k \omega_k a_k^\dagger a_k + \sum_k \phi_k (a_k |a\rangle \langle b| + a_k^\dagger |b\rangle \langle a|), \tag{7}$$

with  $\langle a | a \rangle = \langle b | b \rangle = 1$  and  $[a_k, a_{k'}^\dagger] = \delta_{kk'}$ , other commutators = 0. As is well known, the Fourier-Laplace transform of the survival amplitude  $\mathcal{A}(t)$  in (1) is the expectation value of the resolvent

$$G_a(E) = \int_0^\infty dt e^{iEt} \mathcal{A}(t) = \langle a | \frac{i}{E - H} | a \rangle, \quad \mathcal{A}(t) = \int_B \frac{dE}{2\pi} e^{-iEt} G_a(E), \tag{8}$$

the Bromwich path B being a horizontal line  $\text{Im}E = \text{constant} > 0$  in the half plane of convergence of the Fourier-Laplace transform (upper half plane). By performing Dyson’s re-summation, the resolvent  $G_a$  can be expressed in terms of the self-energy function  $\Sigma_a$

$$G_a(E) = \frac{i}{E - \omega_a - \Sigma_a(E)}, \quad \Sigma_a(E) = \sum_k \frac{|\phi_k|^2}{E - \omega_k} = \int_0^\infty d\omega \frac{\kappa_a(\omega)}{E - \omega}, \tag{9}$$

where  $\kappa_a(\omega) = \langle a | H_1 \delta(\omega - H_0) H_1 | a \rangle = \sum_k |\phi_k|^2 \delta(\omega - \omega_k)$  is the form factor of the interaction (spectral density function).

If  $-\Sigma_a(0) < \omega_a$  (which happens for sufficiently smooth form factors and small coupling), the resolvent is analytic in the whole complex plane cut along the positive real axis (continuous spectrum of  $H$ ). On the other hand, there exists a pole  $E_{\text{pole}}$  located just below the branch cut in the second Riemann sheet, solution of the equation  $E_{\text{pole}} - \omega_a - \Sigma_{\text{all}}(E_{\text{pole}}) = 0$ ,  $\Sigma_{\text{all}}$  being the determination of the self-energy function in the second sheet. The pole has a real and imaginary part  $E_{\text{pole}} = \omega_a + \delta\omega_a - i\gamma/2$  given by

$$\delta\omega_a = \text{Re } \Sigma_{\text{all}}(E_{\text{pole}}) \simeq \text{Re } \Sigma_a(\omega_a + i0^+) = \mathcal{P} \int d\omega \frac{\kappa_a(\omega)}{\omega_a - \omega} = \mathcal{P} \sum_k \frac{|\phi_k|^2}{\omega_a - \omega_k}, \tag{10}$$

$$\gamma = -2 \text{Im } \Sigma_{\text{all}}(E_{\text{pole}}) \simeq -2 \text{Im } \Sigma_a(\omega_a + i0^+) = 2\pi\kappa_a(\omega_a), \tag{11}$$

up to fourth order in the coupling constant. One recognizes the second-order energy shift  $\delta\omega_a$  and the celebrated Fermi “golden” rule  $\gamma$  [3]. The survival amplitude has the general form

$$\mathcal{A}(t) = \mathcal{A}_{\text{pole}}(t) + \mathcal{A}_{\text{cut}}(t), \tag{12}$$

where  $\mathcal{A}_{\text{pole}}(t) = e^{-i(\omega_a + \delta\omega_a)t - \gamma t/2} / [1 - \Sigma'_{\text{all}}(E_{\text{pole}})]$ ,  $\mathcal{A}_{\text{cut}}$  being the branch-cut contribution. At intermediate times, the pole contribution dominates the evolution (Weisskopf-Wigner approximation [4]) and

$$P(t) \simeq |\mathcal{A}_{\text{pole}}(t)|^2 = \mathcal{Z} e^{-\gamma t}, \quad \mathcal{Z} = |1 - \Sigma'_{\text{all}}(E_{\text{pole}})|^{-2}, \tag{13}$$

where  $\mathcal{Z}$ , the intersection of the asymptotic exponential with the  $t = 0$  axis, is the wave function renormalization. As is well known the exponential law is corrected by the cut contribution, which is responsible for a quadratic behavior at short times and a power law at long times.

### 3. Inverse Quantum Zeno Effect

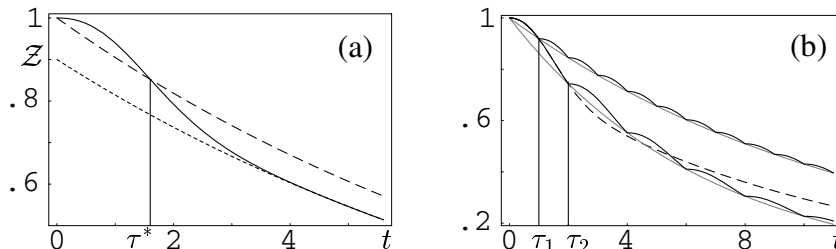
Consider an unstable system with decay rate  $\gamma$  given by (11). By performing a single measurement at a sufficiently long time  $t$ , when the exponential behavior  $P(t) \simeq e^{-\gamma t}$  is dominant, one infers from (4) that the effective decay rate is simply the natural (undisturbed) one

$$\gamma_{\text{eff}}(\tau) \xrightarrow{\text{“long” } \tau} \gamma. \tag{14}$$

We now ask whether it is possible to find a *finite* time  $\tau^*$  such that

$$\gamma_{\text{eff}}(\tau^*) = \gamma. \tag{15}$$

If such a time exists, then by performing measurements at time intervals  $\tau^*$  the system decays according to its undisturbed decay rate  $\gamma$ , as if no measurements were performed. The related concept of “jump” time was considered in [5]. By (5) and (15) we get  $P(\tau^*) = e^{-\gamma\tau^*}$ : the time  $\tau^*$  is the intersection between the curves  $P(t)$  and  $e^{-\gamma t}$ . In the situation depicted in Fig. 2(a) such a time  $\tau^*$  exists: the full line is the survival probability  $P(t)$  and the dashed line the exponential  $e^{-\gamma t}$  [the dotted line is the asymptotic exponential  $\mathcal{Z}e^{-\gamma t}$ , see (13)]. By looking at Fig. 2(b) we realize that  $\tau^*$  represents a *transition time*



**Fig. 2.** (a) Determination of the transition time  $\tau^*$ . The full line is the survival probability  $P(t)$ , the dashed line the exponential  $e^{-\gamma t}$  and the dotted line the asymptotic exponential  $\mathcal{Z}e^{-\gamma t}$  in (13). (b) Quantum Zeno vs inverse Zeno (“Heraclitus”) effect. The dashed line represents the undisturbed survival probability  $P(t)$ . The full lines represent the survival probabilities with measurements at time intervals  $\tau$  and the dotted lines their exponential interpolations (4). For  $\tau_1 < \tau^* < \tau_2$  the effective decay rate  $\gamma_{\text{eff}}(\tau_1)$  [ $\gamma_{\text{eff}}(\tau_2)$ ] is smaller (QZE) [larger (IZE)] than the “natural” decay rate  $\gamma$ . When  $\tau = \tau^*$  one recovers the natural lifetime, according to (15)

from a quantum Zeno to an inverse quantum Zeno regime [2]. Indeed

$$\begin{aligned} \text{if } \tau = \tau_1 < \tau^* &\Rightarrow \gamma_{\text{eff}}(\tau_1) < \gamma && \text{Quantum Zeno Effect (QZE);} \\ \text{if } \tau = \tau_2 > \tau^* &\Rightarrow \gamma_{\text{eff}}(\tau_2) > \gamma && \text{Inverse quantum Zeno Effect (IZE).} \end{aligned}$$

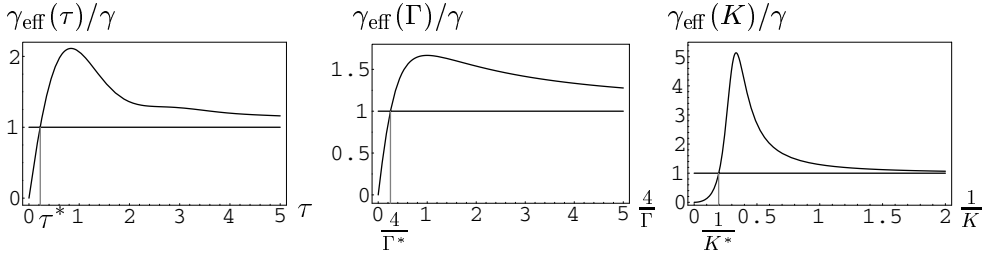
If  $\tau^*$  exists, frequent measurements first accelerate decay (IZE) [2, 6], then, eventually, slow it down (QZE) when the frequency of measurements becomes larger than  $1/\tau^*$  [2, 7]. Note that the existence of such a transition time  $\tau^*$  is related to the value of the wave function renormalization  $\mathcal{Z}$ : if  $\mathcal{Z} < 1$  a finite  $\tau^*$  certainly exists [2] and the system exhibits both QZE and IZE, depending on the frequency of measurements. (This is the case considered in Fig. 2.) The transition from a Zeno to an inverse Zeno regime has been recently confirmed in a beautiful experiment performed by Raizen’s group [7].

#### 4. Pulsed versus Continuous Observation

We now introduce some alternative descriptions of a measurement process and discuss the notion of continuous measurement. This is to be contrasted with the idea of pulsed measurements, discussed in the previous sections and hinging upon von Neumann’s projections. We will show that the use of instantaneous pulsed measurements is not essential to obtain QZE [8, 9] or (possibly) IZE. We will provide a dynamical picture of the measurement process by introducing a Hamiltonian description of the interaction with the detector and show that the detector response time plays a role very similar to that of the period between measurements in the pulsed version [5]. We will also show that irreversibility is not an essential ingredient of this picture. By replacing an irreversible detector with an oscillating one, we show that QZE and IZE are a simple consequence of a strong interaction between the “observed” decaying system and an “observing” agent (the “detector”) which closely “looks” at the system [10].

##### 4.1. Pulsed observation (period $\tau$ )

We start by considering pulsed measurements performed at time intervals  $\tau$ . For simplicity we choose a Lorentzian form factor  $\kappa_a(\omega) = \lambda^2 A/\pi(\omega^2 + A^2)$ , from which an analytical



**Fig. 3.** Effective decay rate as a function of the detector response time: pulsed observation (period  $\tau$ ); continuous observation (decay time  $\Gamma^{-1}$ ); continuous Rabi observation (Rabi period  $2\pi/K$ )

expression of the survival amplitude can be easily obtained. (Notice that the Hamiltonian in this case is not lower bounded and one expects no deviations from exponential behavior at very large times.) We chose  $\lambda = 0.1$ ,  $\mathcal{A} = 1$  and  $\omega_a = 3$ , so that  $\mathcal{Z} = 0.998 < 1$ , a finite  $\tau^*$  exists and the system exhibits a QZE-IZE transition. The effective decay rate (5) is shown in the left frame of Figure 3 as a function of  $\tau$ . Notice the linear behavior (6) for  $\tau \rightarrow 0$ , with slope  $1/\tau_Z^2$ . Observe that for the chosen value of the parameters, the linear approximation (6) is valid well beyond the intersection  $\tau^*$  and one gets  $\tau^* \simeq \tau_Z^2 \gamma = 0.2$ . For  $\tau > \tau^*$  the system decays faster, with a decay rate  $\gamma_{\text{eff}}$  that first increases up to  $2\gamma$ , then decreases and eventually relaxes to the natural decay rate  $\gamma$  according to (14).

**4.2. Continuous observation (response time  $\Gamma^{-1}$ )**

Let us consider now a continuous measurement process. This is accomplished, for instance, by adding to (7) the following interaction Hamiltonian

$$H_{\text{meas}}(\Gamma) = \sqrt{\frac{\Gamma}{2\pi}} \int d\omega (d(\omega) |b\rangle \langle M| + d^\dagger(\omega) |M\rangle \langle a|), \tag{16}$$

with  $[d(\omega), d^\dagger(\omega')] = \delta(\omega - \omega')$ , other commutators = 0. As soon as it becomes populated, state  $|b\rangle$  decays into state  $|M\rangle$ , with a decay rate  $\Gamma$ . This yields a continuous monitoring of the decay process  $|a\rangle \rightarrow |b\rangle$ , with a response time  $1/\Gamma$ . The presence of the interaction Hamiltonian (17) simply modifies the self-energy function in (9) as  $\Sigma_a(E, \Gamma) = \Sigma_a(E - i\Gamma/2)$ , whence, by (11),

$$\gamma_{\text{eff}}(\Gamma) = -2 \text{Im} \Sigma_a \left( \omega_a - i \frac{\Gamma}{2} \right) = \frac{4}{\Gamma} \int d\omega \kappa_a(\omega) \frac{\frac{\Gamma^2}{4}}{(\omega - \omega_a)^2 + \frac{\Gamma^2}{4}}. \tag{17}$$

The effective decay rate (18) is shown in the central frame of Fig. 3 as a function of  $4/\Gamma$ . The behavior is similar to that described in Sec. 4.1. For large values of  $\Gamma$  one gets a linear behavior

$$\gamma_{\text{eff}}(\Gamma) \sim 4/\Gamma \tau_Z^2, \quad \text{for } \Gamma \rightarrow \infty, \tag{18}$$

which, when compared with (6), yields Schulman’s relation  $\tau \simeq 4/\Gamma$  [5]. When  $\Gamma < \Gamma^* = 4/\tau^*$ , i.e. when the response of the apparatus is not very quick, the decay is accelerated (IZE). For  $\Gamma \rightarrow 0$  one recovers the natural decay rate  $\gamma$ .

### 4.3. Continuous “Rabi” observation (response time $K^{-1}$ )

The previous example is nothing but a more refined model of (the first stage of) a detection process than that given by the projection prescription. In this sense one might be led to think that irreversibility is a fundamental requisite for obtaining quantum Zeno effects: the observed system has to be coupled to a *bona fide* detector that irreversibly records its state. This expectation would be incorrect. In order to hinder (or accelerate) decay it is enough to introduce an external agent which couples differently to the initial state  $|a\rangle$  and to the “decay” products  $(1 - |a\rangle\langle a|)|\psi\rangle$  ( $\psi$  being the wave function of the system). In other words, one only needs an interaction which is able to distinguish whether the system is in its initial state or not: in this (very) loose sense the external agent can be viewed as a detector [10]. Let us illustrate this point by adding to (7) the following interaction Hamiltonian

$$H_{\text{meas}}(K) = K(|b\rangle\langle M| + |M\rangle\langle b|), \quad (19)$$

which is probably the simplest way to include an external apparatus: as soon as state  $|b\rangle$  becomes populated it undergoes Rabi oscillations to state  $|M\rangle$  with Rabi frequency  $K$  (detector response time =  $1/K$ ) [11]. The interaction modifies the self-energy function as  $\Sigma_a(E, K) = [\Sigma_a(E + K) + \Sigma_a(E - K)]/2$ , whence the effective decay rate reads [12]

$$\gamma_{\text{eff}}(K) = [\gamma(\omega_a + K) + \gamma(\omega_a - K)]/2 = \pi[\kappa_a(\omega_a + K) + \kappa_a(\omega_a - K)] \quad (20)$$

and is shown in the right frame of Fig. 3 as a function of  $1/K$ . The behavior is similar to those previously described. For large values of  $K$  one gets the behavior

$$\gamma_{\text{eff}}(K) \sim \pi\kappa_a(K) \sim \Lambda/\tau_Z^2 K^2, \quad \text{for } \Lambda \rightarrow \infty. \quad (21)$$

Note, however, that this quadratic law, unlike the linear laws (6) and (18), is not generic, for it depends on the specific asymptotic behavior of the chosen form factor  $\kappa_a$ . As in the previous cases, when  $K < K^*$ , i.e. when the response of the apparatus is not very quick, the decay is accelerated (IZE) and for  $K \rightarrow 0$  the system eventually decays with the natural rate  $\gamma$ .

## 5. Conclusions

We have shown that the only requisite to obtain QZE is a coupling which is able to “pick out” the initial state of the system. For unstable systems this can also give rise to IZE. The recent experiment [7] has proved the existence of a transition from QZE to IZE in the case of pulsed measurements for a *bona fide* unstable system. It would be interesting to check the presence of such a transition also in the other cases envisaged in this paper (continuous and continuous Rabi observation).

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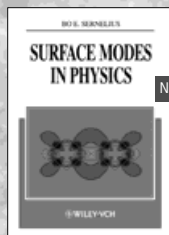
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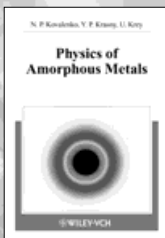
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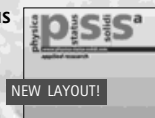
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