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From trivial to non-trivial conformal string backgrounds via O(*d*, *d*) transformations

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Recently proposed extensions of target-space duality and Narain transformations can be used to generate highly non-trivial conformal string backgrounds out of (almost) trivial ones. The example of "inverting" and "boosting" a flat, Milne-type cosmological metric is worked out in detail. Analogously, starting from Rindler's metric, one can generate a sort of 3D black hole.

1. Introduction

Target-space duality is the statement [1,2] that two (more generally a finite number of) apparently different string theories actually coincide after a suitable relabelling of states and operators. Thus, for instance, a closed string moving on a circle of radius R is equivalent to one moving on a circle of radius α'/R. This is known as R duality.

The study of classical propagation in non-trivial (generally anisotropic) cosmological backgrounds [3] has recently suggested [4] a generalization of R duality to the discrete group of inversion of the scale factor(s) defining these geometries. This group was termed in ref. [4] scale-factor-duality (SFD). SFD transformations where shown to be a symmetry not only of classical string motions but also of the (lowest-order) string-modified Einstein–Friedmann equations with or without classical stringy sources [4,5]. A characteristic feature [4,5] of the SFD transformations is that they involve, in a non-trivial way, the dilaton field, explaining why SFD is violated [4] by the usual Einstein–Friedmann equations.

Parallel work [6] on the recently constructed 2D black hole (BH) conformal backgrounds [7] has also employed "duality" transformations to relate different BHs to one another. As emphasized in ref. [4], SFD (and its analog for BH) is not a symmetry of the theory. SFD (or BHD)-related solutions are usually inequivalent, making the use of the term duality somewhat inappropriate. Given a classical solution SFD simply allows to construct other classical solutions very much in analogy with the action of Narain's O(d, d) group [8] on static compactifications in closed string theory. However, unlike in Narain's case, by allowing for time dependence in the spatial part of the metric, SFD is non-trivial even for open strings or in the absence of compactification.

The analogy (and difference) with Narain's work was made even more compelling when SFD was later extended [9] to a full continuous non-compact O(d, d) group ($d=D-1$ being the number of spatial dimensions), i.e., precisely to Narain's group. As in ref. [8], this enlarged group necessarily brings in the "torsion" field B_{ij} even when originally absent.

It was also argued in ref. [9] that there is a "Narain" group associated with any canonical transformation

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which preserves the σ-reparametrization constraint. Only subgroups that are unitarily implementable (i.e. which preserve the spectrum), however, lead to equivalent string theories, while, in general, one obtains from the action of the group other (inequivalent) classical solutions. Consequently, while the nature of Narain's group is rather stable, that of the spectrum-preserving subgroup depends on the details of the model (open or closed strings, compact or non-compact target space, etc.).

It is then clear how to generalize further SFD to the case in which the original solution is independent of any number n of coordinates. The canonical transformations of ref. [9] acting on those n coordinates generate an $O(n, n)$ "Narain" group. This straightforward extension was explicitly pointed out by Sen [10], who also gave string-field-theory arguments for $O(n, n)$ to be preserved to all orders in α', possibly with some higher order modification [4,5,10].

In the following we shall refer to the elements of the $O(d, d)$ group as dynamical Narain transformations (DNT). In order to label physically inequivalent solutions one has to divide this group by the "gauge subgroup", i.e. by all those transformations that leave the theory physically unchanged. One thus arrives [9] at a set of "gauge invariant" parameters labelling inequivalent conformal backgrounds precisely as in Narain's work [8]. In the absence of compactification there are just $d(d-1)/2$ such parameters [9], which appear to belong [10] to an $O(d)\otimes O(d)/O(d)$ coset space.

In this note DNT will be used in a somewhat unexpected direction, i.e. in order to generate out of almost trivial (in particular flat) space–time metrics some highly non-trivial conformal backgrounds, consisting of a non-flat metric and of non-trivial dilaton and antisymmetric-tensor backgrounds. To the extent that a flat metric (with horizons) is an exact solution of the conditions of conformal invariance and that DNT are valid at all orders [10], this method provides new exact conformal string theories, whose possible physical relevance is still to be investigated.

Since we only wish to illustrate here the general ideas, we shall mainly limit our attention to the simple case of applying DNT to "Milne's" metric Almost identical considerations would apply (after replacing time with one space coordinate) to Rindler's metric. This would lead to a new class of "black holes" living in $2+1$ dimensions.

We shall first discuss the Rindler and Milne metrics, showing how they emerge as limiting cases of the $SL(2,\mathbb{R})/U(1)$ Wess–Zumino–Witten (WZW) models as used in ref. [7]. Next we shall discuss their (SFD-related) inverse metrics, showing their non-triviality. We shall finally use the "boosts" of $O(d, d)$ to generate out of Milne and of its inverse a one-parameter family of solutions which interpolates smoothly between the two. We shall compute various properties of the solutions, showing in particular how the presence of a non-trivial B_{ij} appears to avoid the generic curvature singularities found to occur in torsion-free cosmological backgrounds.

2. Minkowski, Rindler, Milne and their duals

Recent work on 2D black holes [7] has brought up the very likely existence of new conformal backgrounds corresponding to highly non-trivial geometries. These models are based on level k, gauged, $SL(2,\mathbb{R})/U(1)$ WZW theory and live in a target space consisting of two non-trivial coordinates and of d_t extra "passive dimensions" which simply make up for the critical value of the central charge. One has

$$d_t=24-6/(k-2)\ , \quad d_t=2/(k-2)\ , \tag{2.1}$$

for bosonic or fermionic strings, respectively.

While for $k=\frac{9}{4}$ the model is genuinely two-dimensional, for $k\to\infty$ one is back to critical dimensions. Perhaps surprisingly, however, the $k\to\infty$ limit does not lead necessarily to the trivial (and obviously conformal) Minkowski metric. Starting from the BH solutions of ref. [7], or from the cosmological solutions of ref. [4], one arrives at line elements of the form

$$ds^2 = -(a+bx_1)^2 dt^2 + dx_1^2 + \sum_j dx_j^2 , \tag{2.2a}$$

$$ds^2 = -dt^2 + (a+bt)^2 dx_1^2 + \sum_j dx_j^2 , \tag{2.2b}$$

respectively. The dilaton, on the other hand, does go to a constant as $k \to \infty$.

For $b=0$ this is just the line element for Minkowski space–time which, in the absence of compactification, is a fixed point of DNT (model by the gauge group). Any DNT acting on Minkowski's metric (plus possibly a constant B) is equivalent [9] to a general coordinate transformation (GCT) combined with a B-gauge transformation. On the other hand, for $b \neq 0$, the parameter a can be gauged to 0, while b itself can be gauged to any chosen value (we shall set $a=0$ in the following).

Although these two metrics do not look like Minkowski, they are completely flat and can be brought globally to Minkowski's form by a GCT. The corresponding transformations are singular, however. They are given respectively by

$$X_1 = x_1 \cosh bt , \quad T = x_1 \sinh bt , \tag{2.3a}$$

$$X_1 = t \sinh bx_1 , \quad T = t \cosh bx_1 . \tag{2.3b}$$

The above transformations clearly map the whole t–x_1 plane into regions of the T–X_1 planes, i.e. into the regions $|X_1| > |T|$ and $|X_1| < |T|$, respectively. We thus recognize, in the above two metrics, a parametrization of Rindler's (accelerated observer) and Milne's (flat space-time with linear scale factor) causally distinct portions of the Minkowski plane. The complete flatness of these sectors ($\mathscr{R}_\pm$ and $\mathscr{M}_\pm$, respectively) readily explains why these, like the full Minkowski manifold, represent exact conformal backgrounds in the critical number of dimensions.

One of the main points of this work is to show that, in spite of their "triviality", $\mathscr{R}$ and $\mathscr{M}$ are not, unlike Minkowski, fixed points of DNT. In other words, DNT do not commute with (singular) GCT in the sense that they transform GCT-related into GCT-unrelated backgrounds.

This can immediately be checked for the discrete SFD subgroup [4–6] which inverts the metric (besides changing the dilaton). The new $\tilde{\mathscr{M}}$ and the new dilaton $\tilde{\phi}$ are simply given by

$$d\tilde{s}^2 = -dt^2 + (bt)^{-2} dx_1^2 + \sum_j dx_j^2 , \quad \tilde{\phi} = -2 \ln bt + \text{constant} . \tag{2.4}$$

For $t<0$, $\tilde{\mathscr{M}}_-$ belongs to the class of Bianchi I type anisotropic, "superinflationary" metrics considered e.g. in refs. [3,4], and describing the expansion of some spatial dimensions from initial flatness at $t=-\infty$ to a final state of arbitrarily large curvature. It is straightforward to check that $\tilde{\mathscr{M}}$ is not flat. We can easily compute e.g. the scalar curvature of $\tilde{\mathscr{M}}$ obtaining

$$\tilde{R} = 4t^{-2} . \tag{2.5}$$

As was the case with the generic (torsion-free) cosmological solution of ref. [4], (2.5) too exhibits a curvature singularity at a finite value of t (the "big bang"). Similarly, the BH solution dual to Rindler would exhibit a curvature singularity a $x_1 = 0$.

As far as the thermal properties of these backgrounds are concerned, it is well known [11] that one can associate a temperature $\hbar a/2\pi$ with hyperbolic observers (whose world-lines span the Rindler sectors $\mathscr{R}_\pm$) having uniform acceleration a. For the Milne sectors a thermal interpretation is not as clear as in Rindler's case. However, if one considers particle production in $\mathscr{M}_-$, one finds a Planck spectrum in the longitudinal momentum, at a proper temperature [12]

$$kT = \hbar/2\pi|t| \tag{2.6}$$

(otherwise stated, the conformal vacuum in Milne coordinates is in a thermal state with respect to the Minkowski vacuum [13]).

The dual (superinflationary) metric of $\tilde{\mathcal{M}}_-$, on the other hand, is characterized by the existence of an event horizon at a proper distance

$$d_{\mathrm{E}}=\tfrac{1}{2}|t| \tag{2.7}$$

from a co-moving observer placed at the origin. The horizon moves away from the observer with an acceleration $a=1/2d_{\mathrm{E}}$. Analogy, with de Sitter (see also ref. [14]) suggests thus a dual temperature $k\tilde{T}\simeq\hbar/4\pi d_{\mathrm{E}}$, i.e. an identical temperature for $\mathcal{M}_-$ and $\tilde{\mathcal{M}}_-$. In such a case the superinflationary phase of increasing curvature and effective density of $\tilde{\mathcal{M}}_-$ corresponds to a growing background temperature (in agreement with recently suggested "pre-big-bang" scenarios [3,4,15]). By contrast, Giveon [6] argued for duality related (i.e. inverse) temperatures for duality-related BHs.

3. A one-parameter family of "boosted" backgrounds

What happens if we apply to $\mathcal{M}$ or to $\tilde{\mathcal{M}}$ the full $\mathrm{O}(d, d)$ group? Obviously, some of these transformations will just give "gauge" equivalent backgrounds. Others will be related to each other by trivial rotations in the $(d-1)$-dimensional space orthogonal to the direction (called x_1) in which the original metrics live. The only genuinely new backgrounds turn out to be generated by "boosts" (in the sense of $\mathrm{O}(d, d)$, of course) which connect the originally non-trivial direction x_1 to a second direction which we identify with x_2. The background obtained this way thus lives in $2+1$ dimensions. This (abelian) boost subgroup is defined by the $\mathrm{O}(d, d)$ matrix

$$\Omega(\gamma)=\frac{1}{2}\begin{pmatrix}1+c & s & c-1 & -s\\ -s & 1-c & -s & 1+c\\ c-1 & s & 1+c & -s\\ s & 1+c & s & 1-c\end{pmatrix},\quad c=\cosh\gamma\,,\quad s=\sinh\gamma\,, \tag{3.1}$$

which depends on the boost parameter γ, with $0<\gamma<\infty$. The boosted backgrounds are then given by [9]

$$M(\gamma)=\Omega^{\mathrm{T}}(\gamma)M\Omega(\gamma)\,,$$

where the $2d\times 2d$ matrix M is given by [2]

$$M=\begin{pmatrix}G^{-1} & -G^{-1}B\\ BG^{-1} & G-BG^{-1}B\end{pmatrix}, \tag{3.2}$$

and G, B stand for the $d\times d$ spatial parts of the metric and torsion [9]. After some straightforward algebra one finally arrives at the following expressions for the boosted metric, torsion (in the two non-trivial spatial dimensions) and dilation:

$$G(\gamma)=\begin{bmatrix}\dfrac{(c-1)+(c+1)b^2t^2}{(c+1)+(c-1)b^2t^2} & \dfrac{s(1+b^2t^2)}{(c+1)+(c-1)b^2t^2}\\ \dfrac{s(1+b^2t^2)}{(c+1)+(c-1)b^2t^2} & 1\end{bmatrix},$$

$$B(\gamma)=\begin{bmatrix}0 & \dfrac{-s(1+b^2t^2)}{(c+1)+(c-1)b^2t^2}\\ \dfrac{s(1+b^2t^2)}{(c+1)+(c-1)b^2t^2} & 0\end{bmatrix},$$

$$\phi(\gamma)=-\ln[1+\tanh^2(\gamma/2)b^2t^2]\,. \tag{3.3}$$

Similar results hold for the boosted version of $\mathcal{M}$. They are simply obtained by replacing bt with $(bt)^{-1}$ everywhere.

We have calculated with a computer program all sort of GCT tenors pertaining to the boosted backgrounds, rechecking most of them analytically. The outcome of the exercise is shown in table 1 for the background (3.3). We have explicitly checked that all one-loop β-functions vanish, a property that, according to ref. [10], should continue to hold at higher orders.

Note that all curvature components depend on t through two parameters: the boost γ and the scale b. The results are expressed in terms of b and of the combination

$$\beta = b\tanh(\gamma/2)\ . \tag{3.4}$$

While the generic tensor components depend both on b and on β, all genuine scalars (R, H^2 $R^2_{\mu\nu}$, etc.) only depend on the latter.

This is in accordance with the parameter counting of refs. [9,10] which predicts a single "gauge invariant" parameter for $d=2$ (which is the effective number of dimensions since $d-2$ dimensions are completely inert).

The properties of $\tilde{\mathcal{M}}(\gamma)$ can be similarly computed. One finds that they can be obtained from those reported in table 1 via the replacements

$$c \to -c\ , \quad s \to -s \ \Rightarrow\ \tanh(\gamma/2) \to \coth(\gamma/2)\ . \tag{3.5}$$

Comparing the properties of $\mathcal{M}(\gamma)$ with those $\tilde{\mathcal{M}}(\gamma)$ we realize that these two families of backgrounds are now connected smoothly to one another through the boost. While for $\gamma=0$ the two metrics are completely different, one being flat and the other being curved and singular, the two backgrounds look more and more similar as γ is increased, until they coincide at infinite boost.

If, as we should, we just look at scalars under GCT, we see that, in a b–γ plane, they are gauge orbits relating equivalent backgrounds, those with the same value of the "physical" parameter (β for $\mathcal{M}(\gamma)$ or a similarly defined $\tilde{\beta}$ for $\tilde{\mathcal{M}}(\gamma)$). The situation is illustrated in fig. 1, where the lower half of the figure refers to $\mathcal{M}(\gamma)$ and the upper half to $\tilde{\mathcal{M}}(\gamma)$. In the middle the gauge orbits meet and the two families of backgrounds go into each other. Thus there is a single parameter describing physically inequivalent backgrounds and smoothly interpolating between $\mathcal{M}$ and $\tilde{\mathcal{M}}$.

Actually, all scalars take the form

$$R_i = (\beta)^{p_i} f_i(z)\ , \quad z = \beta t\ , \tag{3.6}$$

Table 1
Non-vanishing tensor components for $\mathcal{M}(\gamma)$.

$R^0_0 = \dfrac{2\beta^2(\beta^2t^2-2)}{(1+\beta^2t^2)^2}$	$R^2_1 = -R^1_2 = \dfrac{2\beta^3/b}{(1-\beta^2/b^2)}\ \dfrac{1}{(1+\beta^2t^2)}$
$R^1_1 = -\dfrac{2\beta^2}{(1+\beta^2t^2)^2}\left(1+\dfrac{1+\beta^4t^2/b^2}{(1-\beta^2/b^2)}\right)$	$R^2_2 = -\dfrac{2\beta^2}{(1+\beta^2t^2)^2}\left(1-\dfrac{\beta^2(1+b^2t^2)}{b^2(1-\beta^2/b^2)}\right)$
$R = \dfrac{\beta^2(4\beta^2t^2-10)}{(1+\beta^2t^2)^2}$	
$R_{11} = -\dfrac{2\beta^2\{\beta^2/b^2+[1+(2-\beta^4/b^4)b^2t^2]\}}{(1+\beta^2t^2)^3}$	$R_{22} = -\dfrac{2\beta^2}{(1+\beta^2t^2)^2}$
$R_{12} = R_{21} = -\dfrac{2\beta^2[1+(2-\beta^2/b^2)b^2t^2]}{b(1+\beta^2t^2)^3}$	$R^2_{\mu\nu} = 4\beta^4\dfrac{2\beta^4t^4-6\beta^2t^2+9}{(1+\beta^2t^2)^4}$
$R_{1212} = b^2\dfrac{(1-\beta^2/b^2)^2\beta^2t^2}{(1+\beta^2t^2)^4}$	$R_{1010} = -\dfrac{\beta^2[\beta^2/b^2+(3-2\beta^4/b^4)b^2t^2]}{(1+\beta^2t^2)^3}$
$R_{2020} = -\dfrac{\beta^2}{(1+\beta^2t^2)^2}$	$R_{1020} = -\dfrac{\beta^3}{b}\ \dfrac{[1+(3-2\beta^2/b^2)b^2t^2]}{(1+\beta^2t^2)^3}$
$R^2_{\mu\nu\alpha\beta} = 4\beta^2\dfrac{4\beta^4t^4-4\beta^2t^2+11}{(1+\beta^2t^2)^4}$	

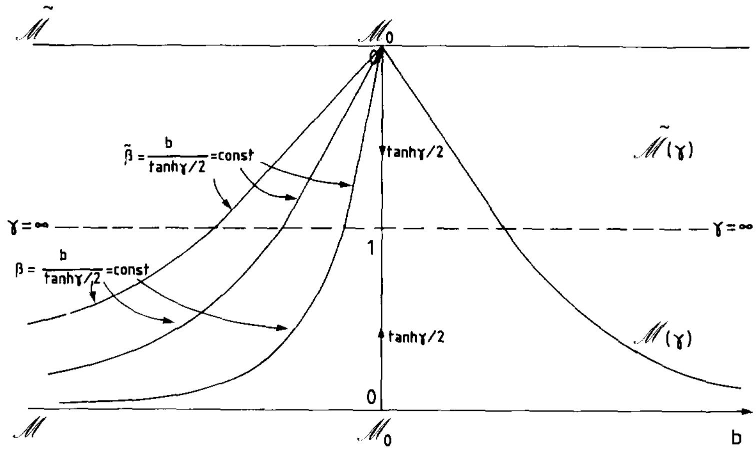

Fig. 1. Gauge orbits in the b–tanh$(\gamma/2)$ plane are shown as solid lines, lower half for $\mathcal{M}$, upper half $\tilde{\mathcal{M}}$. The two sectors join smoothly at $\gamma\to\infty$ (dotted line). The vertical line marked by $\mathcal{M}_0$ denotes Minkowski space–time.

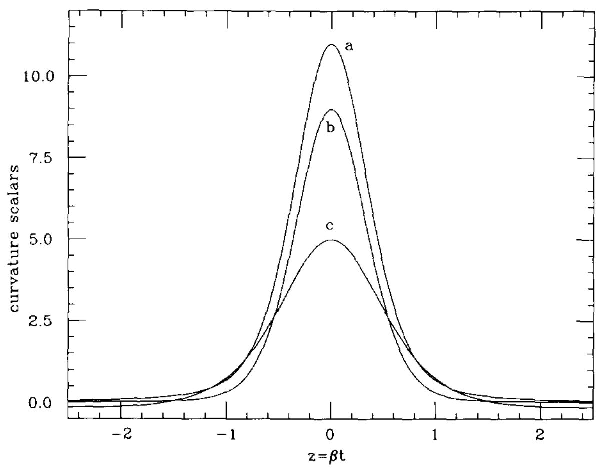

Fig. 2. Plots of curvature scalars as a function of the "scaling variable" z defined in (3.6). Curves (a), (b) and (c) refer to $R^2_{\mu\nu\alpha\beta}/4\beta^4$, $R^2_{\mu\nu}/4\beta^4$, $R/2\beta^2$, respectively.

where p_i is the dimensionality of R_i. The functions $f_i(z)$ are plotted in fig. 2 for the scalar curvature as well as for Ricci and Riemann squared. We note that the Euler–Gauss–Bonnet combination $R^2_{\mu\nu\alpha\beta}-4R^2_{\mu\nu}+R^2$ vanishes identically. All of them go to zero at large z and show some structure at small z, but have no singularity on the real axis. The curvature singularity at $t=0$ in $\tilde{\mathcal{M}}$ is now seen as the $\beta\to\infty$ limit of (3.6) which, instead of being plain infinite, yields a function of t with a singularity. In general, as shown in fig. 2, the maximum of $|f|$ (for the scalar curvature) occurs at $z=0$ and has the value

$$|R|=10\beta^2\,. \tag{3.7}$$

We shall conclude with the description of some general property of the boosted metrics. For finite β our cosmological solutions have nowhere curvature singularities. This is not in contradiction with general results since, in all cases, a non-trivial torsion ($H=\mathrm{d}B$) generated by the boost – together with the dilation – leads to violations of the strong energy condition [16]. This might suggest a crucial role for the antisymmetric tensor not only in evading the no-hair conjecture [17] (see also ref. [18] for the case of stringy black holes), but also in avoiding the initial singularity and in allowing solutions which, being well defined at all times, allow for cosmologies "going through" the big bang.

Moreover, a nonvanishing B is source of shear in the x_1–x_2 plane. Considering a congruence of co-moving geodesics u^μ in $\mathcal{M}(\gamma)$ we find that the squared strength $(\sigma_{\mu\nu})^2$ of the shear tensor [16] (which represents an invariant measure of the anisotropy magnitude) is given by

$$\sigma^2_{\mu\nu}=\frac{2}{3t^2}\frac{1+\beta^2t^2+\beta^4t^4}{(1+\beta^2t^2)^2}\,. \tag{3.8}$$

The shear is rapidly decaying away from the origin, but diverges in the $t\to0$ limit. This represents a singularity of the geodesic congruence (which may occur even in flat space–time), but not a singularity of the space–time structure, as the curvature invariants are bounded. Indeed, if we consider the rate-of-change H_1 of the relative distance between two neighbouring co-moving observers along the x_1 direction (i.e. the analog of the Hubble parameter of isotropic cosmologies), we find for $\mathcal{M}(\gamma)$ (in the gauge $b=1$)

$$H_1\equiv=-\sigma_{\mu\nu}n^\mu n^\nu+\tfrac{1}{3}\theta=\frac{4ct}{(c^2-1)(1+t^4)+2t^2(c^2+1)}\,, \tag{3.9}$$

which is regular at $t=0$ (here n^μ is a unitary vector along x_1, $n^\mu u_\mu=0$, $n^\mu n_\mu=1$, and $\theta=\nabla_\mu u^\mu$ is the volume expansion [16]).

Note that $\mathcal{M}(\gamma)$ describes contraction for $t<0$ and expansion for $t>0$, just like the Milne metric. Unlike Milne, however, one finds superinflation ($H_1>0$, $\dot{H}_1>0$) immediately after the origin, while at large positive t the expansion is decelerated (for the boosted background $\tilde{\mathcal{M}}(\gamma)$ the expansion factor has, of course, the opposite sign, $\tilde{H}_1=-H_1$).

This seems to indicate a role for the antisymmetric field also as a source of anisotropic superinflation. The possible use of DNT-generated torsion in the attempt of constructing some "stringy" cosmological scenario, describing the evolution from a flat initial state through a highly curved (planckian) inflationary phase down to present, decelerated universe [3,4,15], will be discussed elsewhere.

While this work was being written we received a preprint by Sen [19] whose content and conclusions overlap to some extent with ours.

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