SELF-SUSTAINED INFLATION AND DIMENSIONAL REDUCTION
FROM FUNDAMENTAL STRINGS

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In a previous paper we have shown that an ideal gas of fundamental strings is not able to
sustain, by itself, a phase of isotropic inflation of the Universe. We show here that fundamental
strings can sustain, instead, a phase of anisotropic inflation accompanied by the contraction of a
sufficient number of internal dimensions. The conditions to be met for the existence of such a
solution to the Einstein and string equations are derived, and the possibility of a successful
resolution of the standard cosmological problems in the context of this model is discussed.

1. Introduction

It is well known that the classical problems of standard cosmology may be solved
by a primordial phase of accelerated expansion (inflation). Many inflation scenar-
ios, involving just the minimal number of spatial dimensions, have been proposed
with various degrees of success (see ref. [1] for a recent review). In most cases, a
large and positive cosmological constant (vacuum energy) is needed, during a long
enough period of time. Since present bounds on the cosmological constant are very
tight, one is faced with the problem of fine-tuning a potential so that the ratio of
vacuum energy density during and after inflation is enormously large.

In view of this problem it seems worthwhile to consider inflationary scenarios in
which no cosmological constant is actually required in order to drive inflation in
the three physical spatial dimensions. A possible mechanism of this kind is

provided by the simultaneous contraction of some extra dimensions, as first pointed out in some pioneer papers on Kaluza–Klein (KK) cosmology [2].

Consider indeed a cosmological manifold which is the direct product of a (d + 1)-dimensional space-time and an n-dimensional space $M_D = M_{d+1} \otimes M_n$, which exhibits inflationary expansion of three-dimensional space, and simultaneous contraction of the other n dimensions, provided n > 10. We shall denote by D = d + n + 1 the total number of dimensions. Suppose that, in the cosmic-time "gauge", this space is described by the metric

$$G_{AB} = \text{diag}(1, -R^2(t)\delta_{ij}, -r^2(t)\delta_{ab}),$$ (1.1)

where R(t) and r(t) are, respectively, the scale factors of M_d and M_n. (Conventions: $A, B = 0, ..., D - 1; i, j = 1, ..., d; a, b = d + 1, ..., d + n$.) In this metric, the (0,0) component of the Einstein equations reads

$$\frac{d}{R}\frac{d^2R}{dt^2} + \frac{n}{r}\frac{d^2r}{dt^2} = -8\pi G_D\left(T_0^0 - \frac{T}{D-2}\right),$$ (1.2)

where G_D is the D-dimensional gravitational constant, and T is the trace of the energy–momentum of the matter sources. It may be possible, therefore, to realize an inflationary expansion in d dimensions (i.e. $d^2R/dt^2 > 0$) even if the righthand side of eq. (1.2) is negative, provided the evolution of the other n dimensions is characterized by a negative acceleration ($d^2r/dt^2 < 0$).

The consistency, and the efficiency, of a mechanism of dynamical dimensional reduction depends crucially, of course, on the number of dimensions and on the equation of state characterizing the gravitational sources, as shown in detail by a recent phenomenological discussion [3]. As possible models of sources compatible with inflation during the decoupling of internal and external dimensions, the examples so far considered include a relativistic gas of massless particles [2], a perfect fluid with phenomenological equation of state [3–5], the antisymmetric tensor of supergravity theories [6] and, more recently, scalar fields at finite temperature [7].

The problem with KK cosmology is that there is no consistent quantum theory of gravitational or gauge interactions in D > 4. The only exception to this rule appears to reside in string theory. Since string matter may behave differently from point particles it seems worthwhile to reconsider these KK inflationary scenarios within string theory. Also, in a realistic string unification theory, the scale of compactification of the extra dimensions is expected to be not much different from the string mass scale [8–10]. It may be possible, therefore, for the multidimensional phase of our universe to be necessarily a "stringy" phase so that, as stressed in ref. [8], dimensional reduction cannot be adequately described in the field theory limit, but requires instead the direct application of string theory in non-trivial backgrounds.

In this paper we present a particular solution of the Einstein-plus-string equations, showing that it is possible to have a self-sustained string plus gravity system which exhibits inflationary expansion of three-dimensional space, and simultaneous contraction of the other n dimensions, provided n > 10.

This solution describes a phase dominated by unstable strings [11,12], i.e. non-oscillating string configurations whose proper amplitude tends to evolve, asymptotically, like the scale factor (while the co-moving amplitude becomes "frozen"). Their effective pressure is negative in physical three-dimensional space, and positive (but negligible) in internal space. With this source, the internal dimensions contract with a negative acceleration, while the three-dimensional spatial expansion turns out to be of the super-inflationary type [13], i.e. characterized by $d^2R/dt^2 > 0$ and $dH/dt > 0$, where $H = R^{-1}dR/dt$ is the Hubble parameter.

Consequently, the string energy density grows with R and approaches a singularity in a finite (cosmic time) interval. However, this picture of the early universe cannot be extended above a maximal density, where string and other quantum corrections to the Einstein equations cannot be neglected, and where, in any case, a transition to the standard, radiation-dominated scenario may be expected. Nevertheless it seems possible, according to our mechanism, to obtain a significative inflation of the causal horizon of physical space-time already before reaching these maximal (planckian) densities.

The content of the paper is as follows. In sect. 2, considering the equations of motion for a string embedded in an isotropic Friedmann–Robertson–Walker (FRW) background, we review the general form of the leading-order solution valid, in the large-R limit, in the case of inflationary expansion. In sect. 3 we present a new approximate solution to the same string equations, valid in the small-R limit for a negatively accelerated contraction ($dR/dt < 0$, $d^2R/dt^2 < 0$). In sect. 4 we discuss the values of D and n for which the Einstein and string equations can be simultaneously and consistently satisfied by anisotropic configurations in which d dimensions inflate, and the remaining n dimensions contract. The possibility that this scenario, in the particular case of $D = 4 \oplus 22$ dimensions, may represent a viable mechanism to solve at least some of the standard cosmological problems is finally discussed in sect. 5.

2. Asymptotic string configurations in inflationary expanding backgrounds

The equations of motion of a string, coupled to a D-dimensional background metric G_{AB}, can be written [11, 14] (in the gauge in which the world-sheet metric is conformally flat)

$$\ddot{X}^A - X''^A + \Gamma_{BC}^A(\dot{X}^B + X'^B)(\dot{X}^C - X'^C) = 0,$$ (2.1)

where Γ is the Christoffel connection for the metric G_{AB}; a dot and a prime denote, respectively, differentiation with respect to the world-sheet time and space variables, τ and σ. The variation of the action with respect to the world-sheet metric provides, in addition, the two constraints [11,14]

$$G_{AB}(\dot{X}^A\dot{X}^B + X'^A X'^B) = 0, \quad G_{AB}\dot{X}^A X'^B = 0. \quad (2.2)$$

We are interested, in this section, in the case of a homogeneous and isotropic cosmological background, with flat spatial sections, described in the cosmic time gauge ($X^0 = t$) by the FRW metric

$$G_{AB} = \text{diag}(1, -R^2(t)\delta_{ij}), \quad i,j = 1,\ldots,D-1 \quad (2.3)$$

(see ref. [14] for a discussion of the fact that such a manifold does not provide a conformally invariant σ-model, at the quantum level, and hence it is not a candidate string vacuum). The string equations and constraints thus become, explicitly,

$$\ddot{t} - t'' + R\frac{dR}{dt}\left[(\dot{X}^i)^2 - (X'^i)^2\right] = 0, \quad (2.4)$$

$$\ddot{X}^i - X''^i + \frac{2}{R}\frac{dR}{dt}(\dot{t}\dot{X}^i - t'X'^i) = 0, \quad (2.5)$$

$$\dot{t}^2 + t'^2 = R^2\left[(\dot{X}^i)^2 + (X'^i)^2\right], \quad (2.6)$$

$$\dot{t}t' = R^2\dot{X}^i X'^i. \quad (2.7)$$

Consider, in particular, an inflationary expanding background ($dR/dt > 0$, $d^2R/dt^2 > 0$), parametrized by the following scale factor:

$$R(t) = [k(1-\alpha)(t_c - t)]^{-\alpha/(1-\alpha)}, \quad (2.8)$$

where $\alpha > 0$ and k, t_c are positive constant parameters. We have, in particular, super-inflation ($dH/dt > 0$) for $0 < \alpha < 1$, power-law inflation ($dH/dt < 0$) for $\alpha > 1$, while de Sitter (i.e. exponential) inflation ($dH/dt = 0$) corresponds to $\alpha = 1$. The $R \to \infty$ limit corresponds to $t \to t_c$ for super-inflation, and to $t \to \infty$ in the other cases.

For this background, the exact solution to the string equations (2.4)–(2.7), in the

tion described by ($\tau < 0$)

$$t(\sigma, \tau) = t_c - \frac{(-kL\tau)^{1-\alpha}}{k(1-\alpha)}, \quad (2.9)$$

$$X^i(\sigma, \tau) = A^i(\sigma) + \frac{\tau^2 A''^i}{2(1-2\alpha)} + \tau^{1+2\alpha}B^i(\sigma), \quad (2.10)$$

where

$$L^2 = A'^i A'^i, \quad A'^i B^i = 0 \quad (2.11)$$

and, by exploiting σ reparametrization invariance, we have imposed the "gauge" $A'^i A''^i = 0$, which implies $L' = 0$. This configuration, which depends on $2D - 4$ arbitrary functions of σ, represents the general solution of eqs. (2.4)–(2.7), to leading order in τ, as τ goes to zero. (Note that, according to eq. (2.9), the scale factor can be rewritten, in this approximation,

$$R(\tau) = (-kL\tau)^{-\alpha}, \quad (2.12)$$

so that the $\tau \to 0$ limit corresponds indeed to $R \to \infty$.)
In the case $\alpha = 1$ (de Sitter), eq. (2.9) is to be replaced by [11]

$$t(\sigma, \tau) = -k^{-1}\ln(-kL\tau), \quad (2.13)$$

while, for $\alpha = \frac{1}{2}$, eq. (2.10) is to be replaced by [11]

$$X^i(\sigma, \tau) = A^i + \frac{1}{2}\tau^2[B^i + A''^i \ln(-\tau)]. \quad (2.14)$$

We also recall that the general solution for X^i, without the gauge-fixing condition $L' = 0$, has been reported in ref. [11].

It is important to stress that, for these string configurations, the world-sheet time τ turns out to be proportional, asymptotically, to the conformal time coordinate η of the background manifold, defined by $R = dt/d\eta$. Indeed, from eqs. (2.9) and (2.12), $\dot{t} = RL$, so that

$$\eta = \tau L. \quad (2.15)$$

Moreover we note, for future reference, that for these configurations RX'^i behaves asymptotically like R (as $\dot{X}^i \to 0$ for $\tau \to 0$), and that the configurations are characterized by the general properties

$$|\dot{t}| \gg |t'|, \quad |\dot{X}^i| \ll |X'^i|. \quad (2.16)$$

As a consequence, their net contribution to the effective pressure of the gravitational sources is negative. Indeed, by using the constraints (2.2), the string energy-momentum tensor T^{AB} obtained by varying the action with respect to the background metric, i.e.

$$T^{AB}(x) = \frac{1}{\pi\alpha'\sqrt{-G}} \int d\sigma d\tau (\dot{X}^A \dot{X}^B - X'^A X'^B)\delta^D(X-x) \quad (2.17)$$

can be shown to satisfy the identity [11]

$$\pi\alpha'\sqrt{-G}\left[G_{00}T^{00}(x) - G_{ij}T^{ij}(x)\right]$$
$$= -2\int d\sigma d\tau \delta^D(X-x)\left[G_{00}(X^0)^2 + G_{ij}\dot{X}^i\dot{X}^j\right] \quad (2.18)$$

$(\alpha')^{-1}$ is the string tension, and $G = \det G_{AB}$. On the other hand, for a perfect fluid in the FRW background (2.3), one has

$$T_{00} = \rho, \quad T_{ij} = -pG_{ij} = pR^2\delta_{ij}, \quad (2.19)$$

where ρ and p are functions of the cosmic time only. According to eq. (2.16), the equation of state of an ideal gas of strings described asymptotically by eqs. (2.9) and (2.10) can thus be approximated by [11]

$$\rho = -p(D-1), \quad (2.20)$$

which implies $p < 0$.

3. Asymptotic string configurations in backgrounds with accelerated contraction

Consider again a string coupled to the FRW metric (2.3), with a scale factor, however, which parametrizes a negatively accelerated ($d^2R/dt^2 < 0$) contraction ($dR/dt < 0$), i.e.

$$R(t) = [k(1-\delta)(t_c - t)]^{\delta/(1-\delta)}, \quad (3.1)$$

where $0 < \delta < \frac{1}{2}$ and $t < t_c$.

In the small-R limit (i.e. $t \to t_c$), the exact solution to eqs. (2.4)–(2.7) can be expanded again as a power series in τ, around the following leading-order

approximation ($\tau < 0$):

$$X^i(\sigma, \tau) = A^i(\sigma) + \frac{1}{2}\tau^2 D^i(\sigma) + \tau^{1-2\delta}B^i(\sigma), \quad (3.2)$$

where

$$t(\sigma, \tau) = t_c - \frac{(-kL\tau)^{1-\delta}}{k(1-\delta)}, \quad (3.3)$$

$$L^2(\sigma) = [(1-2\delta)^2 B'^i B'^i k^{4\delta}]^{1/(1-2\delta)}, \quad (3.4)$$

$$D^i(\sigma) = \frac{1}{1+2\delta}\left(A''^i + 2\delta\frac{L'}{L}A'^i\right) \quad (3.5)$$

and A^i, B^i are arbitrary functions of σ satisfying the constraint $A'^i B'^i = 0$ (note that, according to eq. (3.2), the $\tau \to 0$ limit corresponds now to $R \to 0$).

Since, for the general solution, only $2D - 4$ arbitrary functions are required, we can further restrict the leading-order approximation (3.2), (3.3) by imposing the convenient gauge condition $B'^i B'^i = 0$ (which implies, as before, $L' = 0$). In any case this approximate solution describes, in the $R \to 0$ limit, string configurations with shrinking proper amplitude, for which $R X^i$ behaves like R, while $R X'^i$ behaves like R^{-1}. These configurations may be regarded as unstable in the sense of refs. [11,14] since they are not oscillating in τ as if the string oscillators would develop, in the small-R regime, imaginary frequencies. Such configurations, moreover, are characterized by the asymptotic properties

$$|\dot{t}| \gg |t'|, \quad |\dot{X}^i| \gg |X'^i| \quad (3.6)$$

which lead, in the perfect fluid approximation, to a radiation-like equation of state. The identity (2.18) can indeed be rewritten as

$$\pi\alpha'\sqrt{-G}\left[G_{00}T^{00}(x) + G_{ij}T^{ij}(x)\right]$$
$$= -2\int d\sigma d\tau \delta^D(X-x)\left[G_{00}(X^0)^2 + G_{ij}X'^i X'^j\right]. \quad (3.7)$$

For an ideal gas of strings satisfying approximately eq. (3.6) at small R we thus have, after using eq. (2.19),

$$\rho = p(D-1) \quad (3.8)$$

which is just the equation of state for a gas of massless particles.

4. String-driven inflation and dimensional reduction

Consider now a D-dimensional anisotropic background, $M_D = M_{d+1} \otimes M_n$, parametrized in the cosmic-time gauge by $X^A = (t, X^i, X^a)$ and by the metric (1.1) (we recall that i,j range from 1 to d, while a,b from $d+1$ to $d+n$). In this background the stress tensor for a perfect fluid has the general form

$$T_A{}^B = \text{diag}(\rho, -p\delta_i{}^j, -q\delta_a{}^b),$$ (4.1)

where ρ, p, q are functions of t only (we have called q the pressure in the "internal" n-dimensional space M_n). The Einstein equations,

$$R_A{}^B = 8\pi G_D\left(T_A{}^B - \frac{T}{D-2}\delta_A{}^B\right)$$ (4.2)

with a perfect fluid as source, become explicitly

$$\frac{d}{R}\frac{d^2R}{dt^2} + \frac{n}{R}\frac{d^2r}{dt^2} = -\frac{8\pi G_D}{D-2}[\rho(D-3)+pd+nq],$$ (4.3)

$$\frac{1}{R}\frac{d^2R}{dt^2} + \frac{d-1}{R^2}\left(\frac{dR}{dt}\right)^2 + \frac{n}{rR}\frac{dR}{dt}\frac{dr}{dt} = \frac{8\pi G_D}{D-2}[\rho - p(1-n)-nq],$$ (4.4)

$$\frac{1}{r}\frac{d^2r}{dt^2} + \frac{n-1}{r^2}\left(\frac{dr}{dt}\right)^2 + \frac{d}{rR}\frac{dr}{dt}\frac{dR}{dt} = \frac{8\pi G_D}{D-2}[\rho - pd + q(d-1)].$$ (4.5)

We take, in particular, a perfect gas of strings as the dominant source of gravity. According to the constraints (2.2), the string stress tensor must satisfy the identity

$$\pi\alpha'\sqrt{-G}\left(G_{00}T^{00} - G_{ij}T^{ij} + G_{ab}T^{ab}\right)$$
$$= -2\int d\sigma d\tau\,\delta^D(X-x)\left[G_{00}(\dot X^0)^2 + G_{ij}\dot X^i\dot X^j + G_{ab}X'^a X'^b\right],$$ (4.6)

where T^{AB} is given in eq. (2.17), and X^A satisfies the string equations (2.1) and constraints (2.2), which in this background become, respectively,

$$\ddot t - t'' + R\frac{dR}{dt}\left[(\dot X^i)^2 - (X'^i)^2\right] + r\frac{dr}{dt}\left[(\dot X^a)^2 - (X'^a)^2\right] = 0,$$ (4.7)

$$\ddot X^i - X''^i + \frac{2}{R}\frac{dR}{dt}(\dot t\dot X^i - t'X'^i) = 0,$$ (4.8)

$$\ddot X^a - X''^a + \frac{2}{r}\frac{dr}{dt}(\dot t\dot X^a - t'X'^a) = 0,$$ (4.9)

$$\dot t^2 + t'^2 = R^2\left[(\dot X^i)^2 + (X'^i)^2\right] + r^2\left[(\dot X^a)^2 + (X'^a)^2\right],$$ (4.10)

$$\dot t t' = R^2\dot X^i X'^i + r^2\dot X^a X'^a.$$ (4.11)

We look for consistent solutions to the coupled Einstein-string equations representing, asymptotically, an accelerated expansion of M_d ($dR/dt > 0$, $d^2R/dt^2 > 0$ for $R \to \infty$), and an accelerated contraction of M_n ($dr/dt < 0$, $d^2r/dt^2 < 0$ for $r \to 0$). Consequently we expect, as shown in sects. 2 and 3, a solution of the string equations characterized by the asymptotic properties

$$|t| \gg |t'|, \quad |\dot X^i| \ll |X'^i|, \quad |\dot X^a| \gg |X'^a|$$ (4.12)

corresponding, according to eq. (4.6), to a perfect fluid with "equation of state"

$$\rho + pd - nq = 0.$$ (4.13)

By making the ansatz

$$q = \gamma p,$$ (4.14)

where γ is a constant parameter, we can eliminate q and p from the Einstein equations (4.3)–(4.5), and we find then the particular exact solution

$$R(t) = [k(t_c - t)]^\beta, \qquad r = R^{-\epsilon},$$ (4.15)

$$\rho = \rho_0[k(t_c - t)]^{-2},$$ (4.16)

where k and t_c are positive integration constants, ρ_0 is a positive constant which can be expressed in terms of d, n, k, and

$$\epsilon = -\frac{2d - \gamma(D-2)}{D-2n},$$

$$\beta = \frac{2(d-n)}{d(d-1-n\gamma) - ne(d-\gamma-n\gamma)}.$$ (4.17)

This solution describes, in the $t \to t_c$ limit, inflationary expansion of M_d, and simultaneous contraction M_n, provided

$$\epsilon > 0, \qquad \beta < 0.$$ (4.18)

(Note that the expansion is always of the super-inflationary type, since

$$\frac{d}{dt}\left(\frac{1}{R}\frac{dR}{dt}\right) > 0$$ (4.19)

for $\beta < 0$.) Moreover, the solution is consistent with a positive energy density provided

$$\frac{a+b(d-ne)}{ne(1+\epsilon)-(d-ne)(d-1-ne)} < 0,$$ (4.20)

where

$$a = \frac{(D-4)(d-n\gamma) - 2n\gamma}{d-n\gamma}, \qquad b = \frac{D-2n}{d-n\gamma}. \qquad (4.21)$$

In order to discuss for which values of ϵ, β, γ the string equations are satisfied, when $t \to t_c$, by configurations consistent with the assumed properties (4.12)–(4.14), we consider separately the three possibilities $|\gamma| = \infty$, $0 < |\gamma| < \infty$ and $\gamma = 0$. Such possibilities define, respectively, a string gas with the asymptotic properties $|p| \ll q$, $|p| \sim q$, and $|p| \gg q$, and correspond, respectively, to string configurations characterized, in addition to eq. (4.12), by the following asymptotic behaviors:

$$|RX''^i| \ll |rX'^a|, \qquad (4.22)$$

$$|RX''^i| \sim |rX'^a|, \qquad (4.23)$$

$$|RX''^i| \gg |rX'^a| \qquad (4.24)$$

(recall eqs. (4.1) and (2.17)). We also know, from our previous discussion of the isotropic case, that RX''^i behaves like R for $R \to \infty$, while rX'^a behaves like r^{-1} for $r \to 0$. The three cases above are thus consistent with the solution (4.15) provided, respectively, $\epsilon > 1$, $\epsilon = 1$ and $\epsilon < 1$.

In the first case, in which the internal pressure q tends to dominate, only the trivial (flat space) solution is asymptotically consistent, since $\beta \to 0$ for $|\gamma| \to \infty$. In the second case in which γ is finite (and non-vanishing) the consistency condition (4.23), which implies $\epsilon = 1$, fixes the value of γ according to eq. (4.16):

$$\gamma = \frac{3D - 4n - 2}{D - 2}. \qquad (4.25)$$

The constraint $q = \gamma p$, which can also be rewritten

$$G_{ab}T^{ab} = \frac{n\gamma}{d} G_{ij}T^{ij} \qquad (4.26)$$

is not compatible, however, with the general form of the leading solution to the string equations; it can be satisfied only by a particular solution, determined by the asymptotic condition

$$r^2(X'^a)^2 = -\frac{n\gamma}{d} R^2 (X'^i)^2. \qquad (4.27)$$

For the last case ($\gamma = 0$, $\epsilon < 1$) we can find instead a general solution which satisfies the required properties. In this case the coefficients ϵ and β become

$$\epsilon = -\frac{2d}{\cdots}, \qquad \beta = \frac{2(D - 2n)}{\cdots}. \qquad (4.28)$$

and the conditions $\beta < 0$, $0 < \epsilon < 1$ imply

$$D < 2n, \qquad 3D - 4n - 2 < 0, \qquad (4.29)$$

which are both fulfilled iff $n > 3d + 1$ (of course $n, d > 0$). For this background the general form of the approximate solution to eqs. (4.7)–(4.11), in the limit $t \to t_c$, is (to leading order in τ, $\tau < 0$)

$$t = t_c - \frac{1}{k}[-kL\tau(1-\beta)]^{1/(1-\beta)}, \qquad (4.30)$$

$$X^i = A^i + \frac{1}{2}\tau^2 D^i + B^i \tau^{-(1-3\beta)/(1-\beta)}, \qquad (4.31)$$

$$X^a = A^a + \frac{1}{2}\tau^2 D^a + B^a \tau^{-(1-\beta+2\epsilon\beta)/(1-\beta)}, \qquad (4.32)$$

where A and B are functions of σ, $L^2 = A'^i A'^i$ and, in the gauge $L' = 0$,

$$D^i = \frac{1-\beta}{1+\beta}A''^i, \qquad D^a = \frac{1-\beta}{1-\beta-2\epsilon\beta}A''^a. \qquad (4.33)$$

Moreover, A and B must satisfy the constraint

$$\left(\frac{1-3\beta}{1-\beta}\right)[-kL(1-\beta)]^{2\beta/(1-\beta)}B'^i A'^i$$
$$+ \left(\frac{1-\beta+2\epsilon\beta}{1-\beta}\right)[-kL(1-\beta)]^{-2\epsilon\beta/(1-\beta)}B'^a A'^a = 0 \qquad (4.34)$$

imposed by eq. (4.11).

In this approximation, the scale factor can be rewritten in terms of τ as

$$R = [-kL\tau(1-\beta)]^{\beta/(1-\beta)}, \qquad r = R^{-\epsilon} \qquad (4.35)$$

and one can easily check that, when the conditions (4.29) are satisfied, the properties (4.12) and (4.24), as well as the energy condition (4.20), and the condition $1 - \beta + 2\epsilon\beta > 0$ required by eq. (4.32), are satisfied. The string configurations (4.30)–(4.32) lead then, in the perfect fluid approximation, to the equation of state

$$p = -\frac{\rho}{d}, \qquad q = 0, \qquad (4.36)$$

which can sustain the inflation-dimensional-reduction scenario of eq. (4.15). For a manifold with $D - n = 4$, in particular, this happens, according to the dimensional bound (4.29), provided $n > 10$.

We note, finally, that the proper amplitude of these string configurations, $\chi^i = R X^i$, $\chi^a = r X^a$, grows in the external space, and shrinks in the internal one, as $\tau \to 0$. In both cases, however, the time evolution of χ tends to follow, asymptotically, that of the scale factor, so we may regard these strings as a generalization to the anisotropic case of the unstable configurations recently discussed for isotropic backgrounds [11,14]. In this sense we can say, therefore, that inflation, and the asymptotic decoupling of internal and external dimensions ($r \to 0$ for $R \to \infty$), are induced by unstable strings.

5. Discussion and conclusions

A gas of unstable strings, which can form and develop at large R in inflationary expanding backgrounds, is characterized by an effective negative pressure. If the expansion is isotropic, the pressure is not negative enough (at least in the perfect fluid approximation) to sustain inflation by itself, as pointed out in a previous paper [11].

It is possible, however, that unstable strings may sustain simultaneously inflation and dimensional reduction, as discussed in sect. 4, provided the number of contracting dimensions is large enough, i.e. $n > 10$ for $D = 4 + n$. In this case, the asymptotic evolution of an $M_4 \otimes M_n$ background is described in particular by these scale factors (according to eqs. (4.15) and (4.28))

$$R = [k(t_c - t)]^{(4-n)/(4+2n)} , \qquad r = [k(t_c - t)]^{6/(4+2n)} . \qquad (5.1)$$

The four-dimensional expansion is then of the super-inflationary type ($dH/dt > 0$), while the internal contraction has, as expected, a negative acceleration ($d^2 r / dt^2 < 0$).

In order to discuss whether this model can provide, during the dimensional decoupling, a significative amount of inflation, let us consider explicitly the interesting case $D = 26$ (i.e. $\beta = -\frac{3}{8}$, $\epsilon = \frac{1}{3}$). In this case our solution (4.15) of the Einstein equations implies

$$8\pi G_D \rho(t_c - t)^2 = 15/16 , \qquad (5.2)$$

and, by denoting by "f" and "i" the end and the beginning of the phase dominated by these unstable strings, we have the relations

$$\frac{r_f}{r_i} = \left(\frac{R_f}{R_i}\right)^{-1/3} , \quad \frac{\rho_f}{\rho_i} = \left(\frac{R_f}{R_i}\right)^{16/3} . \qquad (5.3)$$

Let us assume, moreover, that the typical initial size of causally connected regions, $d_i \simeq (t_c - t_i)$, coincides with the proper length of a typical string, i.e.

$$t_c - t_i = L R_i . \qquad (5.4)$$

For the string solution (4.30) we have indeed $t_c - t = \frac{11}{8} |\tau| L R$, so that a larger size $d_i > L R_i$ would imply $|\tau_i| > 1$ (or, in conformal time, $|\eta_i| > L$), in contrast with the conditions required for the validity of our approximation, which represent an expansion in the $\tau \to 0$ limit.

For $t_c - t \ll t_c - t_i$ the proper size $d(t)$ of the particle horizon in four-dimensional space grows like R,

$$d(t) = R(t) \int_{t_i}^{t} dt' R^{-1}(t') = \frac{(t_c - t)^{\beta}}{1 - \beta} \left[(t_c - t_i)^{1 - \beta} - (t_c - t)^{1 - \beta} \right] \simeq \frac{R(t)}{R_i} d_i \qquad (5.5)$$

$(\beta = -\frac{3}{8})$, as is typical of inflationary backgrounds. The proper size $l(t)$ of the event horizon, on the contrary, shrinks as $t \to t_c$,

$$l(t) = R(t) \int_{t}^{t_c} dt' R^{-1}(t') = \frac{t_c - t}{1 - \beta} \qquad (5.6)$$

(typical behaviour of super-inflation only). As a consequence of this shrinking, unstable configurations may develop [11,12], until finally the Planck density is reached,

$$\rho_f = \rho_P = L_P^{-D} \qquad (5.7)$$

at a Planck curvature scale

$$\left[R \left(\frac{d^2 R}{dt^2} \right)^{-1} \right]^{1/2}_{t = t_f} \simeq t_c - t_f = L_P . \qquad (5.8)$$

Here we suppose that the whole scenario breaks down and the standard adiabatic evolution begins. For a drastic solution of the horizon problem [1,16] one may thus require that the final size of the particle horizon in four-dimensional space, $d_f = d_i (R_f / R_i) = L R_f$, be as large as the present size $d_0 \sim 10^{61} L_P$, rescaled down at the Planck era:

$$L R_f > d_0 \frac{R_f}{R_0} = d_0 \frac{T_0}{T_P} \sim 10^{29} L_P . \qquad (5.9)$$

$(T_P$ and T_0 are, respectively, the Planck temperature and the present temperature of the cosmic microwave background, $T_0 \sim 10^{-13}$ GeV.)

With the four constraints (5.4), (5.7)–(5.9) the model is completely determined. Indeed, using eqs. (5.7) and (5.8) one gets from eq. (5.2) $G_D = L_P^{D-2}$. Moreover, combining eqs. (5.3) and (5.4),

$$\frac{LR_i}{L_P} = \left(\frac{LR_f}{L_P}\right)^{8/11}. \tag{5.10}$$

The condition (5.9) is thus satisfied provided

$$LR_i \geqslant 10^{22} L_P. \tag{5.11}$$

The requirement (5.9) is strong enough* to accomodate the present values of the entropy S_0 and of the density parameter Ω_0 [1]. To be viable, this scenario needs four-dimensionally causally connected regions whose size, at the Planck curvature scale, is very large in natural (planckian) units. However, such an otherwise "unnaturally large" value is not put in by hand, but is the dynamical consequence of a previous super-inflationary expansion. Thus, the basic difference from conventional inflationary models is that the Planck era is the final state of inflation rather than its ancestor.

It is true that the Universe has to be very large in its initial state (see eq. (5.11)), as it is the case in the models of ref. [2]. However, unlike in ref. [2], in the context of string theory a classical, low-curvature regime is naturally expected to occur before the Planck era as a consequence of a symmetry (duality [17]) connecting it to the present epoch.

It should be noted, moreover, that a condition much less stringent than (5.9) is required if one only wants to explain the homogeneity of the cosmic microwave background. In such a case, in fact, one only needs the present size of the visible universe (d_0), rescaled down at the time of last scattering t_{rec} of the cosmic radiation [16] (at a temperature $T_{rec} \sim 10^4 T_0$), to be not larger than the particle horizon at the time. Starting at the Planck scale with a causal region of size $d_i = LR_i$, and assuming radiation-dominated evolution (i.e. $R \sim t^{1/2}$) down to t_{rec}, the homogeneity is thus explained provided

$$d_0 \frac{R_{rec}}{R_0} = d_0 \frac{T_0}{T_{rec}} \leqslant LR_f \left(\frac{t_{rec}}{t_P}\right) = LR_f \left(\frac{T_P}{T_{rec}}\right)^2 \tag{5.12}$$

(we have used the fact that in the standard decelerated scenario the particle

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*A primordial inflation at the Planck scale cannot dilute, however, a possible monopole production during the GUT phase transition.

horizon grows like t, while the scale factor like T^{-1}). Eq. (5.12) is satisfied for

$$LR_f \geqslant 10 L_P. \tag{5.13}$$

which implies, using eq. (5.10), $LR_i \geqslant 10^{8/11} L_P$.

It seems, possible, therefore, to reproduce the desired amount of inflation if one supposes that the universe, before the Planck era, was in a primordial phase dominated by a classical gas of very massive fundamental strings ($M_i \simeq LR_i/\alpha' \geqslant 10^{22} M_P$, if we impose, for example, the condition (5.11)). Note that although initially these strings are closely packed (with an averaged (distance/size) ratio of order $(LR_i/L_P)^{(4-D)/(D-1)} \leqslant 10^{-19}$), they only have a tiny energy density in Planck units, namely (from eq. (5.3))

$$\rho_i \lesssim 10^{-44} \rho_P \tag{5.14}$$

(i.e. they are a highly diluted gas, from a gravitational point of view). This density grew up to ρ_P, after which some violent transition to local physics ("big bang") led to the standard scenario in which the density started to decrease.

This picture of the early universe fits well with the scenario described in ref. [17], where it is suggested that, because of $R \leftrightarrow R^{-1}$ duality, a "stringy" phase before the radiation-dominated expansion is to be expected, and in this phase the temperature should grow (instead of decreasing with R), up to some maximal value of order T_P.

The transition occurring at the Planck scale cannot be described, of course, in the context of the model considered in this paper, since at $T = T_P$ the Einstein equations are no longer valid and moreover, because of the string Hagedorn temperature, a complete thermodynamical treatment is certainly needed (see ref. [18, 19]; and also ref. [20] for a formal regularization procedure which avoids the occurrence, in cosmological backgrounds, of curvatures larger than the Planck one).

The physical interpretation of such a transition as a possible model for the big bang was also previously suggested in ref. [21]. The growth of the density and of the temperature during the string phase were obtained, however, by means of an isotropic contraction of the whole universe (with all the spatial dimensions on the same footing). In our model, on the contrary, the density and the energy scale (i.e. the inverse of the curvature radius) grow together with the size of the particle horizon, which is expanding in the physical four-dimensional space-time. We stress that it is just because of this unconventional property of super-inflation that it becomes possible for the universe to emerge at the Planck time with causally connected regions whose size is much larger than the Planck length [see eqs. (5.9) and (5.13)].

In conclusion, we have shown that the dominance of unstable configurations, in a multidimensional string phase, can drive an anisotropic evolution of the background and realize dynamically an effective decoupling between different spatial dimensions. Unfortunately, this mechanism seems to be incapable of choosing, by itself, both the preferred total dimensionality, and the number of contracting dimensions. However, once these parameters are assigned, one has a model in which the universe reaches the Planck density through a super-inflationary evolution and thus with horizons which may be large enough to avoid causality problems after the transition to the standard scenario.

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