

Spontaneous emission and lifetime modification caused by an intense electromagnetic field

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We study the temporal evolution of a three-level system (such as an atom or a molecule), initially prepared in an excited state, bathed in a laser field tuned at the transition frequency of the other level. The features of the spontaneous emission are investigated and the lifetime of the initial state is evaluated: a Fermi “golden rule” still applies, but the on-shell matrix elements depend on the intensity of the laser field. In general, the lifetime is a decreasing function of the laser intensity. The phenomenon we discuss can be viewed as an “inverse” quantum Zeno effect and can be analyzed in terms of dressed states.

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I. INTRODUCTION

The temporal behavior of quantum-mechanical systems can be strongly influenced by the action of an external agent. A good example is the quantum Zeno effect [1,2], where the quantum-mechanical evolution of a given (not necessarily unstable) state is slowed down (or even halted) by performing a series of measurements that ascertain whether the system is still in its initial state. This peculiar effect is historically associated and usually ascribed to what we could call a “pulsed” quantum-mechanical observation on the system. However, it can also be obtained by performing a “continuous” observation of the quantum state, e.g., by means of an intense field [3,4].

Most experiment that have been performed or proposed in order to modify the quantum-mechanical evolution law make use of oscillating systems [5–9]. On the other hand, it would be interesting to understand whether and to which extent the evolution law of a *bona fide* “unstable” system can be changed. In order to discuss the evolution of genuine unstable systems one usually makes use of the Weisskopf-Wigner approximation [10], that ascribes the main properties of the decay law to a pole located near the real axis of the complex energy plane. This yields the Fermi “golden rule” [11]. In this paper we shall investigate the possibility that the lifetime of an unstable quantum system can be modified by the presence of a very intense electromagnetic field. We shall look at the temporal behavior of a three-level system (such as an atom or a molecule), where level No. 1 is the ground state and levels No. 2, No. 3 are two excited states (see Fig. 1). The system is initially prepared in level No. 2 and if it follows its natural evolution, it will decay to level No. 1. The decay will be (approximately) exponential and characterized by a certain lifetime, that can be calculated from the Fermi golden rule. But if one shines on the system an intense laser field, tuned at the transition frequency 3-1, the evolution can be different. This problem was investigated in Ref. [3], where it was found that the lifetime of the initial state depends on the intensity of the laser field. In the limit of an extremely intense field, the initial state undergoes a “continuous observation” and the decay should be considerably slowed down (quantum Zeno effect). The aim of this paper is to study this effect in more detail and discuss a new phenomenon [12]: we shall see that for physically sensible values of

the intensity of the laser, the decay can be *enhanced*, rather than hindered. This can be viewed as an “inverse” quantum Zeno effect. An important role in this context will be played by the specific properties of the interaction Hamiltonian, in particular by the “form factor” of the interaction.

Other authors have studied physical effects that are related to those we shall discuss. The features of the matrix elements of the interaction Hamiltonian were investigated in the context of the quantum Zeno effect by Kofman and Kurizki [13], who also emphasized that different quantum Zeno regimes are present. Plenio, Knight and Thompson discussed the quantum Zeno effect due to “continuous” measurements and considered several physical systems whose evolution is modified by an external field [14]. There is also work by Kraus on a similar subject [15]. Finally, Zhu, Narducci and Scully [16] investigated the electromagnetic-induced transparency in a context similar to that considered in this paper. In some sense, our present investigation “blends” these studies, by taking into account the important role played by the matrix elements of the interaction. This will enable us to discuss some new features of the evolution that have not been considered before. We shall look at this phenomenon from several perspectives, by first solving the time-dependent Schrödinger equation, then looking at the spectrum of the emitted photons and finally constructing the dressed (Fano) states.

Our analysis will be performed within the Weisskopf-Wigner approximation and no deviations at short [1,17,18] and long [19] times will be considered. The features of the quantum-mechanical evolution are summarized in Ref. [20] and have already been discussed within a quantum field theoretical framework [21–24], where several subtle effects have to be properly taken into account.

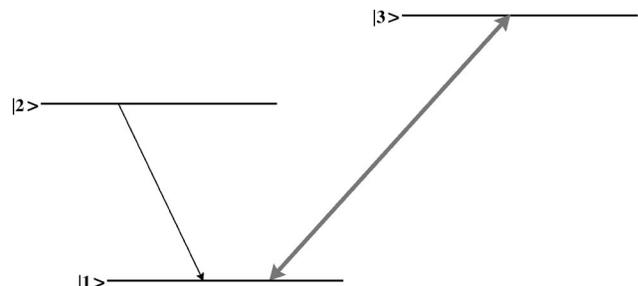


FIG. 1. Level configuration.

This paper is organized as follows. In Sec. II we introduce the three-level system bathed in the laser field. Its temporal evolution is studied in Sec. III. The spectrum of the photons emitted during the evolution is evaluated in Sec. IV. Section V contains a discussion in terms of dressed states, Sec. VI an analysis of the influence of additional levels on the lifetime, and Sec. VII some concluding remarks.

II. PRELIMINARIES AND DEFINITIONS

We consider the Hamiltonian ($\hbar = c = 1$) [3]:

$$\begin{aligned}
 H &= H_0 + H_{\text{int}} \\
 &= \omega_0|2\rangle\langle 2| + \Omega_0|3\rangle\langle 3| + \sum_{\mathbf{k},\lambda} \omega_{\mathbf{k}} a_{\mathbf{k}\lambda}^\dagger a_{\mathbf{k}\lambda} \\
 &\quad + \sum_{\mathbf{k},\lambda} (\phi_{\mathbf{k}\lambda} a_{\mathbf{k}\lambda}^\dagger |1\rangle\langle 2| + \phi_{\mathbf{k}\lambda}^* a_{\mathbf{k}\lambda} |2\rangle\langle 1|) \\
 &\quad + \sum_{\mathbf{k},\lambda} (\Phi_{\mathbf{k}\lambda} a_{\mathbf{k}\lambda}^\dagger |1\rangle\langle 3| + \Phi_{\mathbf{k}\lambda}^* a_{\mathbf{k}\lambda} |3\rangle\langle 1|), \quad (1)
 \end{aligned}$$

where the first two terms are the free Hamiltonian of the three-level atom [whose states $|i\rangle$ ($i=1,2,3$) have energies $E_1=0$, $\omega_0=E_2-E_1>0$, $\Omega_0=E_3-E_1>0$], the third term is the free Hamiltonian of the EM field and the last two terms describe the $1 \leftrightarrow 2$ and $1 \leftrightarrow 3$ transitions in the rotating wave approximation, respectively (see Fig. 1). States $|2\rangle$ and $|3\rangle$ are chosen so that no transition between them is possible (e.g., because of selection rules). The matrix elements of the interaction Hamiltonian read

$$\begin{aligned}
 \phi_{\mathbf{k}\lambda} &= \frac{e}{\sqrt{2\epsilon_0 V \omega}} \int d^3x e^{-i\mathbf{k}\cdot\mathbf{x}} \boldsymbol{\epsilon}_{\mathbf{k}\lambda}^* \cdot \mathbf{j}_{12}(\mathbf{x}), \\
 \Phi_{\mathbf{k}\lambda} &= \frac{e}{\sqrt{2\epsilon_0 V \omega}} \int d^3x e^{-i\mathbf{k}\cdot\mathbf{x}} \boldsymbol{\epsilon}_{\mathbf{k}\lambda}^* \cdot \mathbf{j}_{13}(\mathbf{x}), \quad (2)
 \end{aligned}$$

where $-e$ is the electron charge, ϵ_0 the vacuum permittivity, V the volume of the box, $\omega = |\mathbf{k}|$, $\boldsymbol{\epsilon}_{\mathbf{k}\lambda}$ the photon polarization, and \mathbf{j}_{fi} the transition current of the radiating system. For example, in the case of an electron in an external field, we have $\mathbf{j}_{\text{fi}} = \psi_f^\dagger \boldsymbol{\alpha} \psi_i$ where ψ_i and ψ_f are the wave functions of the initial and final state, respectively, and $\boldsymbol{\alpha}$ is the vector of Dirac matrices. For the sake of generality we are using relativistic matrix elements, but our analysis can also be performed with nonrelativistic ones $\mathbf{j}_{\text{fi}} = \psi_f^* \mathbf{p} \psi_i / m_e$, where \mathbf{p}/m_e is the electron velocity.

We shall concentrate our attention on a three-level system bathed in a continuous laser beam, whose photons have momentum \mathbf{k}_0 ($|\mathbf{k}_0| = \Omega_0$) and polarization λ_0 , and assume, throughout this paper, that

$$\phi_{\mathbf{k}_0\lambda_0} = 0, \quad (3)$$

i.e., the laser does not interact with state $|2\rangle$. Let the laser be in a coherent state $|\alpha_0\rangle$ with a very large average number

$\bar{N}_0 = |\alpha_0|^2$ of \mathbf{k}_0 -photons in volume V [we will eventually consider the thermodynamical limit; see Eq. (26)]. In the picture defined by the unitary operator

$$T(t) = \exp(\alpha_0^* e^{i\Omega_0 t} a_{\mathbf{k}_0\lambda_0} - \alpha_0 e^{-i\Omega_0 t} a_{\mathbf{k}_0\lambda_0}^\dagger), \quad (4)$$

the Hamiltonian (1) reads

$$\begin{aligned}
 H(t) &= THT^\dagger + i\dot{T}T^\dagger = H + (\Phi_{\mathbf{k}_0\lambda_0} \alpha_0^* e^{i\Omega_0 t} |1\rangle\langle 3| \\
 &\quad + \Phi_{\mathbf{k}_0\lambda_0}^* \alpha_0 e^{-i\Omega_0 t} |3\rangle\langle 1|). \quad (5)
 \end{aligned}$$

In this picture, the \mathbf{k}_0 mode is initially in the vacuum state [25] and by noting that for $\bar{N}_0 \gg 1$

$$\begin{aligned}
 |\langle 1; 0_{\mathbf{k}\lambda} | H(t) | 3; 0_{\mathbf{k}\lambda} \rangle| &= \sqrt{\bar{N}_0} |\Phi_{\mathbf{k}_0\lambda_0}| \\
 &\gg |\langle 1; 1_{\mathbf{k}\lambda} | H(t) | 3; 0_{\mathbf{k}\lambda} \rangle| \\
 &= |\Phi_{\mathbf{k}\lambda}|, \quad (6)
 \end{aligned}$$

the Hamiltonian (5) becomes

$$\begin{aligned}
 H &\simeq \omega_0|2\rangle\langle 2| + \Omega_0|3\rangle\langle 3| + \sum_{\mathbf{k},\lambda} \omega_{\mathbf{k}} a_{\mathbf{k}\lambda}^\dagger a_{\mathbf{k}\lambda} \\
 &\quad + \sum'_{\mathbf{k},\lambda} (\phi_{\mathbf{k}\lambda} a_{\mathbf{k}\lambda}^\dagger |1\rangle\langle 2| + \phi_{\mathbf{k}\lambda}^* a_{\mathbf{k}\lambda} |2\rangle\langle 1|) \\
 &\quad + (\Phi_{\mathbf{k}_0\lambda_0} \alpha_0^* e^{i\Omega_0 t} |1\rangle\langle 3| + \Phi_{\mathbf{k}_0\lambda_0}^* \alpha_0 e^{-i\Omega_0 t} |3\rangle\langle 1|), \quad (7)
 \end{aligned}$$

where a prime means that the summation does not include $(\mathbf{k}_0, \lambda_0)$ [due to hypothesis (3)]. In the above equations and henceforth, the vector $|i; n_{\mathbf{k}\lambda}\rangle$ represents a state in which the atom is in state $|i\rangle$ and the electromagnetic field in a state with $n_{\mathbf{k}\lambda}(\mathbf{k}, \lambda)$ photons. We shall analyze the behavior of the system under the action of a continuous laser beam of high intensity. Under these conditions, level configurations similar to that of Fig. 1 give rise to the phenomenon of induced transparency [26], for laser beams of sufficiently high intensities. Our interest, however, will be focused on *unstable* initial states: we shall study the temporal behavior of level No. 2 when the system is shined by a continuous laser of intensity comparable to those used to obtain induced transparency.

Notice that in Eq. (7) the spontaneous decay $3 \rightarrow 1$ has been neglected with respect to the stimulated transition, because of the large factor $\sqrt{\bar{N}_0} \gg 1$ in Eq. (6). However, since our interest is primarily in the first step of this process, namely the decay $2 \rightarrow 1$, these smaller, later effects (of the order of $1/\bar{N}_0$) do not change our conclusions.

The operator

$$\mathcal{N} = |2\rangle\langle 2| + \sum'_{\mathbf{k},\lambda} a_{\mathbf{k}\lambda}^\dagger a_{\mathbf{k}\lambda}, \quad (8)$$

satisfies

$$[H, \mathcal{N}] = 0, \quad (9)$$

which implies the conservation of the total number of photons plus the atomic excitation (Tamm-Dancoff approximation [27]). The Hilbert space splits therefore into sectors that are invariant under the action of the Hamiltonian: in our case, the system evolves in the subspace labeled by the eigenvalue $\mathcal{N}=1$ and the analysis can be restricted to this sector [28].

III. TEMPORAL EVOLUTION

We will study the temporal evolution by solving the time-dependent Schrödinger equation

$$i \frac{d}{dt} |\psi(t)\rangle = H(t) |\psi(t)\rangle, \quad (10)$$

where the states of the total system in the sector $\mathcal{N}=1$ read

$$\begin{aligned} |\psi(t)\rangle = & x(t) |2;0\rangle + \sum'_{k,\lambda} y_{k\lambda}(t) |1;1_{k\lambda}\rangle \\ & + \sum'_{k,\lambda} z_{k\lambda}(t) e^{-i\Omega_0 t} |3;1_{k\lambda}\rangle \end{aligned} \quad (11)$$

and are normalized:

$$\langle \psi(t) | \psi(t) \rangle = |x(t)|^2 + \sum'_{k,\lambda} |y_{k\lambda}(t)|^2 + \sum'_{k,\lambda} |z_{k\lambda}(t)|^2 = 1 \quad (\forall t). \quad (12)$$

By inserting Eq. (11) in Eq. (10) one obtains the equations of motion

$$\begin{aligned} i \dot{x}(t) &= \omega_0 x(t) + \sum'_{k,\lambda} \phi_{k\lambda}^* y_{k\lambda}(t), \\ i \dot{y}_{k\lambda}(t) &= \phi_{k\lambda} x(t) + \omega_k y_{k\lambda}(t) + \alpha_0^* \Phi_{k_0\lambda_0} z_{k\lambda}(t), \\ i \dot{z}_{k\lambda}(t) &= \alpha_0 \Phi_{k_0\lambda_0}^* y_{k\lambda}(t) + \omega_k z_{k\lambda}(t), \end{aligned} \quad (13)$$

where a dot denotes time derivative. At time $t=0$ we prepare our system in the state

$$|\psi(0)\rangle = |2;0\rangle \Leftrightarrow x(0)=1, \quad y_{k\lambda}(0)=0, \quad z_{k\lambda}(0)=0, \quad (14)$$

which is an eigenstate of the free Hamiltonian

$$H_0 |\psi(0)\rangle = H_0 |2;0\rangle = \omega_0 |2;0\rangle. \quad (15)$$

Incidentally, we stress that the choice of the initial state is different from that of Ref. [5], where the three-level atom is initially in the ground state (No. 1) and a Rabi oscillation to level No. 2, provoked by an rf field, is inhibited by a pulsed laser, resonating between levels No. 1 and No. 3, that performs the ‘‘observation’’ of level No. 1. In our case, the atom is initially in level No. 2, so that it can *spontaneously*

decay to level No. 1, and it is ‘‘continuously observed’’ by a continuous laser at the 1-3 frequency [3]: As soon as the system has decayed to level No. 1, the (intense) laser provokes the 1 \rightarrow 3 transition. (The irreversibility inherent in the act of observation is eventually brought in by the spontaneous decay of level No. 3.) This brings us conceptually closer to the seminal formulation [1,2] of quantum Zeno effect.

By Laplace transforming the system of differential equations (13) and incorporating the initial condition (14) we get the algebraic system

$$is \tilde{x}(s) = \omega_0 \tilde{x}(s) + \sum'_{k,\lambda} \phi_{k\lambda}^* \tilde{y}_{k\lambda}(s) + i,$$

$$is \tilde{y}_{k\lambda}(s) = \phi_{k\lambda} \tilde{x}(s) + \omega_k \tilde{y}_{k\lambda}(s) + \alpha_0^* \Phi_{k_0\lambda_0} \tilde{z}_{k\lambda}(s), \quad (16)$$

$$is \tilde{z}_{k\lambda}(s) = \alpha_0 \Phi_{k_0\lambda_0}^* \tilde{y}_{k\lambda}(s) + \omega_k \tilde{z}_{k\lambda}(s),$$

where

$$\tilde{f}(s) = \int_0^\infty dt e^{-st} f(t), \quad f(t) = \frac{1}{2\pi i} \int_B ds e^{ts} \tilde{f}(s), \quad (17)$$

the Bromwich path B being a vertical line $\text{Re } s = \text{const}$ in the half plane of convergence of the Laplace transform. [Very similar equations of motion can be obtained by assuming that the external (laser) field is initially in a number state N_0 , with N_0 very large [12]. See also the discussion in Sec. V.] It is straightforward to obtain

$$\tilde{x}(s) = \frac{1}{s + i\omega_0 + Q(B, s)}, \quad (18)$$

$$\tilde{y}_{k\lambda}(s) = \frac{-i\phi_{k\lambda}(s + i\omega_k)}{(s + i\omega_k)^2 + B^2} \tilde{x}(s), \quad (19)$$

$$\tilde{z}_{k\lambda}(s) = -\frac{\sqrt{N_0} \Phi_{k_0\lambda_0}^* \phi_{k\lambda}}{(s + i\omega_k)^2 + B^2} \tilde{x}(s), \quad (20)$$

with

$$Q(B, s) = \sum'_{k,\lambda} |\phi_{k\lambda}|^2 \frac{s + i\omega_k}{(s + i\omega_k)^2 + B^2} \quad (21)$$

and where

$$B^2 = \bar{N}_0 |\Phi_{k_0\lambda_0}|^2 \quad (22)$$

is proportional to the intensity of the laser field and can be viewed as the ‘‘strength’’ of the observation performed by the laser beam on level No. 2 [3]. See the paragraph following Eq. (15). Note that the coupling B is related to the Rabi frequency by the simple relation $B = \Omega_{\text{Rabi}}/2$.

In the continuum limit ($V \rightarrow \infty$), the matrix elements scale as follows:

$$\lim_{V \rightarrow \infty} \frac{V \omega^2}{(2\pi)^3} \sum_{\lambda} \int d\Omega |\phi_{k\lambda}|^2 \equiv g^2 \omega_0 \chi^2(\omega), \quad (23)$$

where Ω is the solid angle. The (dimensionless) function $\chi(\omega)$ and coupling constant g have the following general properties, discussed in Appendix A:

$$\chi^2(\omega) \propto \begin{cases} \omega^{2j \mp 1} & \text{if } \omega \ll \Lambda, \\ \omega^{-\beta} & \text{if } \omega \gg \Lambda, \end{cases} \quad (24)$$

$$g^2 = \alpha (\omega_0 / \Lambda)^{2j+1 \mp 1}, \quad (25)$$

where j is the total angular momentum of the photon emitted in the $2 \rightarrow 1$ transition, \mp represent electric and magnetic transitions, respectively, $\beta (> 1)$ is a constant, α the fine structure constant and Λ a natural cutoff (of the order of the inverse size of the emitting system, e.g., the Bohr radius for an atom), which determines the range of the atomic or molecular form factor [29].

In order to scale the quantity B , we take the limit of very large cavity, by keeping the density of Ω_0 photons in the cavity constant:

$$V \rightarrow \infty, \quad \bar{N}_0 \rightarrow \infty, \quad \text{with} \quad \frac{\bar{N}_0}{V} = n_0 = \text{const} \quad (26)$$

and obtain from Eq. (22)

$$B^2 = n_0 V |\Phi_{k_0 \lambda_0}|^2 = (2\pi)^3 n_0 |\varphi_{\lambda_0}(\mathbf{k}_0)|^2, \quad (27)$$

where $\varphi \equiv \Phi V^{1/2} / (2\pi)^{3/2}$ is the scaled matrix element of the 1-3 transition. As we shall see, in order to affect significantly the lifetime of level No. 2, we shall need a high value of B , namely, a laser beam of high intensity. It is therefore interesting to consider a 1-3 transition of the dipole type, in which case the above formula reads

$$B^2 = 2\pi\alpha\Omega_0 |\boldsymbol{\epsilon}_{k_0 \lambda_0}^* \cdot \mathbf{x}_{13}|^2 n_0, \quad (28)$$

where \mathbf{x}_{13} is the dipole matrix element.

A. Laser off

Let us first look at the case $B=0$. The laser is off and we expect to recover the well-known physics of the spontaneous emission a two-level system prepared in an excited state and coupled to the vacuum of the radiation field. In this case, $Q(0,s)$ is nothing but the self-energy function

$$Q(s) \equiv Q(0,s) = \sum_{k,\lambda} |\phi_{k\lambda}|^2 \frac{1}{s + i\omega_k}, \quad (29)$$

which becomes, in the continuum limit

$$Q(s) \equiv g^2 \omega_0 q(s) \equiv -i g^2 \omega_0 \int_0^\infty d\omega \frac{\chi^2(\omega)}{\omega - is}, \quad (30)$$

where χ is defined in Eq. (23). The function $\tilde{\chi}(s)$ in Eq. (18) (with $B=0$) has a logarithmic branch cut, extending from 0

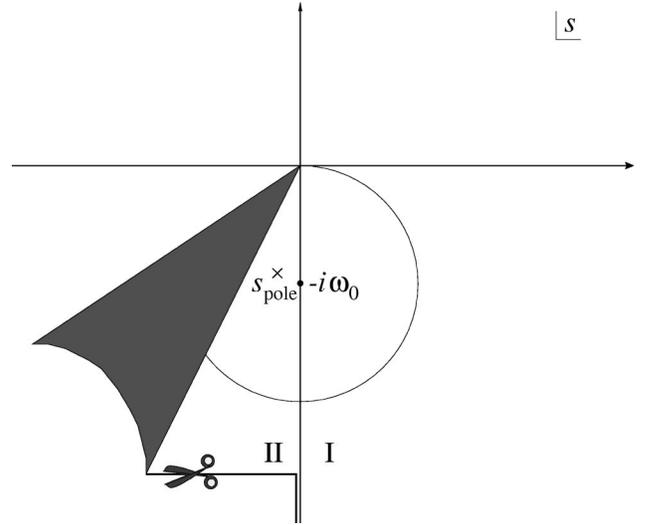


FIG. 2. Cut and pole in the s plane ($B=0$) and convergence circle for the expansion of $Q(s)$ around $s = -i\omega_0$. I and II are the first and second Riemann sheets, respectively. The pole is on the second Riemann sheet, at a distance $O(g^2)$ from $-i\omega_0$.

to $-i\infty$, and no singularities on the first Riemann sheet (physical sheet) [22]. On the other hand, it has a simple pole on the second Riemann sheet, that is the solution of the equation

$$s + i\omega_0 + g^2 \omega_0 q_{\text{II}}(s) = 0, \quad (31)$$

where

$$q_{\text{II}}(s) = q(se^{-2\pi i}) = q(s) + 2\pi\chi^2(is) \quad (32)$$

is the determination of $q(s)$ on the second Riemann sheet. We note that $g^2 q(s)$ is $O(g^2)$, so that the pole can be found perturbatively: by expanding $q_{\text{II}}(s)$ around $-i\omega_0$ we get a power series, whose radius of convergence is $R_c = \omega_0$ because of the branch point at the origin. The circle of convergence lies half on the first Riemann sheet and half on the second sheet (Fig. 2). The pole is well inside the convergence circle, because $|s_{\text{pole}} + i\omega_0| \sim g^2 \omega_0 \ll R_c$, and we can write

$$\begin{aligned} s_{\text{pole}} &= -i\omega_0 - g^2 \omega_0 q_{\text{II}}(-i\omega_0 - 0^+) + O(g^4) \\ &= -i\omega_0 - g^2 \omega_0 q(-i\omega_0 + 0^+) + O(g^4), \end{aligned} \quad (33)$$

because $q_{\text{II}}(s)$ is the analytic continuation of $q(s)$ below the branch cut. By using the formula

$$\lim_{\varepsilon \rightarrow 0^+} \frac{1}{x \pm i\varepsilon} = \text{P} \frac{1}{x} \mp i\pi \delta(x), \quad (34)$$

one gets from Eq. (30)

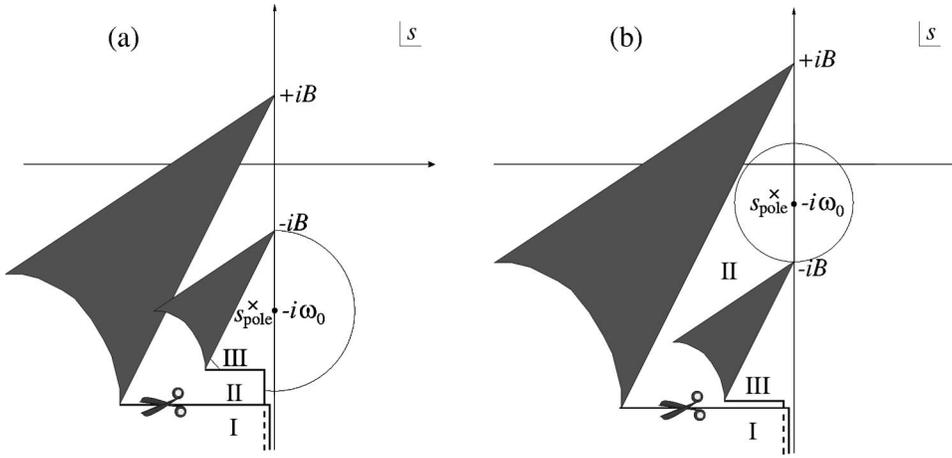


FIG. 3. Cuts and pole in the s -plane ($B \neq 0$) and convergence circle for the expansion of $Q(B, s)$ around $s = -i\omega_0$. I, II, and III are the first, second, and third Riemann sheets, respectively. (a) $B < \omega_0$. (b) $B > \omega_0$. In both cases, the pole is at a distance $O(g^2)$ from $-i\omega_0$.

$$\begin{aligned} q(-i\eta + 0^+) &= -i \int_0^\infty d\omega \chi^2(\omega) \frac{1}{\omega - \eta - i0^+} \\ &= \pi \chi^2(\eta) \theta(\eta) - i \text{P} \int_0^\infty d\omega \chi^2(\omega) \frac{1}{\omega - \eta} \end{aligned} \quad (35)$$

and by setting

$$s_{\text{pole}} = -i\omega_0 + i\Delta E - \frac{\gamma}{2}, \quad (36)$$

one obtains

$$\begin{aligned} \gamma &= 2\pi g^2 \omega_0 \chi^2(\omega_0) + O(g^4), \\ \Delta E &= g^2 \omega_0 \text{P} \int_0^\infty d\omega \frac{\chi^2(\omega)}{\omega - \omega_0} + O(g^4), \end{aligned} \quad (37)$$

which are the Fermi ‘‘golden rule’’ and the second order correction to the energy of level No. 2.

The Weisskopf-Wigner approximation [10] consists in neglecting all branch cut contributions and approximating the self-energy function with a constant (its value in the pole), that is

$$\tilde{x}(s) = \frac{1}{s + i\omega_0 + Q(s)} \simeq \frac{1}{s + i\omega_0 + Q_{\text{II}}(s_{\text{pole}})} = \frac{1}{s - s_{\text{pole}}}, \quad (38)$$

where in the last equality we used the pole equation (31). This yields a purely exponential behavior, $x(t) = \exp(s_{\text{pole}} t)$, without short-time (and long-time) corrections. As is well known, the latter are all contained in the neglected branch cut contribution.

B. Laser on

We turn now our attention to the situation with the laser switched on ($B \neq 0$) and tuned at the 1-3 transition frequency Ω_0 . The self energy function $Q(B, s)$ in Eq. (21) depends on B and can be written in terms of the self energy function

$Q(s)$ in absence of laser field [Eq. (29)], by making use of the following remarkable property:

$$\begin{aligned} Q(B, s) &= \frac{1}{2} \sum_{k,\lambda} |\phi_{k\lambda}|^2 \left(\frac{1}{s + i\omega_k + iB} + \frac{1}{s + i\omega_k - iB} \right) \\ &= \frac{1}{2} [Q(s + iB) + Q(s - iB)]. \end{aligned} \quad (39)$$

Notice, incidentally, that in the continuum limit ($V \rightarrow \infty$), due to the above formula, $Q(B, s)$ scales just as $Q(s)$. The position of the pole s_{pole} (and as a consequence the lifetime $\tau_E \equiv \gamma^{-1} = -1/2 \text{Re } s_{\text{pole}}$) depends on the value of B . There are now two branch cuts in the complex s plane, due to the two terms in Eq. (39). They lie over the imaginary axis, along $(-i\infty, -iB]$ and $(-i\infty, +iB]$.

The pole satisfies the equation

$$s + i\omega_0 + Q(B, s) = 0, \quad (40)$$

where $Q(B, s)$ is of order g^2 , as before, and can again be expanded in power series around $s = -i\omega_0$, in order to find the pole perturbatively. However, this time one has to choose the right determination of the function $Q(B, s)$. Two cases are mathematically possible: (a) The branch point $-iB$ is situated above $-i\omega_0$, so that $-i\omega_0$ lies on both cuts, see Fig. 3(a); (b) the branch point $-iB$ is situated below $-i\omega_0$, so that $-i\omega_0$ lies only on the upper branch cut, see Fig. 3(b). We notice that, although mathematically conceivable, the latter case ($B > \omega_0$) cannot be tackled within our approximations, for a number of additional effects would then have to be considered: multiphoton processes would take place, the other atomic levels would start to play an important role and our approach (three-level atom in the rotating wave approximation) would no longer be valid. We therefore restrict our attention to values of B that are high (of the same order of magnitude as those utilized in electromagnetic induced transparency), but not extremely high, so that our starting approximations still apply.

In case (a), i.e., for $B < \omega_0$, the pole is on the third Riemann sheet (under both cuts) and the power series converges in a circle lying half on the first and half on the third Riemann sheet, within a convergence radius $R_c = \omega_0 - B$, which

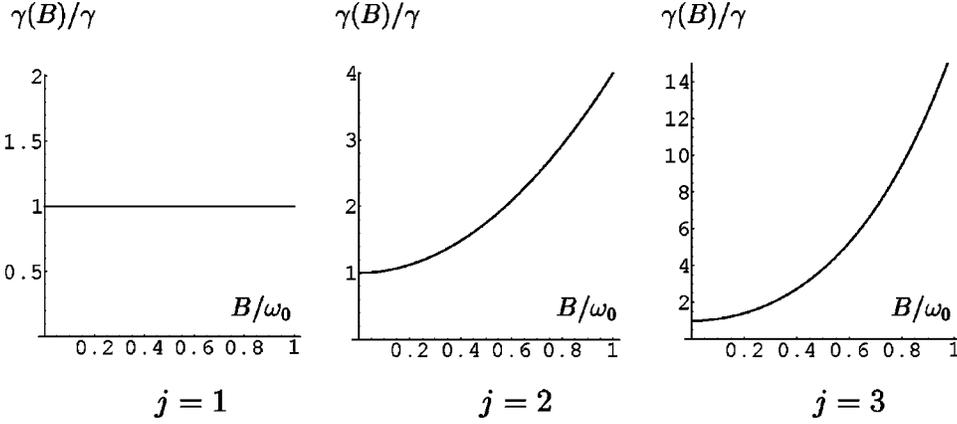


FIG. 4. The decay rate $\gamma(B)$ vs B , for electric transitions with $j=1,2,3$; $\gamma(B)$ is in units γ and B in units ω_0 . Notice the different scales on the vertical axis.

decreases as B increases [Fig. 3(a)]. For the sake of completeness we also notice that in case (b), i.e., for $B > \omega_0$, the pole would be on the second Riemann sheet (under the upper cut only) and the power series would converge in a circle lying half on the first and half on the second Riemann sheet, within a convergence radius $R_c = B - \omega_0$, which increases with B [Fig. 3(b)].

In either cases we can write, for $|s_{\text{pole}} + i\omega_0| < R_c = |B - \omega_0|$,

$$\begin{aligned}
 s_{\text{pole}} &= -i\omega_0 - \frac{1}{2} \{ Q[-i(\omega_0 + B) + 0^+] \\
 &\quad + Q[-i(\omega_0 - B) + 0^+] \} + O(g^4) \\
 &= -i\omega_0 - \frac{1}{2} g^2 \omega_0 \{ q[-i(\omega_0 + B) + 0^+] \\
 &\quad + q[-i(\omega_0 - B) + 0^+] \} + O(g^4). \quad (41)
 \end{aligned}$$

Equation (41) enables us to analyze the temporal behavior of state No. 2.

C. Decay rate vs B

We write, as in Eq. (36),

$$s_{\text{pole}} = -i\omega_0 + i\Delta E(B) - \frac{\gamma(B)}{2}. \quad (42)$$

Substituting Eq. (35) into Eq. (41) and taking the real part, one obtains the following expression for the decay rate:

$$\begin{aligned}
 \gamma(B) &= \pi g^2 \omega_0 [\chi^2(\omega_0 + B) + \chi^2(\omega_0 - B) \theta(\omega_0 - B)] \\
 &\quad + O(g^4). \quad (43)
 \end{aligned}$$

On the other hand, by Eq. (37), one can write

$$\gamma(B) = \gamma \frac{\chi^2(\omega_0 + B) + \chi^2(\omega_0 - B) \theta(\omega_0 - B)}{2\chi^2(\omega_0)} + O(g^4). \quad (44)$$

This is the central result of this paper and involves no approximations: Equation (44) expresses the ‘‘new’’ lifetime $\gamma(B)^{-1}$, when the system is bathed in an intense laser field

B , in terms of the ‘‘ordinary’’ lifetime γ^{-1} , when there is no laser field. By taking into account the general behavior (24) of the matrix elements $\chi^2(\omega)$ and substituting into Eq. (44), one gets to $O(g^4)$

$$\gamma(B) \simeq \frac{\gamma}{2} \left[\left(1 + \frac{B}{\omega_0} \right)^{2j\mp 1} + \left(1 - \frac{B}{\omega_0} \right)^{2j\mp 1} \theta(\omega_0 - B) \right] \quad (B \ll \Lambda), \quad (45)$$

where \mp refers to 1-2 transitions of electric and magnetic type, respectively. Observe that, since $\Lambda \sim$ inverse Bohr radius, only the case $B < \omega_0 \ll \Lambda$ is the physically relevant one [12]. The decay rate is profoundly modified by the presence of the laser field. Its behavior is shown in Fig. 4 for a few values of j . In general, for $j > 1$ (1-2 transitions of electric quadrupole, magnetic dipole or higher), the decay rate $\gamma(B)$ increases with B , so that the lifetime $\gamma(B)^{-1}$ decreases as B is increased. If one looks at B as the strength of the ‘‘observation’’ performed by the laser beam on level No. 2 [3], one can view this phenomenon as an ‘‘inverse’’ quantum Zeno effect, for decay is *enhanced* (rather than suppressed) by observation.

As we shall see in Secs. IV and V, the emitted photons have different frequencies [for they correspond to decay onto different dressed (Fano) states]. By selecting the photon wavelength (i.e., by means of filters), one could therefore also measure the different contributions to the inverse lifetime in Eqs. (44),(45). We shall come back to this point later.

As already emphasized, Eq. (45) is valid for $B \ll \Lambda$. In the opposite (unphysical) case $B \gg \Lambda$, by Eqs. (24) and (44), one gets to $O(g^4)$

$$\gamma(B) \simeq \frac{\gamma}{2} \frac{\chi^2(B)}{\chi^2(\omega_0)} \propto (B/\Lambda)^{-\beta} \quad (B \gg \Lambda). \quad (46)$$

This result is similar to that obtained in Ref. [3]. If such high values of B were experimentally obtainable, the decay would be considerably hindered and B could be properly viewed as the ‘‘strength’’ of the observation performed by the laser field on level no. 2 (quantum Zeno effect). However, in such a case, many additional effects would have to be considered and our analysis should be modified in order to take them

into account. A similar remark was made by Kofman and Kurizki in a different context [13].

A final remark is now in order. If one would use the approximation (38) in Eq. (39), in order to evaluate the new lifetime, i.e., if one sets $Q(s) = Q(s_{\text{pole}}) = \text{const}$, one would obtain $Q(B, s) = Q(s) = Q(s_{\text{pole}})$, i.e., no B dependence. Therefore, the effect we are discussing is ultimately due to the nonexponential contributions arising from the cut. In particular, viewed from the perspective of the time domain, this effect is ascribable to the quadratic short-time behavior of the $2 \rightarrow 1$ decay.

D. Estimates

We saw in the previous subsection that the ratio B/ω_0 is the relevant quantity in the evaluation of the modified lifetime. Let us therefore try to get a rough feeling for the magnitude of the relevant physical parameters. In order to affect significantly the lifetime of level No. 2, we have to look at rather large values of B : for instance at 1-3 transition of the electric dipole type. In such a case, Eq. (28) applies

$$B^2 = 2\pi\alpha\Omega_0 |\boldsymbol{\epsilon}_{k\theta\lambda_0}^* \cdot \mathbf{x}_{13}|^2 n_0. \quad (47)$$

Considering the angle average

$$\langle |\boldsymbol{\epsilon}_{k\theta\lambda_0}^* \cdot \mathbf{x}_{13}|^2 \rangle = \frac{1}{3} |\mathbf{x}_{13}|^2 \quad (48)$$

and remembering that the decay rate is

$$\Gamma_{13} = \frac{4}{3} \alpha |\mathbf{x}_{13}|^2 \Omega_0^3, \quad (49)$$

we obtain

$$B^2 = \frac{\pi}{2} n_0 \frac{\Gamma_{13}}{\Omega_0^2}, \quad (50)$$

which, reinserting c 's and \hbar 's, reads

$$B^2 = \frac{\pi}{2} n_0 \hbar \Omega_0 \frac{c^3}{\Omega_0^3} \hbar \Gamma_{13} = (n_0 \hbar \Omega_0) \frac{\lambda_L^3}{16\pi^2} (\hbar \Gamma_{13}), \quad (51)$$

where $\lambda_L = 2\pi c/\Omega_0$. The quantity B^2 has dimensions of squared energy and is given by the product of the energy of the laser field contained in the volume $\lambda_L^3/16\pi^2$ times the energy spread of the 1-3 transition Ω_0 . Therefore B depends on both laser and atomic system. Observe that $n_0 \lambda_L^3$ is the number of laser photons contained in the volume λ_L^3 .

In terms of laser power P and laser spot area A , Eq. (51) reads

$$B^2 = \frac{P}{cA} \frac{\lambda_L^3}{16\pi^2} (\hbar \Gamma_{13}) = 132 \frac{P \lambda_L^3}{A} (\hbar \Gamma_{13}) \text{ eV}^2, \quad (52)$$

where P is expressed in Watt, λ_L in μm , A in μm^2 and $\hbar\Gamma$ in eV. In Eq. (52) the quantity B is expressed in suitable units and can be easily compared to ω_0 [the ratio B/ω_0 being

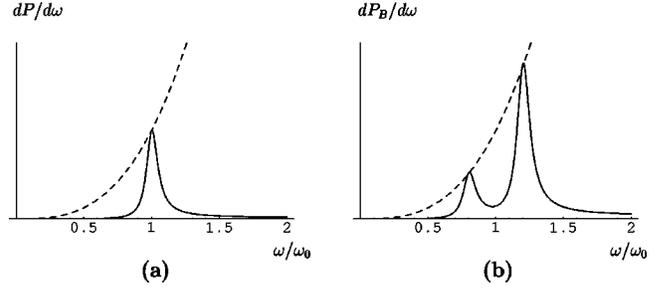


FIG. 5. The spectrum (57) of the emitted photons. The height of the Lorentzians is proportional to the matrix element $\chi^2(\omega)$ (dashed line). We chose an electric quadrupole transition, with $j=2$ and $\gamma = 10^{-1} \bar{\omega}_0$, and used arbitrary units on the vertical axis. (a) $B=0$; (b) $B = \bar{\omega}_0/5$; note that from Eq. (45) $\gamma(B) = (28/25) \gamma$.

the relevant quantity in Eq. (45)]. For laser intensities that are routinely used in the study of electromagnetic induced transparency, the effect should be experimentally observable. For a quick comparison remember that B is just half the Rabi frequency of the resonant transition 1-3 [see paragraph following Eq. (22)].

IV. PHOTON SPECTRUM

It is interesting to look at the spectrum of the emitted photons. It is easy to check that, in the Weisskopf-Wigner approximation, the survival probability $|\langle \psi(0) | \psi(t) \rangle|^2 = |x(t)|^2$ decreases exponentially with time. The standard way to obtain this result is to neglect the cut contribution in the complex s plane, or equivalently, to substitute in Eq. (18) the pole determination of the self-energy function

$$\tilde{x}(s) = \frac{1}{s + i\omega_0 + Q(B, s)} \approx \frac{1}{s + i\omega_0 + Q(B, s_{\text{pole}})} = \frac{1}{s - s_{\text{pole}}}, \quad (53)$$

from which one gets

$$x(t) = \exp(s_{\text{pole}} t) = \exp\left(-i\bar{\omega}_0 t - \frac{\gamma(B)}{2} t\right), \quad (54)$$

where $\bar{\omega}_0 = \omega_0 - \Delta E(B)$. In this approximation, for any value of B , the spectrum of the emitted photons is Lorentzian. The proof is straightforward and is given in Appendix B. One finds that, for $B=0$, the probability to emit a photon in the range $(\omega, \omega + d\omega)$ reads

$$dP_{B=0} = g^2 \omega_0 \chi^2(\omega) f_L(\omega - \bar{\omega}_0; \gamma) d\omega, \quad (55)$$

where

$$f_L(\omega; \gamma) = \frac{1}{\omega^2 + \gamma^2/4}. \quad (56)$$

On the other hand, when $B \neq 0$ one gets

$$dP_B = g^2 \omega_0 \chi^2(\omega) \frac{1}{2} \{f_L[\omega - \bar{\omega}_0 - B; \gamma(B)] + f_L[\omega - \bar{\omega}_0 + B; \gamma(B)]\} d\omega. \quad (57)$$

The emission probability is given by the sum of two Lorentzians, centered in $\bar{\omega}_0 \pm B$. We see that the emission probability of a photon of frequency $\bar{\omega}_0 + B$ ($\bar{\omega}_0 - B$) increases (decreases) with B (Fig. 5). The linewidths are modified according to Eq. (45). When B reaches the ‘‘threshold’’ value $\bar{\omega}_0$, only the photon of higher frequency ($\bar{\omega}_0 + B$) is emitted (with increasing probability vs B).

Photons of different frequencies are therefore emitted with different rates. We shall understand better the features of the emission in the next section, by looking at the dressed states of the system.

V. DRESSED STATES AND LINKS WITH INDUCED TRANSPARENCY

It is useful and interesting to look at our results from a different perspective, by analyzing the modifications of the energy levels provoked by the laser field. For simplicity, since the average number \bar{N}_0 of \mathbf{k}_0 photons in the total volume V can be considered very large, we shall perform our analysis in terms of number (rather than coherent) states of the electromagnetic field. In this limit,

$$\begin{aligned} & \langle 1; 0_{k\lambda}, N_0 | H_{\text{int}} | 3; 0_{k\lambda}, N_0 - 1 \rangle \\ &= \sqrt{N_0} \Phi_{\mathbf{k}_0 \lambda_0} \\ &\gg \langle 1; 1_{k\lambda}, N_0 - 1 | H_{\text{int}} | 3; 0_{k\lambda}, N_0 - 1 \rangle \\ &= \Phi_{k\lambda}, \end{aligned} \quad (58)$$

$\forall (\mathbf{k}, \lambda) \neq (\mathbf{k}_0, \lambda_0)$. [This is equivalent to Eq. (6).] In the above equation and henceforth, the vector $|i; n_{k\lambda}, M_0\rangle$ represents an atom in state $|i\rangle$, with $n_{k\lambda}(\mathbf{k}, \lambda)$ photons and M_0 laser photons.

In the above approximation, the Hamiltonian (1) becomes

$$\begin{aligned} H \simeq & \omega_0 |2\rangle\langle 2| + \Omega_0 |3\rangle\langle 3| + \sum_{\mathbf{k}, \lambda} \omega_{\mathbf{k}} a_{\mathbf{k}\lambda}^\dagger a_{\mathbf{k}\lambda} \\ & + \sum'_{\mathbf{k}, \lambda} (\phi_{\mathbf{k}\lambda} a_{\mathbf{k}\lambda}^\dagger |1\rangle\langle 2| + \phi_{\mathbf{k}\lambda}^* a_{\mathbf{k}\lambda} |2\rangle\langle 1|) \\ & + (\Phi_{\mathbf{k}_0 \lambda_0} a_{\mathbf{k}_0 \lambda_0}^\dagger |1\rangle\langle 3| + \Phi_{\mathbf{k}_0 \lambda_0}^* a_{\mathbf{k}_0 \lambda_0} |3\rangle\langle 1|), \end{aligned} \quad (59)$$

where a prime means that the summation does not include $(\mathbf{k}_0, \lambda_0)$ [due to hypothesis (3)]. In addition to Eq. (8), there is now another conserved quantity: indeed the operator

$$\mathcal{N}_0 = |3\rangle\langle 3| + a_{\mathbf{k}_0 \lambda_0}^\dagger a_{\mathbf{k}_0 \lambda_0} \quad (60)$$

satisfies

$$[H, \mathcal{N}_0] = [\mathcal{N}_0, \mathcal{N}] = 0. \quad (61)$$

In this case, the system evolves in the subspace labeled by the two eigenvalues $\mathcal{N}=1$ and $\mathcal{N}_0=N_0$, whose states read

$$\begin{aligned} |\psi(t)\rangle = & x(t) |2; 0, N_0\rangle + \sum'_{\mathbf{k}, \lambda} y_{\mathbf{k}\lambda}(t) |1; 1_{\mathbf{k}\lambda}, N_0\rangle \\ & + \sum'_{\mathbf{k}, \lambda} z_{\mathbf{k}\lambda}(t) |3; 1_{\mathbf{k}\lambda}, N_0 - 1\rangle. \end{aligned} \quad (62)$$

By using the Hamiltonian (59) and the states (62) and identifying N_0 with $\bar{N}_0 = |\alpha_0|^2$, the Schrödinger equation yields again the equations of motion (13), obtained by assuming a coherent state for the laser mode. Our analysis is therefore independent of the statistics of the driving field, provided it is sufficiently intense, and the (convenient) use of number states is completely justified.

Energy conservation implies that if there are two emitted photons with different energies (as we saw in the previous section), there are two levels of different energies to which the atom can decay. This can be seen by considering the laser-dressed (Fano) atomic states [30]. The shift of the dressed states can be obtained directly from the structure of the Hamiltonian (59). In the sector $\mathcal{N}_0=N_0$, the operator \mathcal{N}_0 is proportional to the unit operator, the constant of proportionality being its eigenvalue. Hence one can write the Hamiltonian in the following form:

$$H = H - \Omega_0 \mathcal{N}_0 + \Omega_0 N_0, \quad (63)$$

which, by the setting $E_1 + N_0 \Omega_0 = 0$, reads

$$\begin{aligned} H = & H_0 + H_{\text{int}} \\ = & \omega_0 |2\rangle\langle 2| + \sum'_{\mathbf{k}, \lambda} \omega_{\mathbf{k}} a_{\mathbf{k}\lambda}^\dagger a_{\mathbf{k}\lambda} \\ & + \sum'_{\mathbf{k}, \lambda} (\phi_{\mathbf{k}\lambda} a_{\mathbf{k}\lambda}^\dagger |1\rangle\langle 2| + \phi_{\mathbf{k}\lambda}^* a_{\mathbf{k}\lambda} |2\rangle\langle 1|) \\ & + (\Phi_{\mathbf{k}_0 \lambda_0} a_{\mathbf{k}_0 \lambda_0}^\dagger |1\rangle\langle 3| + \Phi_{\mathbf{k}_0 \lambda_0}^* a_{\mathbf{k}_0 \lambda_0} |3\rangle\langle 1|). \end{aligned} \quad (64)$$

On the other hand, in the sector $\mathcal{H}_{\mathcal{N}\mathcal{N}_0}$ with $\mathcal{N}=1$ and $\mathcal{N}_0=N_0$, the last term becomes

$$\begin{aligned} & (\Phi_{\mathbf{k}_0 \lambda_0} a_{\mathbf{k}_0 \lambda_0}^\dagger |1\rangle\langle 3| + \Phi_{\mathbf{k}_0 \lambda_0}^* a_{\mathbf{k}_0 \lambda_0} |3\rangle\langle 1|) \\ = & (\Phi_{\mathbf{k}_0 \lambda_0} \sqrt{N_0} |1\rangle\langle 3| + \Phi_{\mathbf{k}_0 \lambda_0}^* \sqrt{N_0} |3\rangle\langle 1|). \end{aligned} \quad (65)$$

Let us diagonalize this operator, i.e. let us look for two non-interacting states $|+\rangle$ and $|-\rangle$ which are linear combinations of the old states $|1\rangle$ and $|3\rangle$. To this end we write

$$\begin{aligned} |1\rangle = & \frac{1}{\sqrt{2}} (|+\rangle + e^{i\delta} |-\rangle), \\ |3\rangle = & \frac{e^{i\alpha}}{\sqrt{2}} (|+\rangle - e^{i\delta} |-\rangle), \end{aligned} \quad (66)$$

with $|+\rangle$ and $|-\rangle$ orthonormal:

$$\langle +|+\rangle = \langle -|- \rangle = 1, \quad \langle +|- \rangle = 0. \quad (67)$$

Plugging Eq. (66) into Eq. (64), the interaction term becomes

$$\begin{aligned} H_{\text{int}} = & \sum'_{k,\lambda} \left[\left(\frac{\phi_{k\lambda}}{\sqrt{2}} a_{k\lambda}^\dagger |+\rangle \langle 2| + \frac{\phi_{k\lambda}^*}{\sqrt{2}} a_{k\lambda} |2\rangle \langle +| \right) \right. \\ & + \left. \left(\frac{\phi_{k\lambda}}{\sqrt{2}} e^{i\delta} a_{k\lambda}^\dagger |-\rangle \langle 2| + \frac{\phi_{k\lambda}^*}{\sqrt{2}} e^{-i\delta} a_{k\lambda} |2\rangle \langle -| \right) \right] \\ & + B \left[\frac{e^{i\beta}}{2} e^{-i\alpha} (|+\rangle + e^{i\delta} |-\rangle) (\langle +| - e^{-i\delta} \langle -|) + \text{H.c.} \right], \end{aligned} \quad (68)$$

where we have set $\Phi_{k_0\lambda_0} \sqrt{N_0} = B e^{i\beta}$. Rearranging the last term

$$\begin{aligned} B \left[\frac{e^{i(\beta-\alpha)}}{2} (|+\rangle \langle +| - |-\rangle \langle -| + e^{i\delta} |-\rangle \langle +| \right. \\ \left. - e^{-i\delta} |+\rangle \langle -|) + \frac{e^{-i(\beta-\alpha)}}{2} (|+\rangle \langle +| \right. \\ \left. - |-\rangle \langle -| + e^{-i\delta} |+\rangle \langle -| - e^{i\delta} |-\rangle \langle +|) \right] \\ = B \cos(\beta-\alpha) (|+\rangle \langle +| - |-\rangle \langle -|) \\ + iB \sin(\beta-\alpha) (e^{i\delta} |-\rangle \langle +| - e^{-i\delta} |+\rangle \langle -|), \end{aligned} \quad (69)$$

and setting $\alpha = \beta$ the two states $|+\rangle$ and $|-\rangle$ decouple and one gets

$$\begin{aligned} H_{\text{int}} = & \sum'_{k,\lambda} \left[\left(\frac{\phi_{k\lambda}}{\sqrt{2}} a_{k\lambda}^\dagger |+\rangle \langle 2| + \frac{\phi_{k\lambda}^*}{\sqrt{2}} a_{k\lambda} |2\rangle \langle +| \right) \right. \\ & + \left. \left(\frac{\phi_{k\lambda}}{\sqrt{2}} e^{i\delta} a_{k\lambda}^\dagger |-\rangle \langle 2| + \frac{\phi_{k\lambda}^*}{\sqrt{2}} e^{-i\delta} a_{k\lambda} |2\rangle \langle -| \right) \right] \\ & + B |+\rangle \langle +| - B |-\rangle \langle -|. \end{aligned} \quad (70)$$

Therefore we can write

$$H_0 + H_{\text{int}} = H'_0 + H'_{\text{int}}, \quad (71)$$

where the primed free and interaction Hamiltonians read, respectively,

$$\begin{aligned} H'_0 = & \omega_0 |2\rangle \langle 2| + B |+\rangle \langle +| - B |-\rangle \langle -| + \sum'_{k,\lambda} \omega_k a_{k\lambda}^\dagger a_{k\lambda}, \\ H'_{\text{int}} = & \sum'_{k,\lambda} \left[\left(\frac{\phi_{k\lambda}}{\sqrt{2}} a_{k\lambda}^\dagger |+\rangle \langle 2| + \frac{\phi_{k\lambda}^*}{\sqrt{2}} a_{k\lambda} |2\rangle \langle +| \right) \right. \\ & + \left. \left(\frac{\phi_{k\lambda}}{\sqrt{2}} a_{k\lambda}^\dagger |-\rangle \langle 2| + \frac{\phi_{k\lambda}^*}{\sqrt{2}} a_{k\lambda} |2\rangle \langle -| \right) \right] \end{aligned} \quad (72)$$

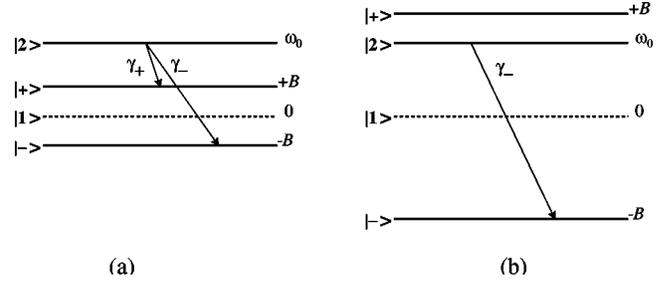


FIG. 6. Shift of the dressed states $|+\rangle$ and $|-\rangle$ vs B . (a) For $B < \omega_0$ there are two decay channels, with $\gamma_- > \gamma_+$. (b) For $B > \omega_0$ level $|+\rangle$ is above level $|2\rangle$ and only the γ_- decay channel remains.

and we set $\delta = 0$. We see that the laser dresses the states $|1\rangle$ and $|3\rangle$, which (if one includes the Ω_0 photon) are degenerate [with energy $E = 0$, due to the choice of the zero of energy: see line after Eq. (63)]. The dressed states $|+\rangle$ and $|-\rangle$ have energies $+B$ and $-B$ and interact with state $|2\rangle$ with a coupling $\phi_{k\lambda}/\sqrt{2}$. Since $2B = \Omega_{\text{Rabi}}$ these are the well-known Autler-Townes doublet [31].

Therefore, by applying the Fermi golden rule, the decay rates into the dressed states read

$$\gamma_+ = 2\pi g^2 \omega_0 \frac{\chi^2(\omega_0 - B)}{2}, \quad \gamma_- = 2\pi g^2 \omega_0 \frac{\chi^2(\omega_0 + B)}{2} \quad (73)$$

and the total decay rate of state $|2\rangle$ is given by their sum

$$\gamma = \gamma_+ + \gamma_-, \quad (74)$$

which yields Eq. (43). One sees why there is a threshold at $B = \omega_0$: For $B < \omega_0$, the energies of both dressed states $|\pm\rangle$ are lower than that of the initial state $|2\rangle$ [Fig. 6(a)]. The decay rate γ_- increases with B , whereas γ_+ decreases with B ; their sum γ increases with B . These two decays (and their lifetimes) could be easily distinguished by selecting the frequencies of the emitted photons, e.g., by means of filters.

We also notice, for completeness, that when $B > \omega_0$, the energy of the dressed state $|+\rangle$ is larger than that of state $|2\rangle$ and this decay channel disappears [Fig. 6(b)]. As repeatedly emphasized, this situation would require a different analysis, for additional effects would play an important role.

Finally, let us emphasize that if state $|2\rangle$ were *below* state $|1\rangle$, our system would become a three-level system in a ladder configuration, and the shift of the dressed states would give rise to electromagnetically induced transparency [26]. The situation we consider and the laser power required to bring these effects to light are therefore similar to those used in induced transparency.

VI. INFLUENCE OF OTHER LEVELS

Let us now see how our results are modified by the presence of off-resonant levels. To this end we generalize the three-level Hamiltonian (7) by including other off-resonant levels $|j\rangle$ ($j = 4, \dots, N$) in our analysis:

$$\begin{aligned}
 H = & \omega_0|2\rangle\langle 2| + \sum_{j=3}^N \Omega_j|j\rangle\langle j| + \sum_{k,\lambda} \omega_k a_{k\lambda}^\dagger a_{k\lambda} \\
 & + \sum_{k,\lambda}' (\phi_{k\lambda} a_{k\lambda}^\dagger |1\rangle\langle 2| + \phi_{k\lambda}^* a_{k\lambda} |2\rangle\langle 1|) \\
 & + \sum_{j=3}^N (\Phi_j \alpha_0^* e^{i\Omega_3 t} |1\rangle\langle j| + \Phi_j^* \alpha_0 e^{-i\Omega_3 t} |j\rangle\langle 1|),
 \end{aligned} \tag{75}$$

where $\Phi_j = \Phi_{j,k_0\lambda_0}$ are the matrix elements of the $1 \leftrightarrow j$ transitions and $\Omega_j = E_j - E_1$ the energy of level $|j\rangle$ [in particular, $\Phi_{3,k_0\lambda_0} = \Phi_{k_0\lambda_0}$ and $\Omega_3 = \Omega_0$ in Eq. (7)].

By a calculation similar to that used in Sec. III one gets again the expression (18) for the Laplace transform of the survival amplitude, with the new self-energy function modified by the presence of other levels

$$Q(B,s) = \sum_{k,\lambda} \frac{|\phi_{k\lambda}|^2}{s + i\omega_k + B^2 \sum_{j=3}^N [f_j / (s + i\delta_j + i\omega_k)]}, \tag{76}$$

where $f_j = |\Phi_j|^2 / |\Phi_3|^2$ and $\delta_j = \Omega_j - \Omega_3$.

The denominator of $Q(B,s)$ is now a polynomial of order $N-1$ [when $N=3$ one reobtains Eq. (21) with a quadratic polynomial]. Hence the new $Q(B,s)$ in Eq. (76) has $N-1$ branching points and the property (39) is generalized to

$$\begin{aligned}
 Q(B,s) = & c_+ Q(s + i\sigma_+) + c_- Q(s + i\sigma_-) \\
 & + \sum_{j=4}^N c_j Q(s + i\sigma_j),
 \end{aligned} \tag{77}$$

where $\{-i\sigma_+, -i\sigma_-, -i\sigma_j\}$ ($j=4, \dots, N$) are the branching points, i.e., the zeroes of the denominator of $Q(B,s)$. In this case one has to solve an algebraic equation of $(N-1)$ th order, whose zeroes do not have in general an analytical expression. We seek a perturbative solution in B . It is lengthy, but straightforward, to obtain up to second order in B

$$\begin{cases} \sigma_\pm = \pm B - B^2 \sum_{j=4}^N \frac{f_j}{2\delta_j}, \\ \sigma_j = \delta_j + B^2 \frac{f_j}{\delta_j}, \end{cases} \tag{78}$$

$$\begin{cases} c_\pm = \frac{1}{2} \mp B \sum_{j=4}^N \frac{f_j}{4\delta_j} - B^2 \sum_{j=4}^N \frac{f_j}{2\delta_j^2}, \\ c_j = B^2 \frac{f_j}{\delta_j^2}. \end{cases} \tag{78}$$

From the above equations we see that the presence of off-resonant levels modifies the energies $\sigma_\pm = \pm B$ of the two dressed states by a shift of order B^2 and creates $N-3$ new

dressed states with energies $\delta_j \approx \Omega_j - \Omega_3$, whose contribution to the self-energy function is of order B^2 .

By starting with the self-energy function (77) and looking for the location of the pole one obtains instead of Eq. (45) the following expression for the modified decay rate:

$$\begin{aligned}
 \gamma_{\text{many}}(B) = & \gamma \left[c_- \left(1 - \frac{\sigma_-}{\omega_0} \right)^\kappa + c_+ \left(1 - \frac{\sigma_+}{\omega_0} \right)^\kappa \theta(\omega_0 - \sigma_+) \right. \\
 & \left. + \sum_{\ell=4}^N c_\ell \left(1 - \frac{\sigma_\ell}{\omega_0} \right)^\kappa \theta(\omega_0 - \sigma_\ell) \right],
 \end{aligned} \tag{79}$$

where $\kappa = 2j \mp 1$.

By substituting the expressions (78) for the zeroes and the coefficients, valid up to second order in B , into Eq. (79) one gets

$$\begin{aligned}
 \gamma_{\text{many}}(B) = & \gamma \left\{ 1 + \kappa \frac{B^2}{\omega_0^2} + \frac{B^2}{\omega_0^2} \sum_{\ell=4}^N f_\ell \left[\left(\kappa \frac{\omega_0}{\delta_\ell} - \frac{\omega_0^2}{\delta_\ell^2} \right) \right. \right. \\
 & \left. \left. - \left(\frac{\omega_0}{\delta_\ell} - \frac{\omega_0^2}{\delta_\ell^2} \right) \theta(\omega_0 - \delta_\ell) \right] \right\} \\
 = & \gamma(B) + \gamma \frac{B^2}{\omega_0^2} \sum_{\ell=4}^N f_\ell \left[\left(\kappa \frac{\omega_0}{\delta_\ell} - \frac{\omega_0^2}{\delta_\ell^2} \right) \right. \\
 & \left. - \left(\frac{\omega_0}{\delta_\ell} - \frac{\omega_0^2}{\delta_\ell^2} \right) \theta(\omega_0 - \delta_\ell) \right] + O(B^3),
 \end{aligned} \tag{80}$$

where $\gamma(B)$ is the decay rate (45) evaluated in the three-level approximation.

The above general expression can be evaluated in practical cases of interest. For instance, by assuming that the off-resonant levels are well separated from the three main levels, that is by assuming $\delta_\ell = \Omega_\ell - \Omega_3 > \omega_0$, all dressed states other than $|\pm\rangle$ do not enter in Eq. (79), because their energies are larger than the energy ω_0 of level $|2\rangle$, and Eq. (80) reads

$$\gamma_{\text{many}}(B) \approx \gamma \left[1 + \kappa \frac{B^2}{\omega_0^2} \left(1 + \sum_{\ell=4}^N f_\ell \frac{\omega_0}{\delta_\ell} \right) \right] = \gamma(B^*), \tag{81}$$

where

$$B^* = B \left[1 + \sum_{\ell=4}^N f_\ell \frac{\omega_0}{2\delta_\ell} \right]. \tag{82}$$

This is the correction sought: the effect of sufficiently off-resonant levels, $\delta_\ell > \omega_0$, modifies the decay rate (45), calculated in the three-level approximation, simply by changing B into B^* . Observe that f_ℓ is a rapidly decreasing function of ℓ (polynomial fall-off in atomic systems). Notice also that $B^* > B$, hence the presence of the other levels enhances the effect discussed in Sec. III.

VII. CONCLUDING REMARKS

We have studied the evolution of an unstable system under the action of an external (laser) field. The dynamical evolution of level No. 2 (initial state) is modified by the laser field, tuned at the transition frequency 1-3. For physically sensible values of the parameters, the decay of level No. 2 is *faster* when the laser is present. Equations (44),(45) (valid to fourth order in the coupling constant) express the new lifetime as a function of the “natural” one and other parameters characterizing the physical system. The initial state decays to the laser-dressed states with different lifetimes. We have obtained Eq. (44) in three different ways, deriving the Fermi golden rule from the time-dependent Schrödinger equation, by making use of Laplace transforms, as in Sec. III C, or starting from the dressed states, as in Eqs. (73),(74), or as a consequence of a normalization condition, as in Eq. (B13). We also computed, in Sec. VI, the corrections due to off-resonant levels. We emphasize that, since we always work in the Weisskopf-Wigner approximation, the conceptual problems related to state preparation [22] and deviation from exponential behavior [20,18,19] were not considered.

In which sense is the phenomenon discussed in this paper an “inverse” quantum Zeno effect? If the situation $B \gg \Lambda$ were experimentally attainable, then decay would be hindered and one could reasonably speak of a quantum Zeno effect provoked by the “continuous” observation performed on the system by the laser beam. On the other hand, when $B \ll \Lambda$, one can still think in terms of a “continuous gaze” of the laser on the system, but this enhances (rather than hinders) decay. One should also notice that the inclusion of the spontaneous decay of level No. 3 in the Hamiltonian (7) would not change our conclusion (up to order Γ_{13}/B). The interpretation in terms of an “inverse” quantum Zeno effect is appealing and enables one to look at the problem from a different perspective.

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APPENDIX A

We discuss here some general properties of the matrix elements and derive Eqs. (24),(25) of the text. An exhaustive analysis of some general features of the matrix elements can be found in Ref. [29], but we will focus here on the behavior at small and large values of ω . The matrix elements (2) of the 1-2 interaction Hamiltonian read

$$\phi_{k\lambda} = \sqrt{\frac{2\pi\alpha}{V\omega}} \int d^3x e^{-ik \cdot x} \boldsymbol{\epsilon}_{k\lambda}^* \cdot \mathbf{j}_{12}(\mathbf{x}), \quad (\text{A1})$$

where $\alpha = e^2/4\pi\epsilon_0$ is the fine-structure constant. If the wavelength of the radiation is large compared to the size a of the system (i.e., $\omega \ll \Lambda = a^{-1}$) the main contribution to the integral (A1) comes from small values of $r = |\mathbf{x}|$ ($\omega r \ll 1$). Expanding the exponential ($\mathbf{k} = \mathbf{n}\omega$)

$$\begin{aligned} \exp(-i\mathbf{k} \cdot \mathbf{x}) &\equiv \exp(-i\omega \mathbf{n} \cdot \mathbf{x}) \\ &= 1 - i\omega(\mathbf{n} \cdot \mathbf{x}) + \frac{(-i\omega)^2}{2!}(\mathbf{n} \cdot \mathbf{x})^2 + \dots \end{aligned} \quad (\text{A2})$$

and integrating term by term one obtains the asymptotic series

$$\phi_{k\lambda} \sim \sqrt{\frac{2\pi}{V}} \sqrt{\frac{\alpha}{\omega}} \sum_{s=0}^{\infty} q_{n\lambda}^{(s)} \omega^s, \quad (\text{A3})$$

where

$$q_{n\lambda}^{(s)} \equiv \frac{(-i)^s}{s!} \int d^3x \boldsymbol{\epsilon}_{n\lambda}^* \cdot \mathbf{j}_{12}(\mathbf{x}) (\mathbf{n} \cdot \mathbf{x})^s \quad (\text{A4})$$

($\boldsymbol{\epsilon}_{k\lambda} = \boldsymbol{\epsilon}_{n\lambda}$ depends only on the direction of \mathbf{k}). Notice that we explicitly wrote every ω dependence and that $q^{(s)}$ does not depend on ω . Observe that $q^{(0)}$ corresponds to electric-dipole transitions $E1$, $q^{(1)}$ to electric quadrupole $E2$ and/or magnetic dipole transitions $M1$, and so on. Hence $s = j - \lambda$, where $\lambda = 0$ ($\lambda = 1$) stands for magnetic (electric) transition $Mj(Ej)$. Since the dominant contribution to the integral in Eq. (A4) comes from a region of size a and the current j_{12} is essentially ω_0/a^2 , we get

$$q_{n\lambda}^{(s)} \propto \omega_0 a^{s+1}, \quad \omega_0 \equiv E_2 - E_1. \quad (\text{A5})$$

If $\omega a \ll 1$ the dominant term in the series (A3) is the first nonvanishing one, namely,

$$\phi_{k\lambda} \sim q_{n\lambda}^{(r)} \omega^r \propto (\omega_0 a)^r (\omega a)^r, \quad (\text{A6})$$

for some $s = r$. In the continuum limit one gets

$$\begin{aligned} \sum_{k,\lambda} |\phi_{k\lambda}|^2 &\rightarrow \frac{V}{(2\pi)^3} \sum_{\lambda} \int d^3k |\phi_{k\lambda}|^2 \\ &= \int_0^{\infty} d\omega \omega^2 \frac{V}{(2\pi)^3} \sum_{\lambda} \int d\Omega |\phi_{k\lambda}|^2 \\ &= \int_0^{\infty} d\omega g^2 \omega_0 \chi^2(\omega), \end{aligned} \quad (\text{A7})$$

where we have defined

$$g^2 \omega_0 \chi^2(\omega) \equiv \lim_{V \rightarrow \infty} \frac{\omega^2 V}{(2\pi)^3} \sum_{\lambda} \int d\Omega |\phi_{k\lambda}|^2, \quad (\text{A8})$$

as in Eq. (23). From Eq. (A6) we obtain

$$|\phi_{k\lambda}|^2 = \frac{2\pi}{V} \frac{\alpha}{\omega} \left| \sum_{r=0}^{\infty} q_{n\lambda}^{(r)} \omega^r \right|^2 \sim \frac{2\pi}{V} \alpha |q_{n\lambda}^{(r)}|^2 \omega^{2r-1} \quad (\text{A9})$$

and therefore

$$g^2 \omega_0 \chi^2(\omega) \sim \frac{\alpha}{(2\pi)^2} \left(\sum_{\lambda} \int d\Omega |q_{n\lambda}^{(r)}|^2 \right) \omega^{2r+1} \propto [\alpha(\omega_0 a)^{2r+2}] \omega_0 \left(\frac{\omega}{\omega_0} \right)^{2r+1}. \quad (\text{A10})$$

Remembering that $2r+1=2j-2\lambda+1=2j\mp 1$, we obtain the first equation in Eq. (24) and Eq. (25).

On the other hand, if the wavelength is much smaller than a (i.e. $\omega \gg \Lambda$), we first rewrite Eq. (A1) in the form

$$\begin{aligned} \phi_{k\lambda} &= \sqrt{\frac{2\pi\alpha}{V\omega}} \int d^3x e^{-i\omega n \cdot x} \mathbf{e}_{k\lambda}^* \cdot \mathbf{j}_{12}(x) \\ &= \sqrt{\frac{2\pi\alpha}{V\omega}} \int dx_{\parallel} e^{-i\omega x_{\parallel}} j_{n\lambda,12}(x_{\parallel}), \end{aligned} \quad (\text{A11})$$

where

$$j_{n\lambda,12}(x_{\parallel}) \equiv \int d^2x_{\perp} \mathbf{e}_{n\lambda}^* \cdot \mathbf{j}_{12}(x) \quad (\text{A12})$$

and $\mathbf{x} \equiv x_{\parallel} \mathbf{n} + \mathbf{x}_{\perp}$. According to the Riemann-Lesbegue lemma, the integral in Eq. (A11) vanishes in the $\omega \rightarrow \infty$ limit. In particular, if $j_{n\lambda,12}(x_{\parallel})$ is N times differentiable, integrating by parts we get

$$\phi_{k\lambda} = \sqrt{\frac{2\pi\alpha}{V\omega}} \frac{1}{(i\omega)^N} \int dx e^{-i\omega x} \frac{d^N}{dx^N} j_{n\lambda,12}(x) \quad (\text{A13})$$

and we can write

$$\phi_{k\lambda} = o(\omega^{-N-1/2}), \quad (\omega \gg \Lambda) \quad (\text{A14})$$

which yields the large ω behavior of the second equation in Eq. (24). It goes without saying that if $j_{n\lambda,12}(x_{\parallel})$ is an analytic function, then $\phi_{k\lambda} \rightarrow 0$ more rapidly than any power. The second equation in Eq. (24) is therefore a conservative estimate.

APPENDIX B

In this appendix we shall analyze the spectrum of the emitted photons. We start by substituting Eq. (53) into Eqs. (19) and (20), to obtain

$$\tilde{y}_{k\lambda}(s) = \frac{-i\phi_{k\lambda}(s+i\omega_k)}{(s+i\omega_k)^2+B^2} \frac{1}{s-s_{\text{pole}}}, \quad (\text{B1})$$

$$\tilde{z}_{k\lambda}(s) = -\frac{\sqrt{N_0} \Phi_{k_0\lambda_0}^* \phi_{k\lambda}}{(s+i\omega_k)^2+B^2} \frac{1}{s-s_{\text{pole}}}. \quad (\text{B2})$$

Closing the Bromwich path with a semicircle in the half plane $\text{Re } s < 0$, we get

$$y_{k\lambda}(t) = \frac{1}{2\pi i} \int_{\Gamma} ds e^{ts} \tilde{y}_{k\lambda}(s),$$

$$z_{k\lambda}(t) = \frac{1}{2\pi i} \int_{\Gamma} ds e^{ts} \tilde{z}_{k\lambda}(s), \quad (\text{B3})$$

which can be evaluated by summing over the integrand residues. The quantity $|y_{k\lambda}(t)|^2 (|z_{k\lambda}(t)|^2)$ represents the probability that, at time t , the transition $2 \rightarrow 1$ ($2 \rightarrow 1 \rightarrow 3$) has taken place. When $t \rightarrow \infty$, the contribution of s_{pole} (that has a finite negative real part) is exponentially damped. This leaves only the contributions of the poles in $-i(\omega_k \pm B)$.

We look first at the case $B=0$ (laser off). One gets ($z_{k\lambda} = 0, \forall t$)

$$|y_{k\lambda}(+\infty)|^2 = \frac{|\phi_{k\lambda}|^2}{(\omega_k - \bar{\omega}_0)^2 + \gamma^2/4} \quad (\text{B4})$$

and, in the continuum limit (23), the probability to emit a photon in the frequency range $(\omega, \omega+d\omega)$ reads

$$dP_{B=0} = g^2 \omega_0 \chi^2(\omega) f_L(\omega - \bar{\omega}_0; \gamma) d\omega, \quad (\text{B5})$$

where f_L is the Lorentzian profile

$$f_L(\omega; \gamma) = \frac{1}{\omega^2 + \gamma^2/4}. \quad (\text{B6})$$

This is Eq. (55) of the text. The quantity P must be normalized to unity: imposing this condition one gets the Fermi golden rule (37).

On the other hand, when $B \neq 0$, the total emission probability is given by the sum

$$|y_{k\lambda}(\infty)|^2 + |z_{k\lambda}(\infty)|^2 \quad (\text{B7})$$

and it is straightforward to derive the following expressions [$\nu_k = \omega_k - \bar{\omega}_0$ and we write for simplicity $\gamma(B) = \gamma$]

$$\begin{aligned} |y_{k\lambda}(\infty)|^2 &= \frac{|\phi_{k\lambda}|^2}{\left| \left(\nu_k + i\frac{\gamma}{2} \right)^2 - B^2 \right|^2} \left[\left(\nu_k^2 + \frac{\gamma^2}{4} \right) \cos^2(Bt) \right. \\ &\quad \left. + B^2 \sin^2(Bt) + \frac{\gamma B}{2} \sin(2Bt) \right], \\ |z_{k\lambda}(\infty)|^2 &= \frac{|\phi_{k\lambda}|^2}{\left| \left(\nu_k + i\frac{\gamma}{2} \right)^2 - B^2 \right|^2} \left[\left(\nu_k^2 + \frac{\gamma^2}{4} \right) \sin^2(Bt) \right. \\ &\quad \left. + B^2 \cos^2(Bt) - \frac{\gamma B}{2} \sin(2Bt) \right], \end{aligned} \quad (\text{B8})$$

which yield

$$|y_{k\lambda}(\infty)|^2 + |z_{k\lambda}(\infty)|^2 = \frac{|\phi_{k\lambda}|^2}{\left| \left(\nu_k + i\frac{\gamma}{2} \right)^2 - B^2 \right|^2} \left(\nu_k^2 + \frac{\gamma^2}{4} + B^2 \right). \quad (\text{B9})$$

Therefore, in the continuum limit, we can write

$$dP_B = g^2 \omega_0 \chi^2(\omega) \times \frac{(\omega - \bar{\omega}_0)^2 + \frac{\gamma^2}{4} + B^2}{\left[(\omega - \bar{\omega}_0 - B)^2 + \frac{\gamma^2}{4} \right] \left[(\omega - \bar{\omega}_0 + B)^2 + \frac{\gamma^2}{4} \right]} d\omega. \quad (\text{B10})$$

This formula can be rewritten in the following form:

$$dP_B = g^2 \omega_0 \chi^2(\omega) \frac{1}{2} [f_L(\omega - \bar{\omega}_0 - B; \gamma) + f_L(\omega - \bar{\omega}_0 + B; \gamma)] d\omega. \quad (\text{B11})$$

This is Eq. (57) of the text. We see that the emission probability is the sum of two Lorentzians, centered in $\bar{\omega}_0 - B$ and $\bar{\omega}_0 + B$ and weighted by $g^2 \omega_0 \chi^2(\omega)$. This result is in agreement with that obtained in Refs. [16,32]. Incidentally, we

notice that the value (43) of $\gamma(B)$ can be readily estimated by imposing the normalization of the emission probability

$$\int dP_B = \int_0^\infty g^2 \omega_0 \chi^2(\omega) \frac{1}{2} [f_L(\omega - \bar{\omega}_0 - B; \gamma) + f_L(\omega - \bar{\omega}_0 + B; \gamma)] d\omega = 1. \quad (\text{B12})$$

Performing the integration one obtains [$\gamma \ll \bar{\omega}_0$, hence one can integrate over the whole real axis and take $\chi^2(\omega)$ equal to its value on each Lorentzian peak]

$$1 = \int dP_B \approx \frac{1}{2} g^2 \omega_0 [\chi^2(\bar{\omega}_0 + B) + \chi^2(\bar{\omega}_0 - B)] \frac{2\pi}{\gamma(B)}, \quad (\text{B13})$$

which yields Eq. (43) of the text.

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