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ABSTRACTS

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Relaxation of Nelson processes toward quantum states

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The Nelson Stochastic Mechanics is a model which can simulate the quantum behaviour of a system by means of classical stochastic processes. In recent years it has also been used as a basis for the Thermal Wave Model for charged particle beam propagation in particle accelerators: a quantum-like description of the optics and the dynamics of such beams by means of a Schrödinger-like equation. We consider in the present paper the problem of the asymptotic convergence (in time) of non-equilibrium solutions of the classical Kolmogorov equations of these Nelson processes toward the quantum mechanical densities $|\psi|^2$ derived from the Schrödinger equation. This idea, suggested by a time-honored paper by Bohm and Vigier about the stochastic interpretation of quantum mechanics, is relevant since these non-equlibrium solutions are, in general, not observable and the required ergodicity property can help to understand how the *physical* solutions are selected by quantum mechanics among all the possible solutions. We choose the L^1 metrics, we prove a few general propositions about the time behaviour of the solutions and finally we elaborate e few examples by solving the Fokker-Planck equation. Besides some well known evolutions related to the Ornstein-Uhlenbeck process, we analyze a few less common cases by generalizing the classical results about expansions in orthogonal polynomials summarized, for example, in R.I.Cukier, K.Lakatos-Lindenberg e K.E.Shuler (J.Stat.Phys. 9 (1973) 137) to the case of singular velocity fields. We find that the choice of the L^1 metrics and of the Nelson stochastic flux are correct only for a particular class of quantum states, but can not be adopted in general. However it can be shown that by a suitable choice of a time-dependence of the diffusion parameter even the pathological examples can be made convergent. This artifice, which could be considered disturbing when the stochastic mechanics is used as a model for quantum mechanics (where the diffusion parameter is connected to the Plack constant), is surprisingly well suited for the Thermal Wave Model since the Schrödinger-like equation ruling the particle beam shows a time-dependent diffusion parameter connected to the beam emittance.