Hot and Dense QCD on the lattice

Frithjof Karsch, BNL

- Introduction:

  T, gT, g^2T,...

  screening and the running coupling

- Bulk thermodynamics

  T_c and the equation of state in (2+1)-flavor QCD

  with an almost realistic quark mass spectrum

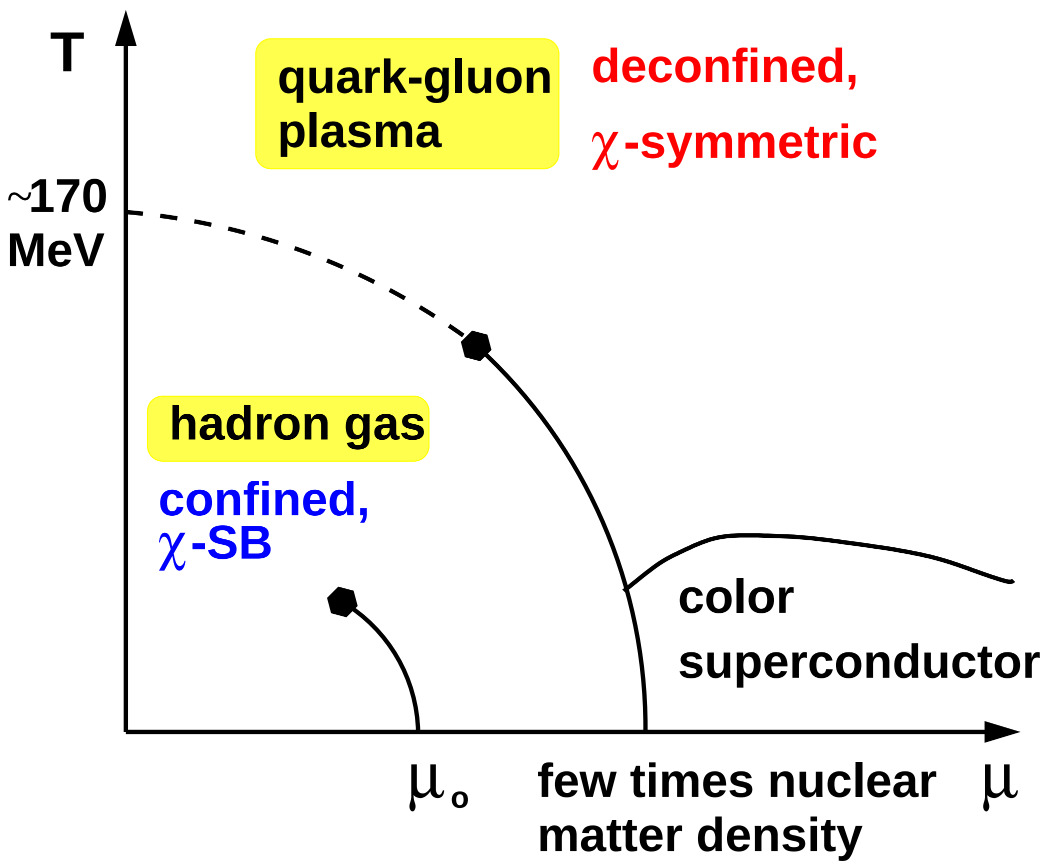
- Thermodynamics at non-zero baryon number density

  hadronic fluctuations

  isentropic equation of state

- Conclusions

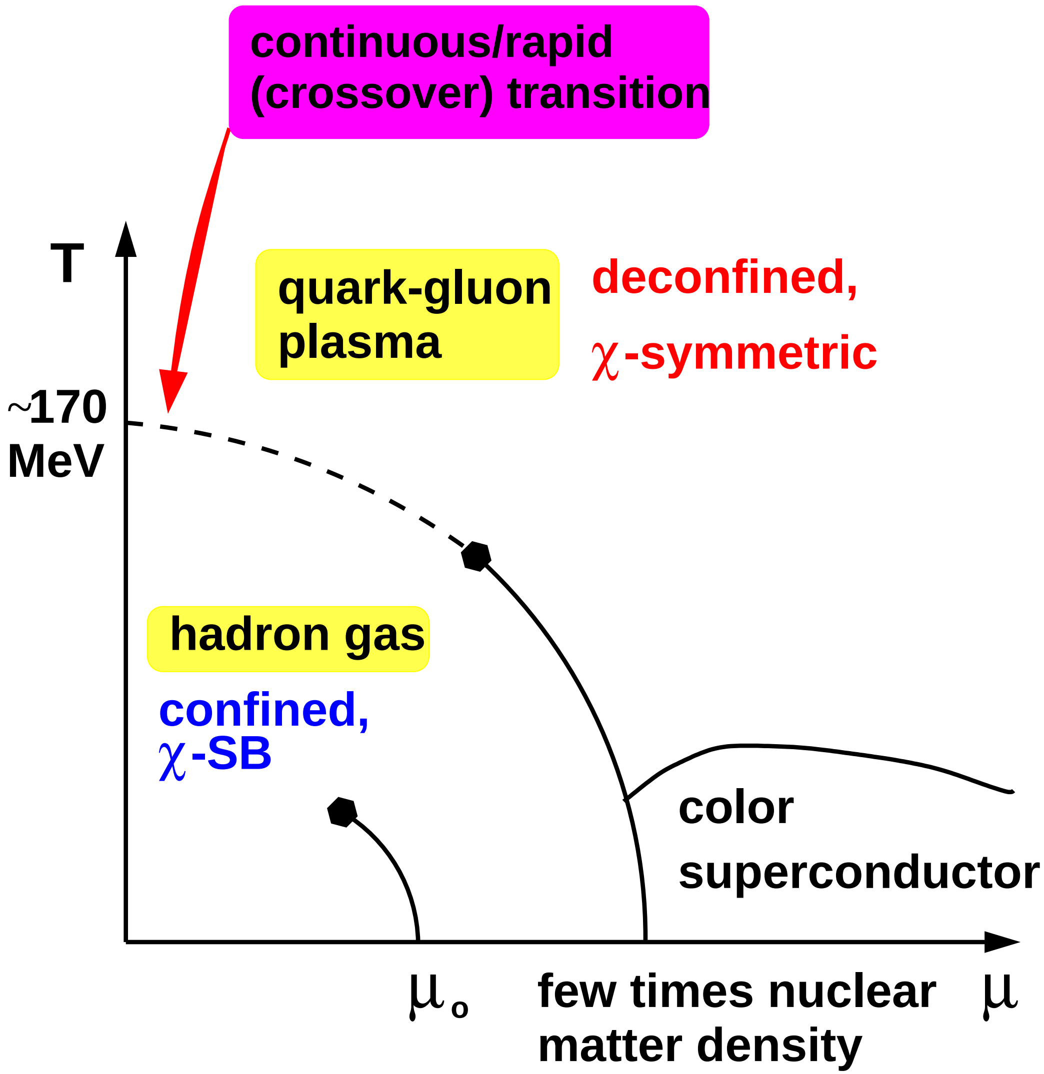
Critical behavior in hot and dense matter: QCD phase diagram



crossover vs.

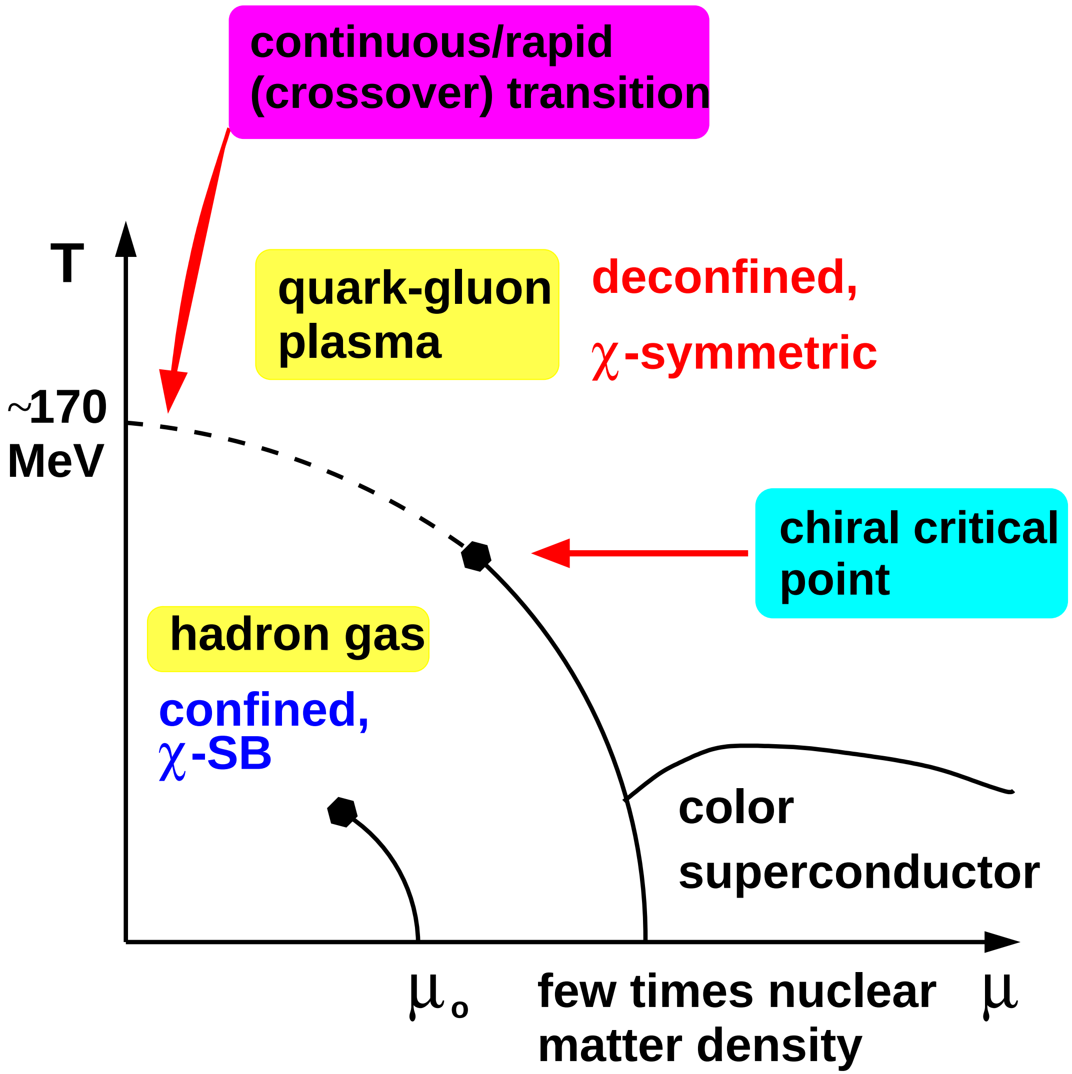
phase transition

Critical behavior in hot and dense matter: QCD phase diagram



continuous transition for
small chemical potential
and small quark masses

Critical behavior in hot and dense matter: QCD phase diagram

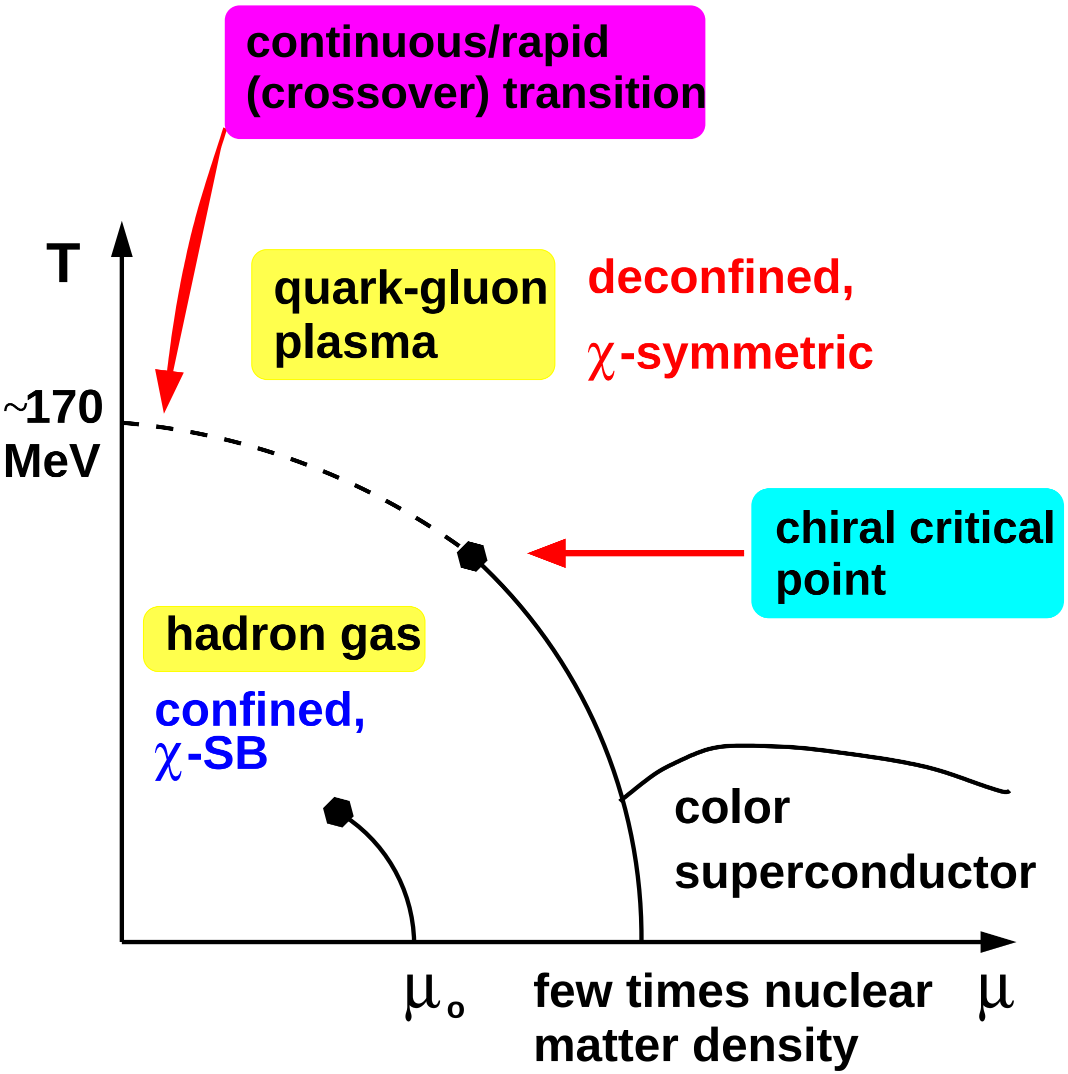


continuous transition for small chemical potential and small quark masses

2nd order phase transition;
Ising universality class
$T_c(\mu)$ under investigation

location of CCP uncertain:
volume and quark mass dependence

Critical behavior in hot and dense matter: QCD phase diagram



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location of CCP uncertain:
volume and quark mass dependence

improving accuracy on T_c, ϵ_c, $\epsilon(p)$ and the phase boundary is mandatory to make contact to HIC phenomenology

Non-perturbative QGP

- Perturbation theory provides a hierachy of length scales

  $T \gg gT \gg g^2T$...$\Rightarrow$ guiding principle for effective theories,

  resummation, dimensional reduction...

- Early lattice results show that $g^2(T) > 1$ even at $T \sim 5T_c$

  G. Boyd et al, NP B469 (1996) 419: SU(3) thermodynamics..

  ...one has to conclude that the temperature dependent running coupling has to be large, $g^2(T) \simeq 2$ even at $T \simeq 5T_c$

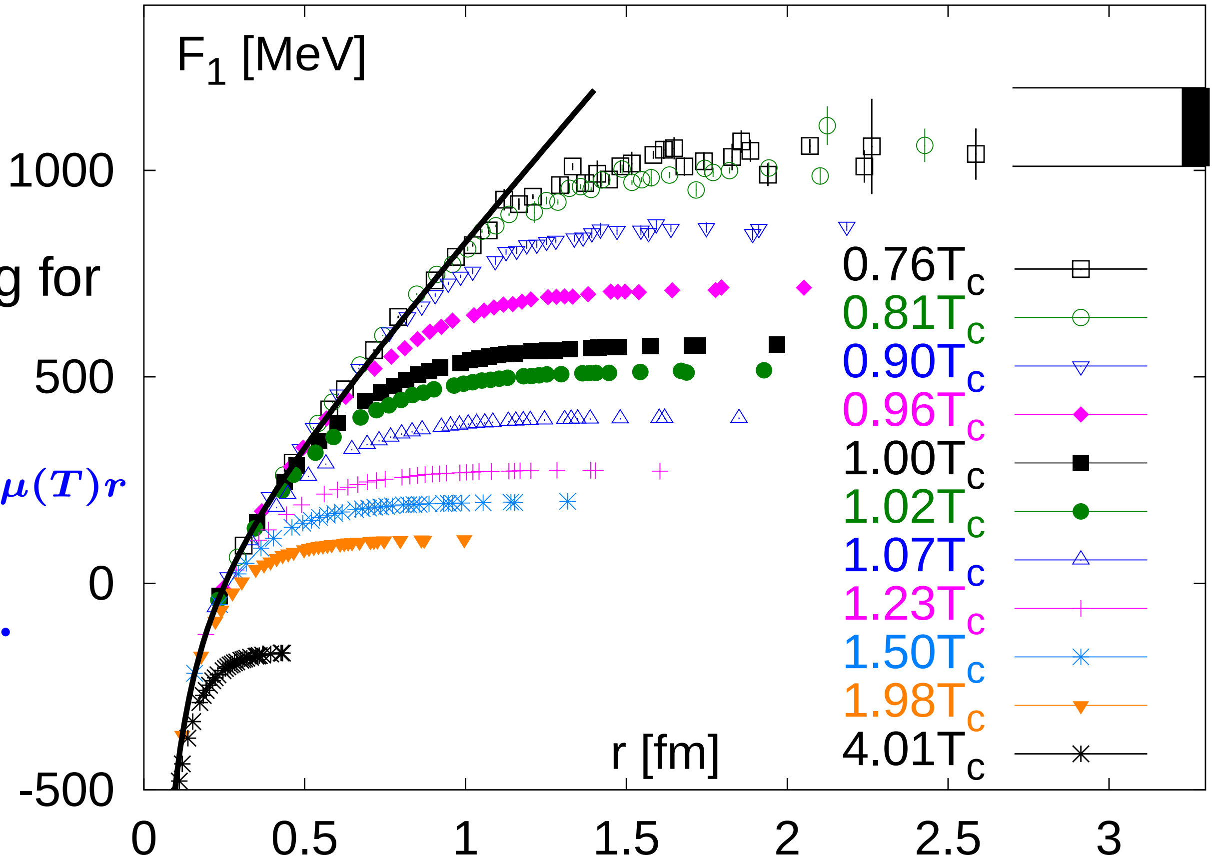
- the Debye screening mass is large close to T_c
- the spatial string tension does not vanish above T_c

  $\sqrt{\sigma_s} \neq 0 \Rightarrow$ the QGP is ”non-perturbative” up to very high T

Screening of heavy quark free energies
– remnant of confinement above T_c –

pure gauge: O.Kaczmarek, FK, P. Petreczky, F. Zantow, PRD70 (2005) 074505
2-flavor QCD: O.Kaczmarek, F. Zantow, Phys. Rev. D71 (2005) 114510

- singlet free energy



- $T \simeq T_c$: screening for $r \gtrsim 0.5$fm

$$F_1(r,T) \sim \frac{\alpha(T)}{r} e^{-\mu(T)r} + const.$$

- $F_1(r,T)$ follows linear rise of $V_{\bar{q}q}(r,T=0) = -\frac{4\alpha(r,T=0)}{3r} + \sigma r$ for $T \lesssim 1.5 T_c$, $r \lesssim 0.3$ fm

Singlet free energy and asymptotic freedom

pure gauge: O.Kaczmarek, FK, P. Petreczky, F. Zantow, PRD70 (2005) 074505
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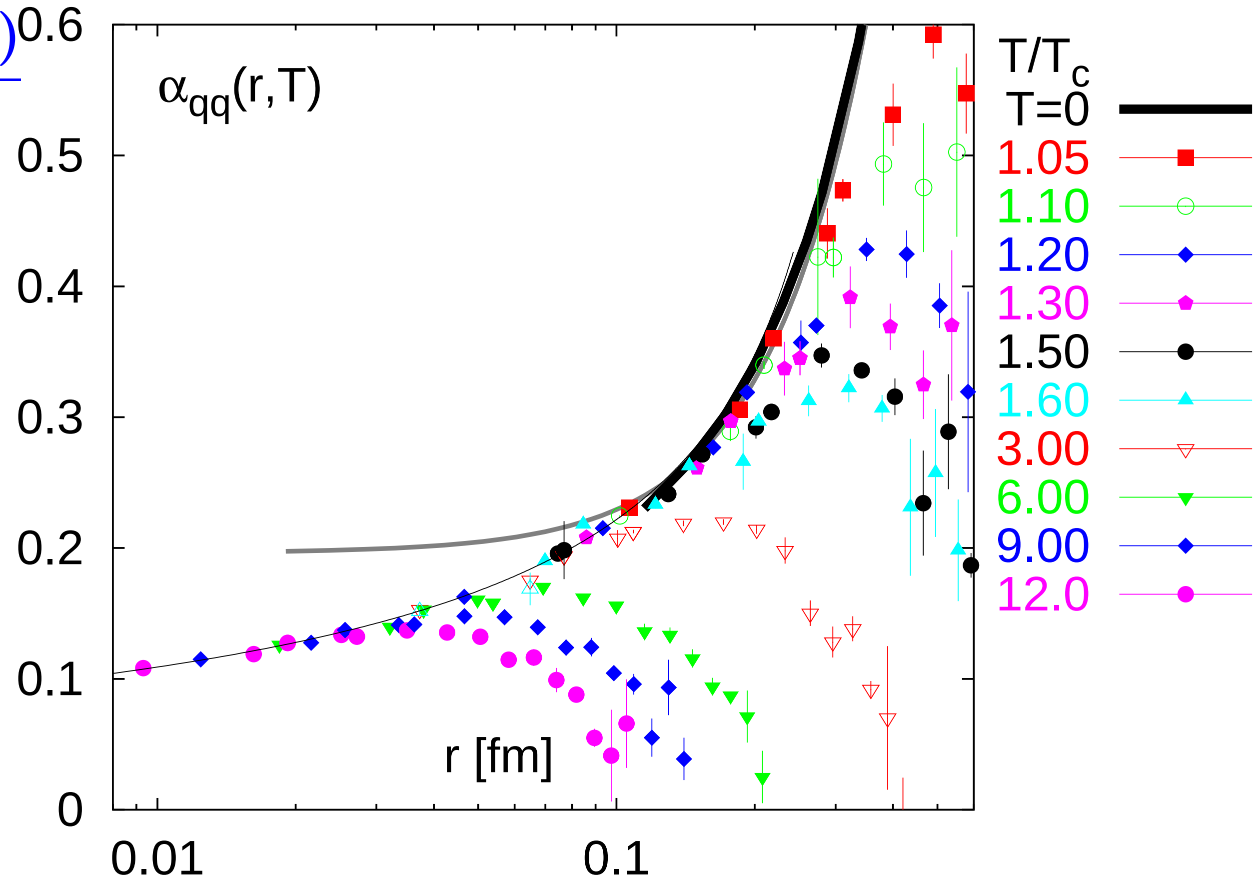
- singlet free energy defines a running coupling:

$$\alpha_{\mathrm{eff}} = \frac{3r^2}{4}\,\frac{\mathrm{d}F_1(r,T)}{\mathrm{d}r}$$

(in Coulomb gauge)



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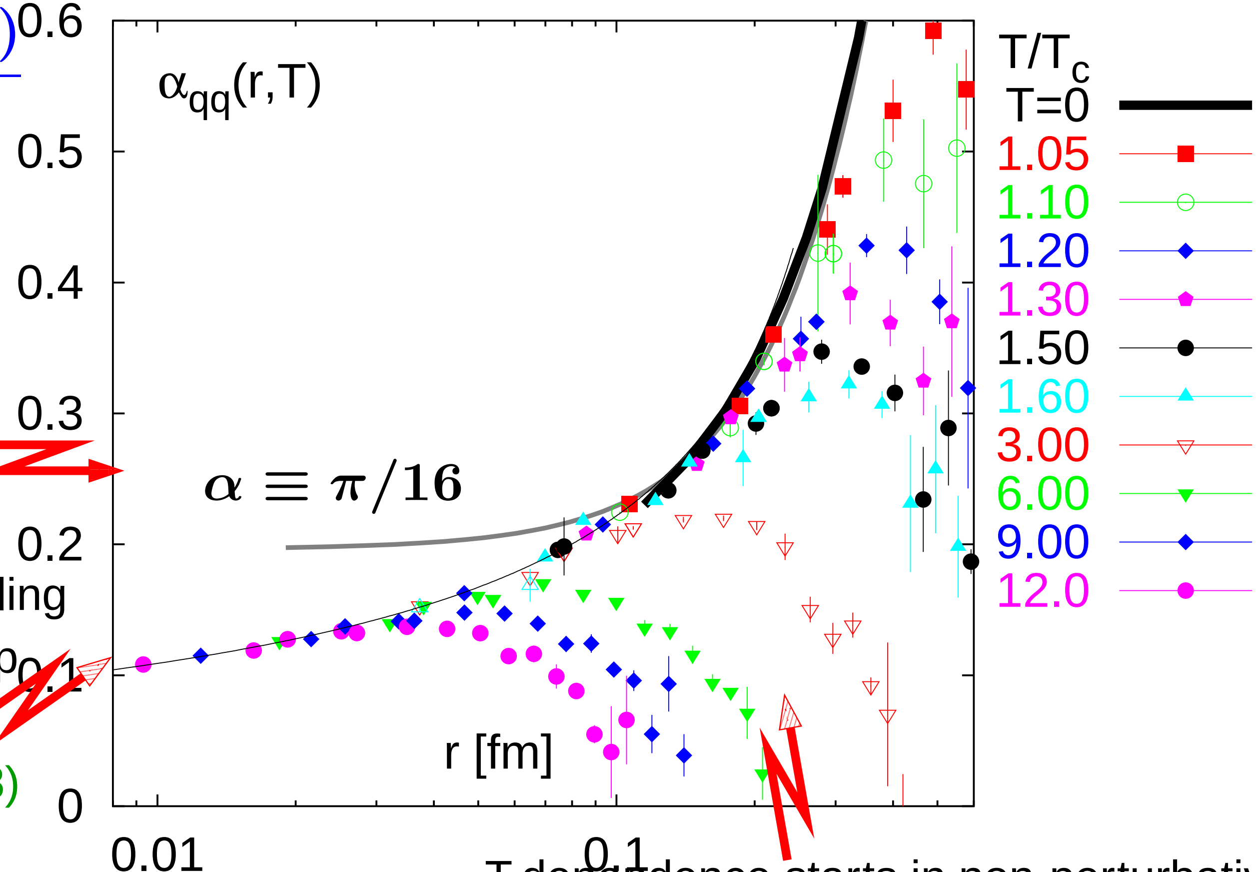
large distance: constant Coulomb term (string model)

short distance: running coupling $\alpha(r)$ from $(T=0)$, 3-loop (S. Necco, R. Sommer, Nucl. Phys. B622 (2002) 328)



T-dependence starts in non-perturbative regime for $T \lesssim 3\ T_c$

- short distance physics ⇔ vacuum physics

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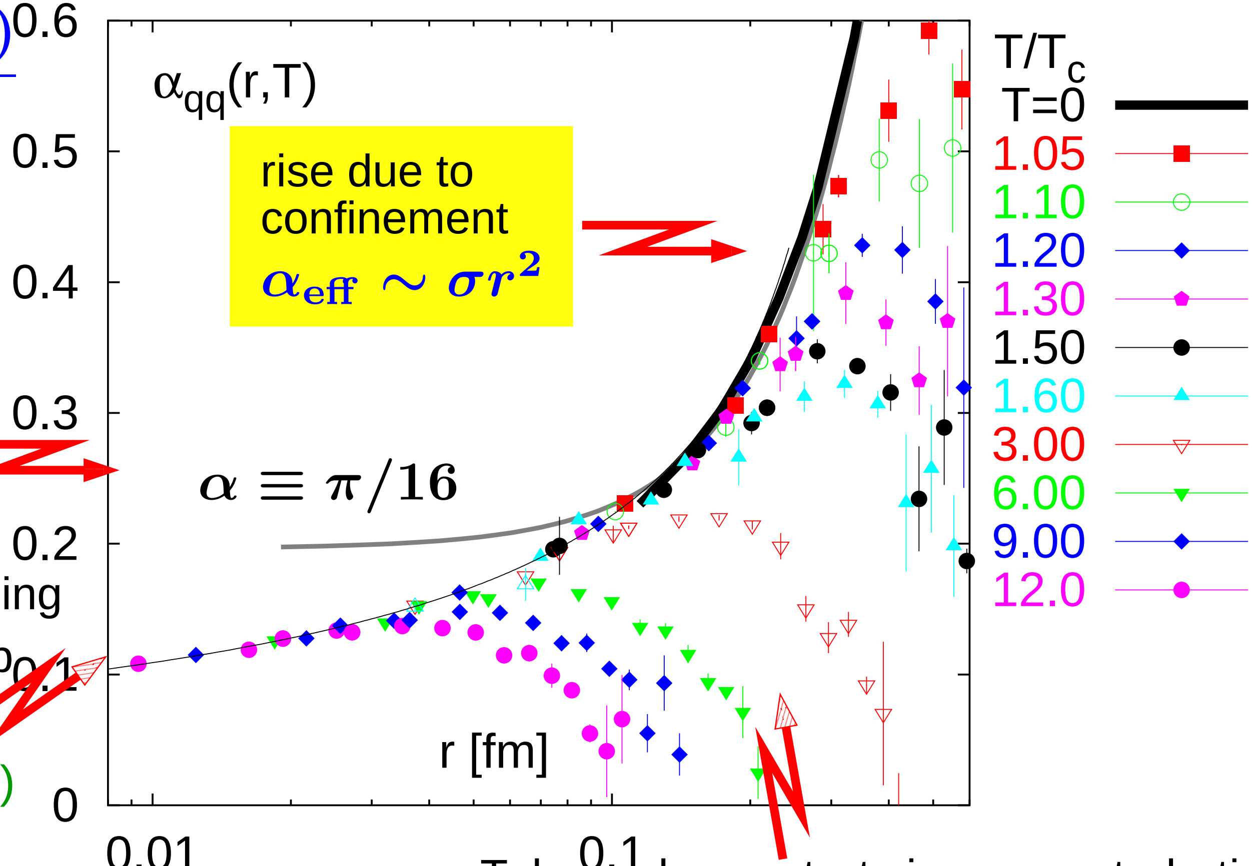

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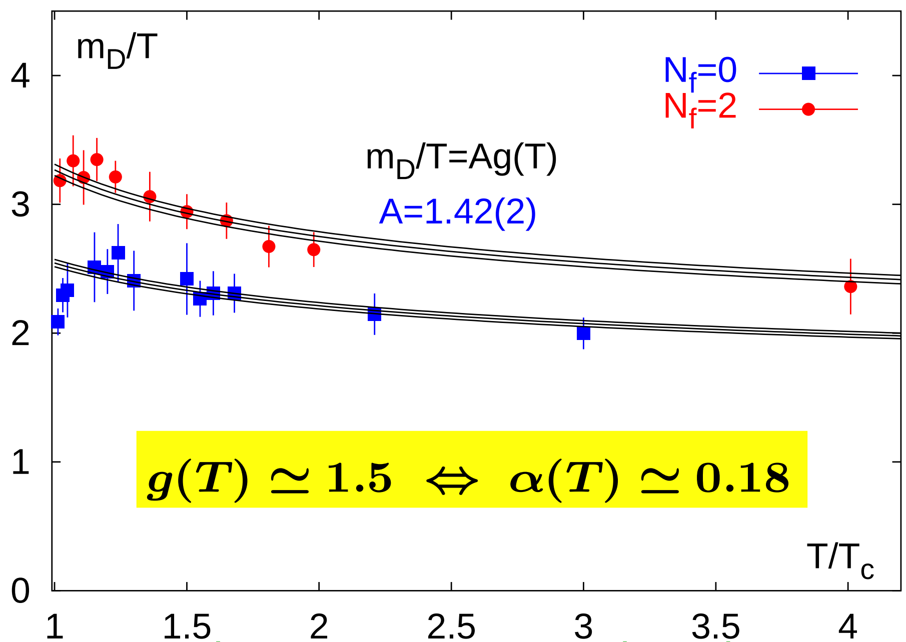
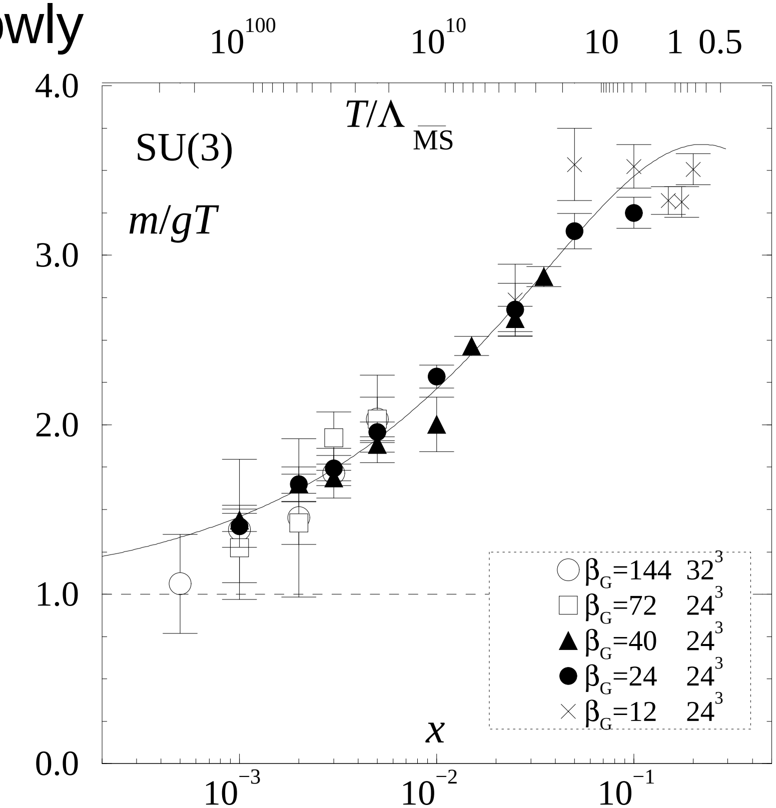
Non-perturbative Debye screening

- leading order perturbation theory: $m_D = g(T)T\sqrt{1+\frac{n_f}{6}}$
- $T_c < T \lesssim 10T_c$: non-perturbative effects are well represented by an "A-factor": $m_D \equiv Ag(T)T,\ A \simeq 1.5$
- perturbative limit is reached very slowly (logarithms at work!!)



O.Kaczmarek,F.Zantow, PRD 71 (2005) 114510



K.Kajantie et al, PRL 79 (1997) 3130

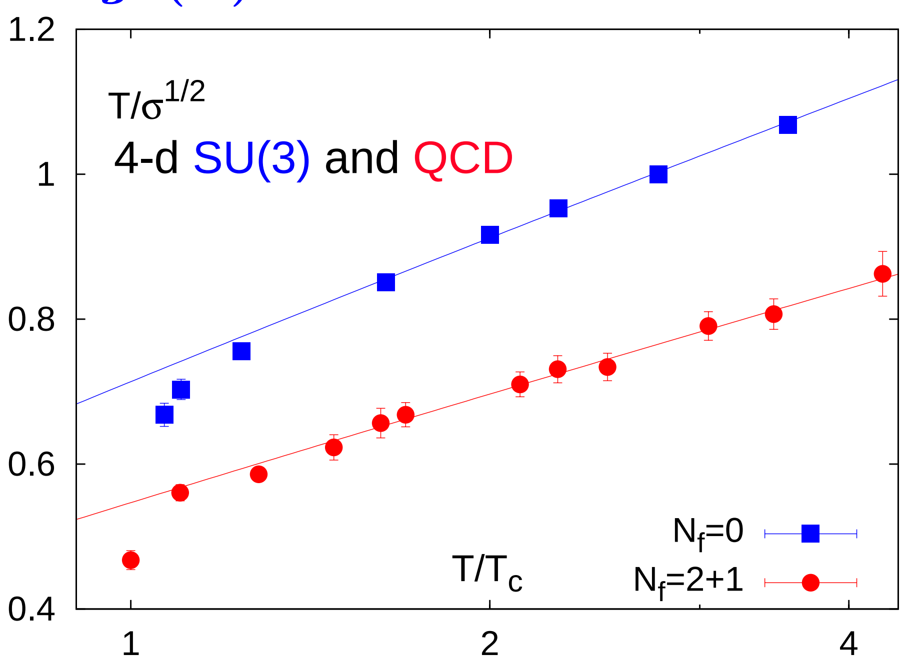
The spatial string tension

- Non-perturbative, vanishes in high-T perturbation theory:

$$\sqrt{\sigma_s} = - \lim_{R_x, R_y \to \infty} \ln \frac{W(R_x, R_y)}{R_x R_y}$$

- $$\frac{\sqrt{\sigma_s}}{g^2(T)T} = c_M f_M(g(T)) \ , \ c_M = 0.553(1)$$

c_M: 3-d SU(3), LGT
$g_M \equiv g^2 f_M$: dim. red. pert. th.



$g^2(T) \simeq 2 \ \Leftrightarrow \ \alpha(T) \simeq 0.16$

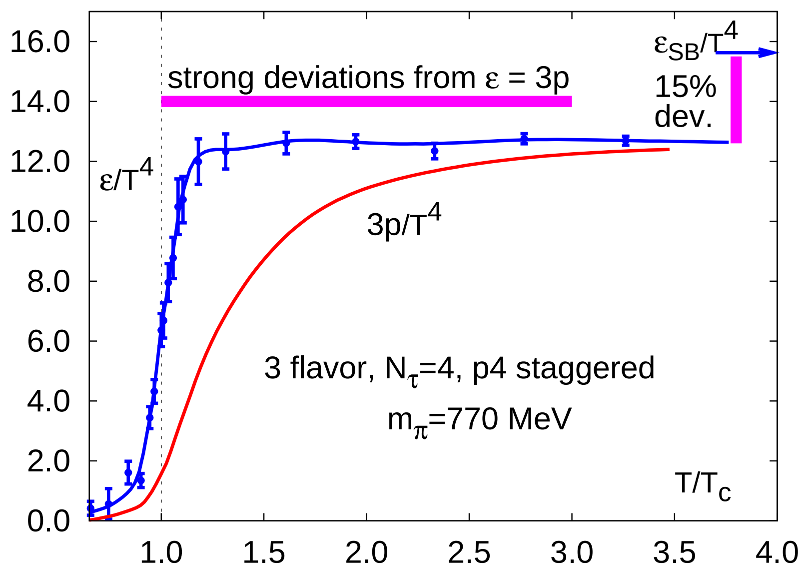
dimensional reduction works for $T \gtrsim 2T_c$

- c_M (almost) flavor independent
- $g^2(T)$ shows 2-loop running

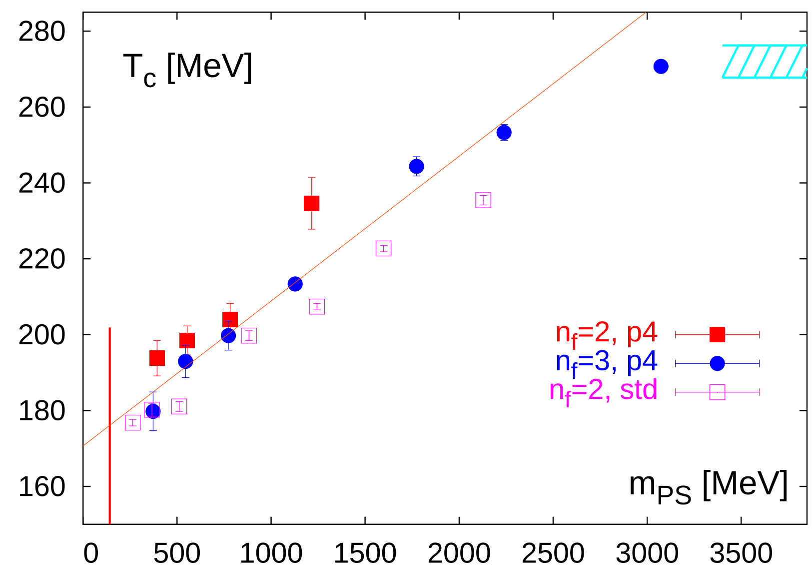
$c = 0.566(13)$ [SU(3)]
$c = 0.594(39)$ [QCD]

G. Boyd et al. NP B469 (1996) 419
RBC-Bielefeld, preliminary

$\mu = 0$: Equation of State and T_c



QCD EoS



transition temperature

- strong deviations from ideal gas behavior ($\epsilon = 3p$) for $T_c \leq T \sim 3T_c$ and even at high T
- $T_c = (173 \pm 8 \pm sys)$ MeV weak quark mass and flavor dependence

- improved staggered fermions but still on rather coarse lattices: $N_\tau = 4$, i.e. $a^{-1} \simeq 0.8$ GeV with moderately light quarks

FK, E. Laermann, A. Peikert, Nucl. Phys. B605 (2001) 579

EoS and T_c

Goal: QCD thermodynamics with realistic quark masses and controlled extrapolation to the continuum limit T_c, EoS, $\mu_q > 0$, ...

- use an improved staggered fermion action that removes $\mathcal{O}(a^2)$ errors in bulk thermodynamic quantities and reduces flavor symmetry breaking inherent to the staggered formulation

  RBC-Bielefeld choice: p4-action + 3-link smearing (p4fat3)

  MILC: Naik-action + (3,5,7)-link smearing (asqtad);

  Wuppertal: standard staggered + exponentiated 3-link smearing (stout)

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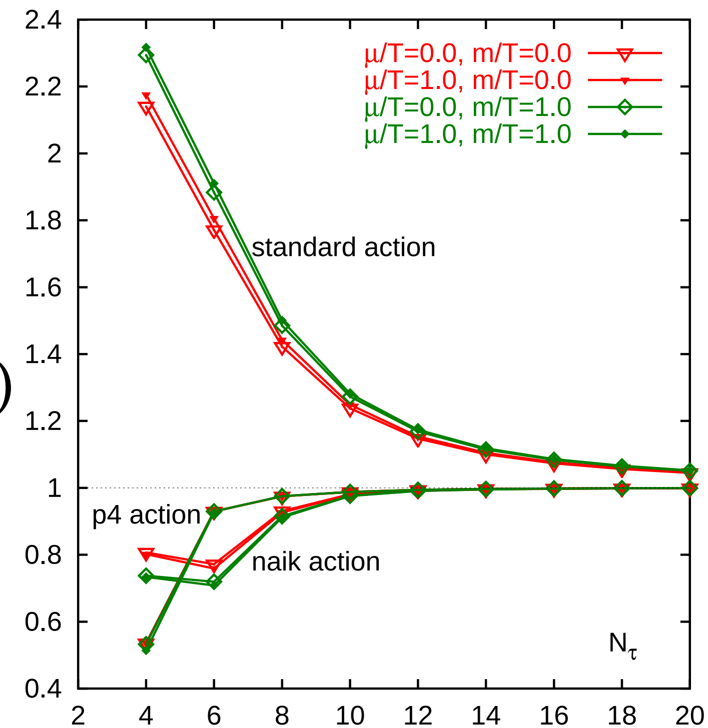
- p4-action: smooth high-T behavior for bulk thermodynamics on lattice with temporal extent N_τ

$$p(N_\tau)/T^4 = p_{SB}^{cont}/T^4 + \mathcal{O}(N_\tau^{-4})$$

- p4&Naik: similarly small cut-off dependence of renormalized Polyakov loops and quark number susceptibilities



Thermodynamics on QCDOC and apeNEXT

US/RBRC QCDOC

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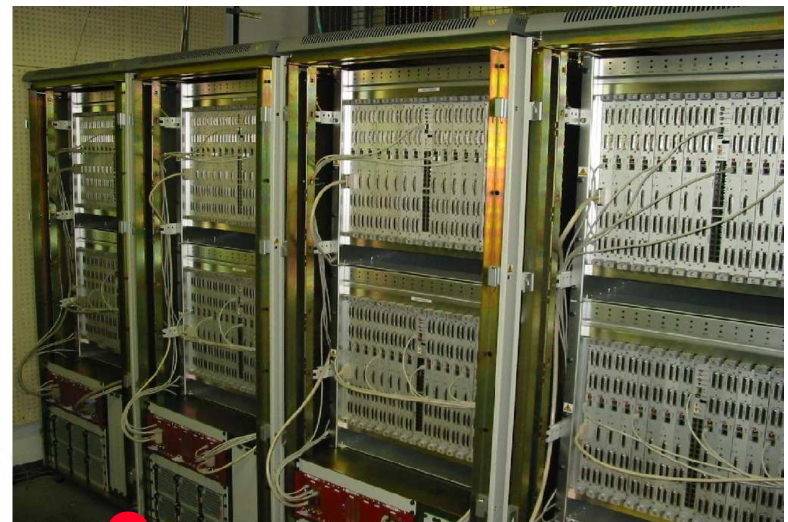
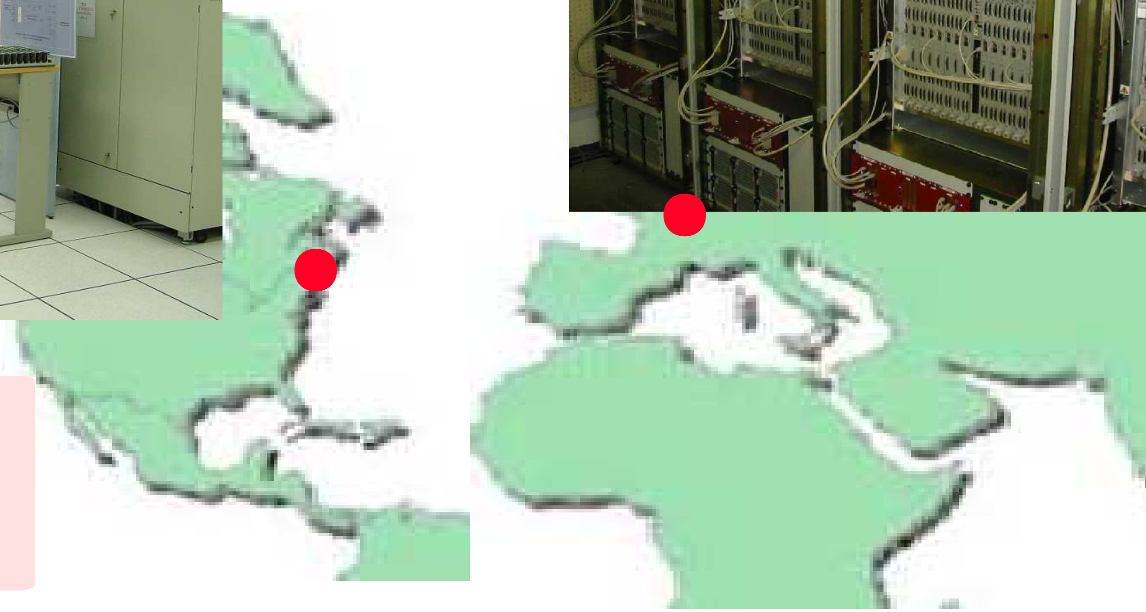
BI – apeNEXT

5.000.000.000.000 ops/sec





- critical temperature
- equation of state
- finite density QCD

EoS and T_c

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  RBC-Bielefeld choice: p4-action + 3-link smearing (p4fat3)

- use the newly developed RHMC algorithm to remove 'step-size errors' in the numerical simulation

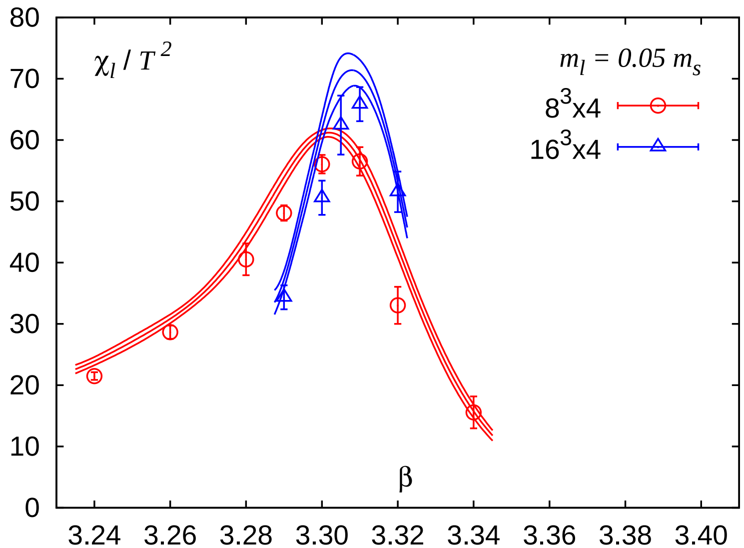
- perform simulations with (3-4) different light quark masses corresponding to $150\ \mathrm{MeV} \lesssim m_\pi \lesssim 500\ \mathrm{MeV}$ at 2 different values of the lattice cut-off controlled by the spatial lattice size $N_\tau = 4,\ 6$ to perform the chiral and continuum extrapolation

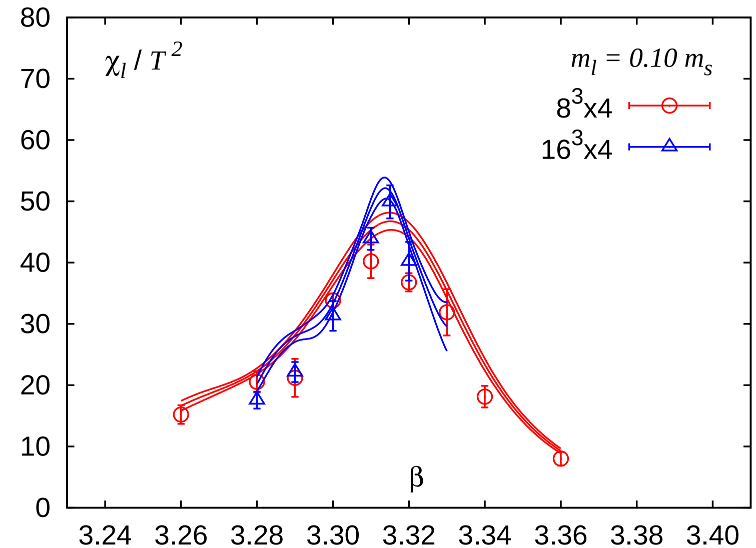
previous results with p4-action:
2-flavor QCD: $N_\tau = 4,\ m_\pi \simeq 770$ MeV

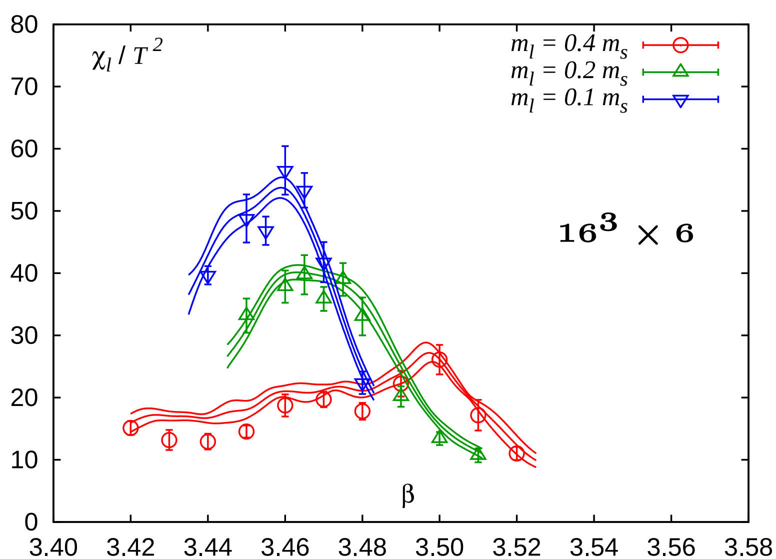
Transition temperature

- crossover rather than phase transition:
  need to determine location of the transition from various susceptibilities:
  (disconnected part of the) light and strange quark chiral susceptibility; Polyakov loop and quark number susceptibility,...
- thermodynamic limit:
  need to control finite volume effects;
- continuum limit:
  need to analyze cut-off dependence in $T > 0$ and $T = 0$ calculations;
  - large statistics; several ten thousand trajectories
  - find little volume dependence of location of transition point
  - overall scale setting using $T = 0$ potential parameter; find weak cut-off dependence

Chiral susceptibility, $N_\tau = 4,\ 6$

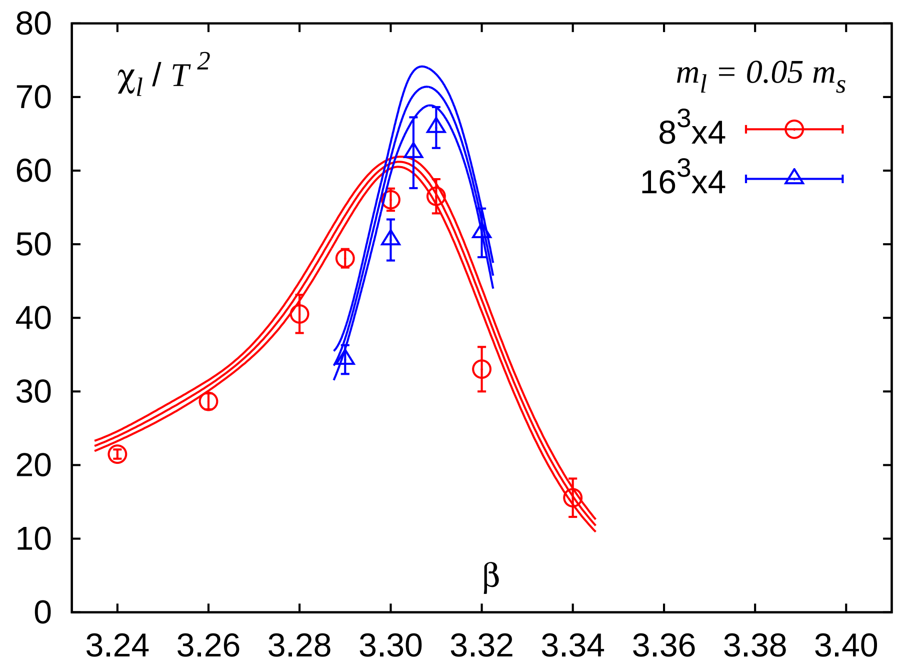






- weak volume dependence
- peak location consistent with that of Polyakov loop susceptibility and maximum of quartic fluctuation of quark number density

Chiral and L susceptibility, $N_\tau = 4$

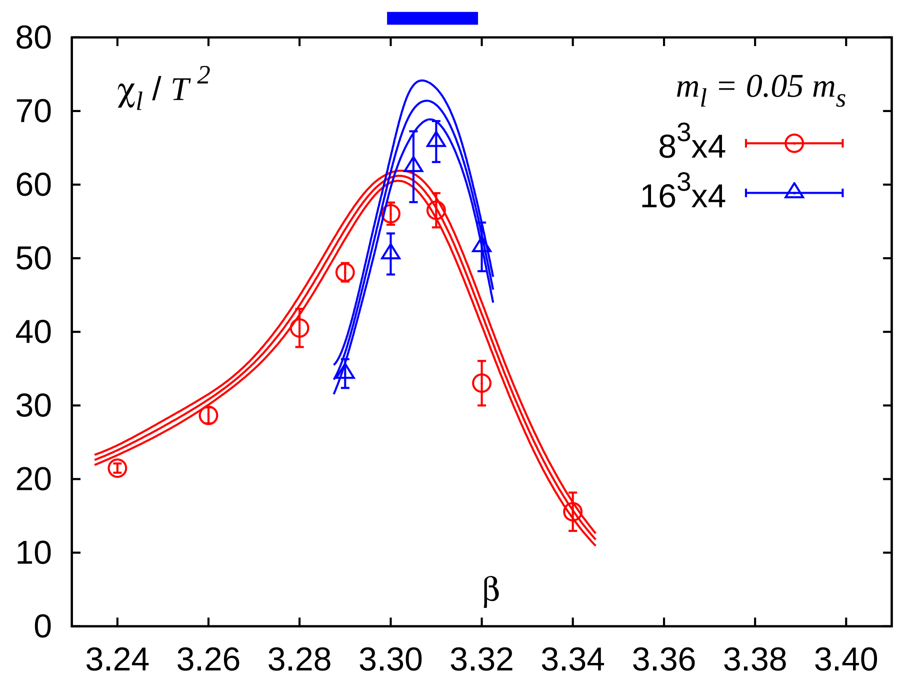


Chiral and L susceptibility, $N_\tau = 4$

data sample for smallest quark mass on $\mathbf{16^3} \times \mathbf{4}$ lattice

β	no. of conf.
3.2900	38960
3.3000	40570
3.3050	32950
3.3100	42300
3.3200	39050

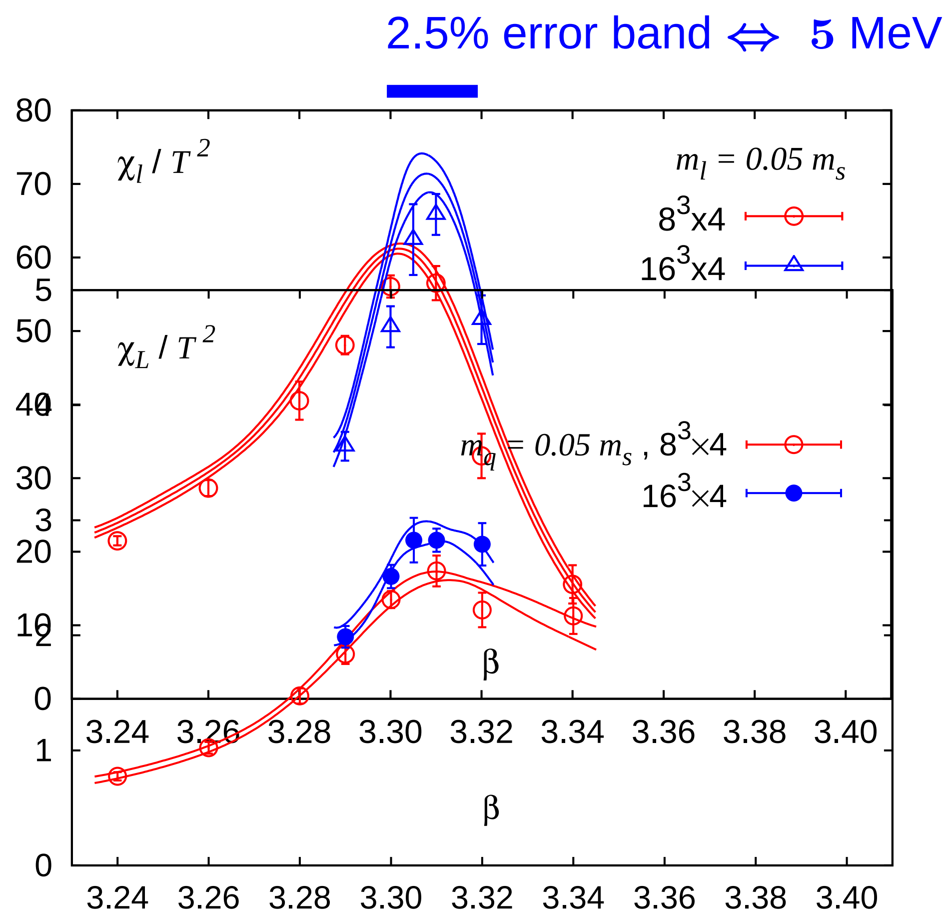
2.5% error band $\Leftrightarrow$ $\mathbf{5}$ MeV



200,000/10 trajectories enter Ferrenberg-Swendsen sample

Chiral and L susceptibility, $N_\tau = 4$

2.5% error band $\Leftrightarrow$ **5** MeV



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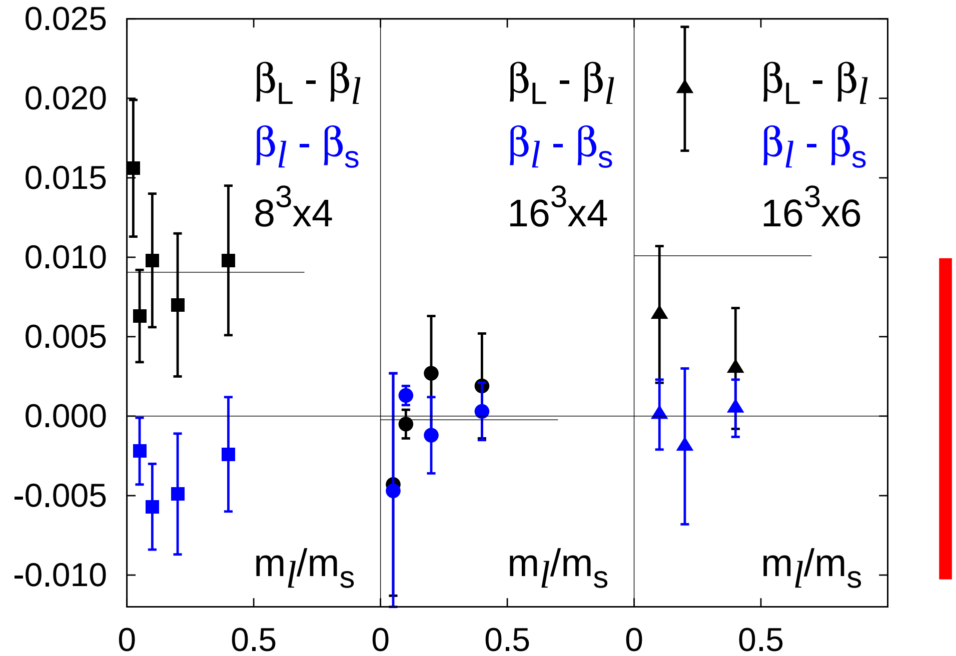
Ambiguities in locating the crossover point

differences of pseudo-critical couplings locating peaks in light (β_l), strange (β_s) and Polyakov loop (β_L) susceptibilities



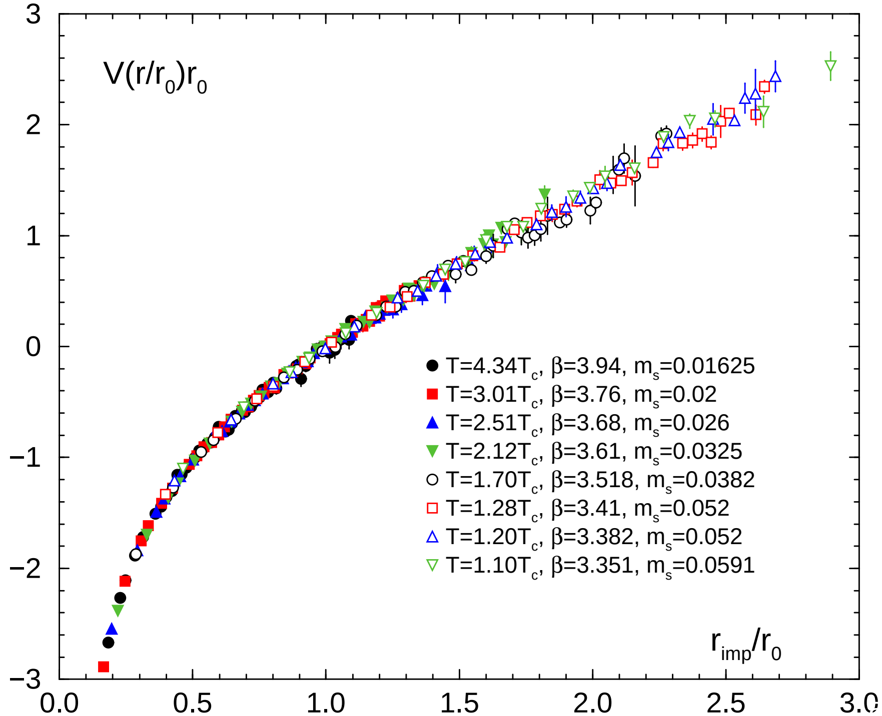
2.5% ($N_\tau = 4$) or 4% ($N_\tau = 6$) error band $\Leftrightarrow$ 5 or 8 MeV

differences in the location of pseudo-critical couplings are taken into account as systematic error

$T = 0$ scale setting using the heavy quark potential

use r_0 or string tension to set the scale for $T_c = 1/N_\tau a(\beta_c)$

$$V(r) = -\frac{\alpha}{r} + \sigma r \quad , \quad r^2 \frac{\mathrm{d}V(r)}{\mathrm{d}r}\Big|_{r=r_0} = 1.65$$



no significant cut-off dependence when cut-off varies by a factor 4

i.e. from the transition region on $N_\tau = 4$ lattices to that on $N_\tau = 16$ lattices !!

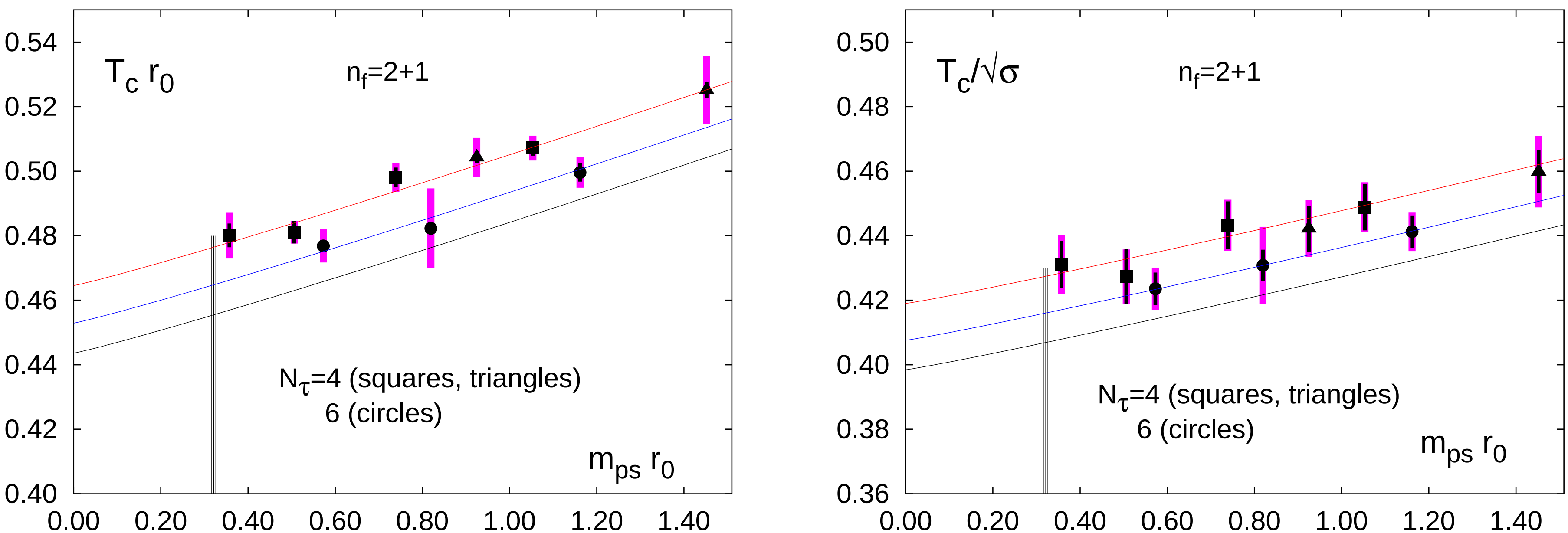
we use $r_0 = 0.469(7)$ fm determined from quarkonium spectroscopy
A. Gray et al, Phys. Rev. D72 (2005) 094507

$\Rightarrow \quad T_c r_0,\ T_c/\sqrt{\sigma}$

extrapolation to chiral and continuum limit

$$(r_0 T_c)_{N_\tau} = (r_0 T_c)_{cont.} + b\,(m_{PS} r_0)^d + c/N_\tau^2$$

(d=1.08 (O(4), 2nd ord.), d=2 (1st ord.))



$\Rightarrow \quad r_0 T_c = 0.456(7)^{+3}_{-1} \quad , \quad T_c/\sqrt{\sigma} = 0.408(7)^{+3}_{-1}$ at phys. point

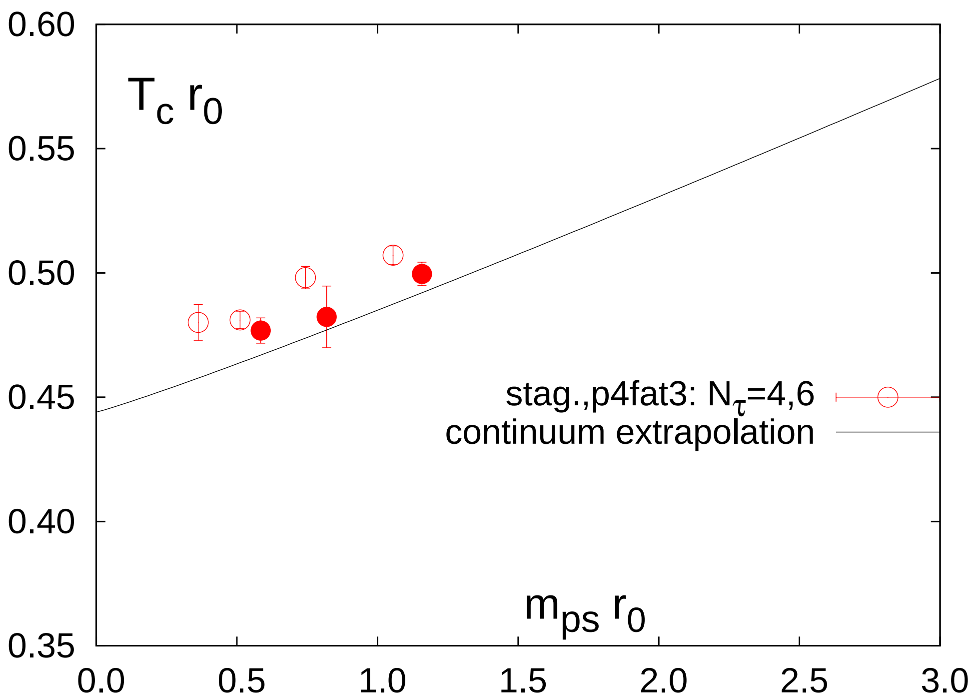
$\Rightarrow T_c = 192(7)(4)$ MeV

(1st error: stat. error on β_c and r_0; 2nd error: N_τ^{-2} extrapolation)

Transition temperature

staggered fermions $N_\tau = 4, 6$

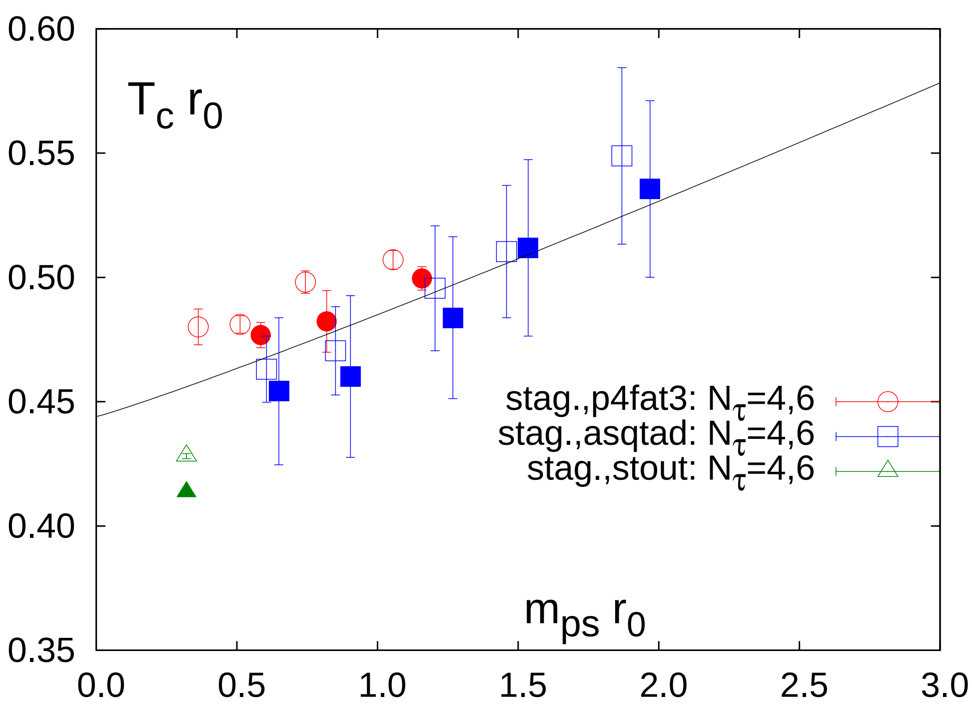
- RBC-Bielefeld (p4fat3 (p4))



Transition temperature

staggered fermions $N_\tau = 4, 6$

- RBC-Bielefeld (p4fat3 (p4)) vs. MILC (asqtad (Naik)) and Wuppertal (stout (stand. staggered))
- asqtad results for $N_\tau = 4$ and 6 agree with p4 results within statistical errors; (C.Bernard et al., PR D71, 034504 (2005))
- results obtained with stout action for $N_\tau = 4$ and 6 are about 15% lower; β_c from $N_\tau = 8, 10$ covers $(151 - 176)$ MeV; (Y. Aoki et al., hep-lat/0609068)



asqtad data for $T_c r_1$ rescaled with $r_0/r_1 = 1.4795$

asqtad: continuum extrapolation:

quoted T_c from $m_q/m_s \le 1$ and fit in m_π/m_ρ yields

$T_c = 169(12)(4)$ MeV

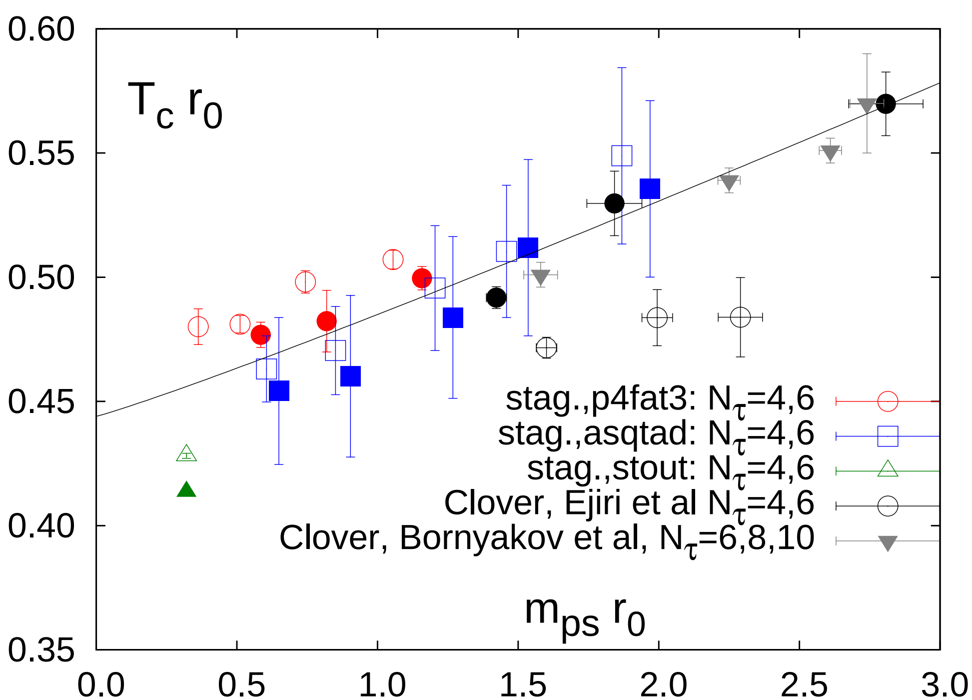
using $m_q/m_s \le 0.4$ and fit in $m_\pi r_0$ yields

$T_c = 173(13)(4)$ MeV

Transition temperature

staggered fermions $N_\tau = 4, 6$ and Wilson fermions $N_\tau = 6 - 10$

- RBC-Bielefeld (p4fat3 (p4)) vs. MILC (asqtad (Naik)) and Wuppertal (stout (stand. staggered))
- T_c from Wilson/Clover fermions so far only for $m_{ps} r_0 > 1.5$; consistent with staggered results
- Wilson for $N_\tau \geq 6$ show no significant cout-off effects (V.G. Bornyakov et al., hep-lat/0509122)



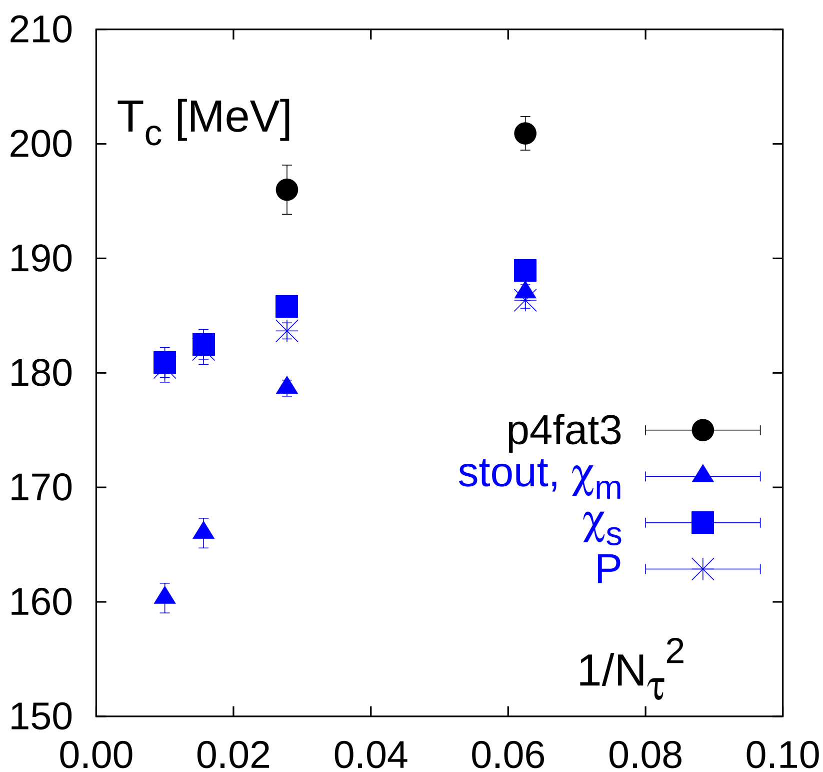
scale setting uncertainties:

staggered: $r_0 = 0.469(7)$ fm
(MILC + heavy quark spec.)

Clover: $r_0 = 0.516(21)$ fm
(CP-PACS+JLQCD, light quark spec.)

extrapolations to phys. point

- RBC-Bielefeld (p4fat3 (p4)) vs. Wuppertal (stout (stand. staggered))
- results for $N_\tau = 4, 6$ differ by 15% but show similar cut-off dependence
- stout results for different observables no longer consistent with each other for $N_\tau = 8, 10$



overall scale set with

$r_0 = 0.469$ fm

Calculating the EoS on lines of constant physics (LCP)

- The pressure

$$\frac{p}{T^4}\Big|_{\beta_0}^{\beta} = N_\tau^4 \int_{\beta_0}^{\beta} d\beta' \Big[\frac{1}{N_\sigma^3 N_t} (\langle S_g\rangle_0 - \langle S_g\rangle_T)$$

$$- \left(2(\langle\bar\psi\psi\rangle_{l0} - \langle\bar\psi\psi\rangle_{lT}) + \frac{\hat m_s}{\hat m_l}(\langle\bar\psi\psi\rangle_{s0} - \langle\bar\psi\psi\rangle_{sT}) \right) \left(\frac{\partial \hat m_l}{\partial\beta'}\right)_{\hat m_s/\hat m_l}$$

$$- \hat m_l \left(\langle\bar\psi\psi\rangle_{s0} - \langle\bar\psi\psi\rangle_{sT}\right) \left(\frac{\partial \hat m_s/\hat m_l}{\partial\beta'}\right)_{\hat m_l} \Big]$$

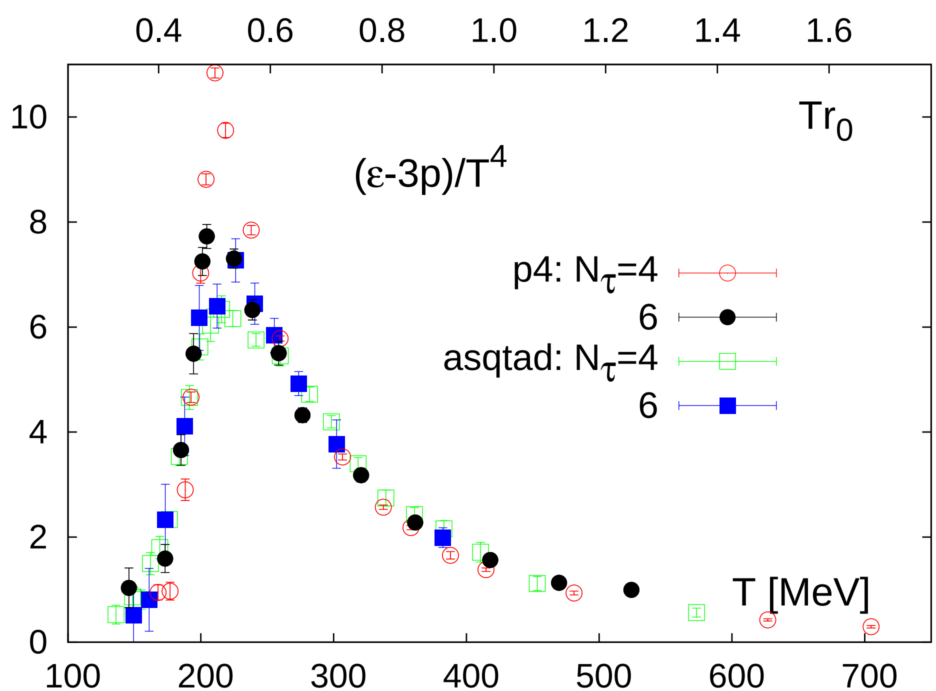
- The interaction measure for $N_f = 2+1$

$$\frac{\epsilon - 3p}{T^4} = T\frac{\mathrm{d}}{\mathrm{d}T}\left(\frac{p}{T^4}\right) = \left(a\frac{\mathrm{d}\beta}{\mathrm{d}a}\right)_{LCP} \frac{\partial p/T^4}{\partial\beta}$$

$$= \left(\frac{\epsilon-3p}{T^4}\right)_{gluon} + \left(\frac{\epsilon-3p}{T^4}\right)_{fermion} + \left(\frac{\epsilon-3p}{T^4}\right)_{\hat m_s/\hat m_l}$$

$(\epsilon - 3p)/T^4$ on LCP

- Using an RG-inspired 2-loop β-function underestimates $(\epsilon - 3p)/T^4$ in the transition region and stretches the temperature interval in the low temperature regime artifically, i.e. makes the transition region look broader than it is.



RBC-Bielefeld, preliminary

differences in the transition region partly arise from differences in the β-functions used in the crossover region

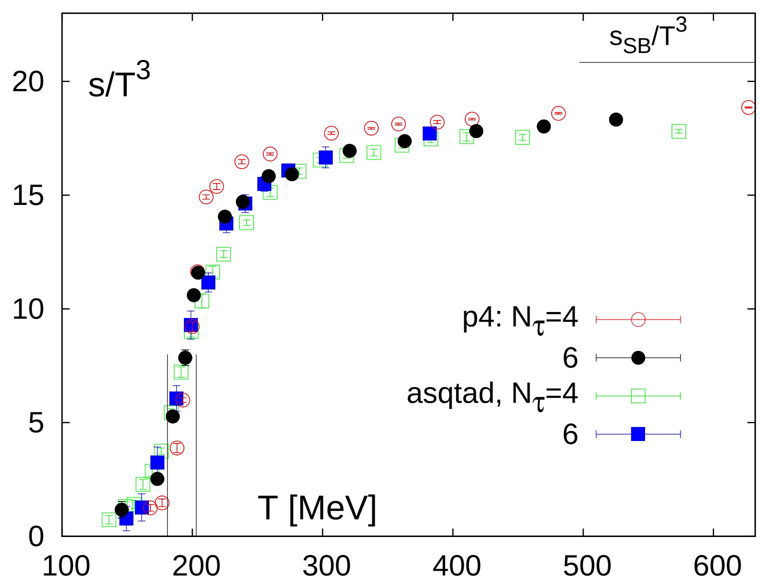
- overall good agreement

Note:
T-scale is not dependent on T_c determination

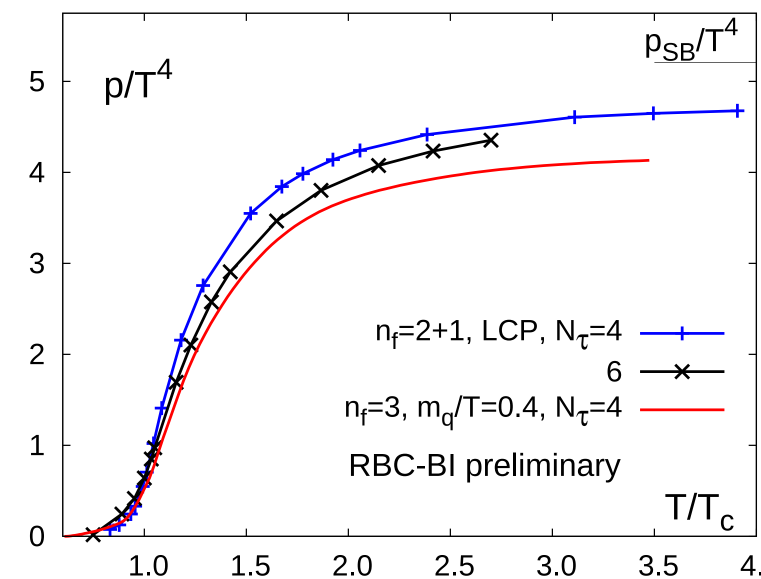
asqtad data:
C. Bernard et al., hep-lat/0611031

Energy density and pressure $N_\tau = 4,\ 6$

- RBC-Bielefeld vs. MILC: the RBC-Bi energy/entropy density on $N_\tau = 4$ lattices rises more steeply;
  direct consequence of the use of a non-perturbative β-function directly deduced from calculated r_0/a values
- overall good agreement for $N_\tau = 4,\ 6$,
  Note: T-scale does not depend on T_c determination!!



band marks $T = (192 \pm 11)$ MeV
RBC-Bielefeld, preliminary



pressure increased slightly with smaller quark mass

Lattice EoS: energy density $\Leftrightarrow$ temperature $\Longrightarrow$ conditions for heavy $q\bar{q}$ bound states

LGT: $T_c \simeq 190$ MeV

$T = T_c$: $\epsilon_c/T_c^4 \simeq 6 \;\Rightarrow\; \epsilon_c \simeq 1$ GeV/fm^3

$T \;\geq\; 1.5T_c$: $\epsilon/T^4 \simeq (13-14)$

$T = 1.5T_c$: $\epsilon \simeq 11$ GeV/fm^3

$T = 2.0T_c$: $\epsilon \simeq 35$ GeV/fm^3

$\Downarrow$

observable consequences:

J/ψ suppression

RHIC

$R_{Au} \simeq 7$ fm;

$\tau_0 \simeq 1$ fm

$\langle E_T \rangle \simeq 1$ GeV

$dN/dy \simeq 1000$

$\Downarrow$

$\epsilon_{Bj} \simeq 7$ GeV/fm^3

maybe: $\tau_0 \simeq 0.5$ fm

$\Downarrow$

$\epsilon_{Bj} \simeq 14$ GeV/fm^3

Lattice EoS: energy density $\Leftrightarrow$ temperature
$\Rightarrow$ conditions for heavy $q\bar{q}$ bound states

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$\Downarrow$

χ_c, ψ' suppression at RHIC

direct J/ψ suppression unlikely

$\Downarrow$

$S(J/\psi) \simeq 0.6 + 0.4S(\chi_c)$

(assume $S(\chi_c) \simeq S(\psi'))$

RHIC

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$\Downarrow$

$\epsilon_{Bj} \simeq 7$ GeV/fm^3

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$\epsilon_{Bj} \simeq 14$ GeV/fm^3

Bulk thermodynamics with non-vanishing chemical potential

$$Z(V,T,\mu) = \int \mathcal{D}\mathcal{A}\mathcal{D}\psi\mathcal{D}\bar{\psi}\ \mathrm{e}^{-S_E(V,T,\mu)}$$

$$= \int \mathcal{D}\mathcal{A}\ \left[det\ M(\mu)\right]^f\ \mathrm{e}^{-S_G(V,T)}$$

$\Uparrow$ complex fermion determinant;

Bulk thermodynamics with non-vanishing chemical potential

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$$= \int \mathcal{D}\mathcal{A}\ \left[det\ M(\mu)\right]^f\ \mathrm{e}^{-S_G(V,T)}$$

⇑complex fermion determinant;

ways to circumvent this problem:

- reweighting: works well on small lattices; requires exact evaluation of $detM$
  Z. Fodor, S.D. Katz, JHEP 0203 (2002) 014
- Taylor expansion around $\mu = 0$: works well for small μ;
  C. R. Allton et al. (Bielefeld-Swansea), Phys. Rev. D66 (2002) 074507
  R.V. Gavai, S. Gupta, Phys. Rev. D68 (2003) 034506
- imaginary chemical potential: works well for small μ; requires analytic continuation
  Ph. deForcrand, O. Philipsen, Nucl. Phys. B642 (2002) 290
  M. D'Elia and M.P. Lombardo, Phys. Rev. D64 (2003) 014505

Energy and Entropy density for $\mu_q > 0$

S. Ejiri, F. Karsch, E. Laermann and C. Schmidt, hep-lat/0512040

Thermodynamics: (NB: continuum $\hat{m} \equiv m_q$
lattice $\hat{m} \equiv m_q a$, implicit T-dependence)

- pressure $$\frac{p}{T^4} \equiv \frac{1}{VT^3} \ln Z(T, \mu_q) = \sum_{n=0}^{\infty} c_n(T, \hat{m}) \left(\frac{\mu_q}{T}\right)^n$$

- energy density from "interaction measure"

$$\frac{\epsilon - 3p}{T^4} = \sum_{n=0}^{\infty} c_n'(T, \hat{m}) \left(\frac{\mu_q}{T}\right)^n \ , \ c_n'(T, \hat{m}) \equiv T \frac{\mathrm{d}c_n(T, \hat{m})}{\mathrm{d}T}$$

- entropy density

$$\frac{s}{T^3} \equiv \frac{\epsilon + p - \mu_q n_q}{T^4} = \sum_{n=0}^{\infty} \left((4-n) c_n(T, \hat{m}) + c_n'(T, \hat{m})\right) \left(\frac{\mu_q}{T}\right)^n$$

Bulk thermodynamics for small μ_q/T on $16^3 \times 4$ lattices

- Taylor expansion of pressure up to $\mathcal{O}\left((\mu_q/T)^6\right)$

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln \mathcal{Z} = \sum_{n=0}^{\infty} c_n(T) \left(\frac{\mu_q}{T}\right)^n \simeq c_0 + c_2 \left(\frac{\mu_q}{T}\right)^2 + c_4 \left(\frac{\mu_q}{T}\right)^4 + c_6 \left(\frac{\mu_q}{T}\right)^6$$

quark number density
$$\frac{n_q}{T^3} = 2c_2 \frac{\mu_q}{T} + 4c_4 \left(\frac{\mu_q}{T}\right)^3 + 6c_6 \left(\frac{\mu_q}{T}\right)^5$$

quark number susceptibility
$$\frac{\chi_q}{T^2} = 2c_2 + 12c_4 \left(\frac{\mu_q}{T}\right)^2 + 30c_6 \left(\frac{\mu_q}{T}\right)^4$$

an estimator for the radius of convergence

$$\left(\frac{\mu_q}{T}\right)_{crit} = \lim_{n\to\infty} \left| \frac{c_{2n}}{c_{2n+2}} \right|^{1/2}$$

$c_n > 0$ for all n;
singularity for real μ

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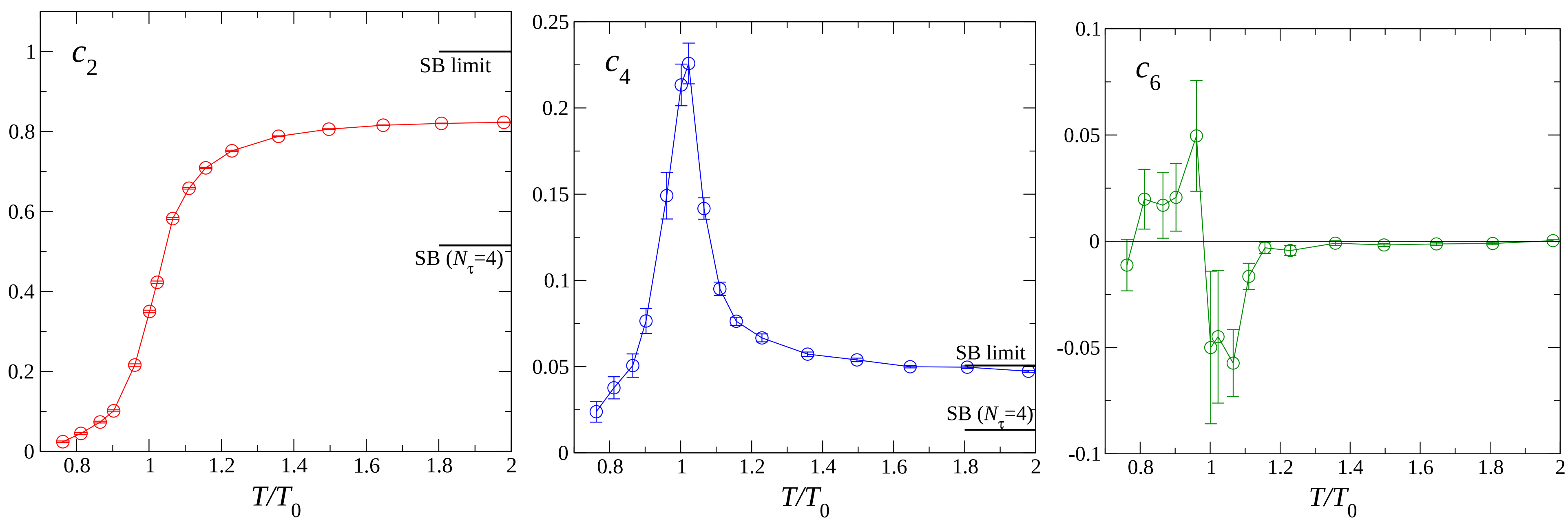

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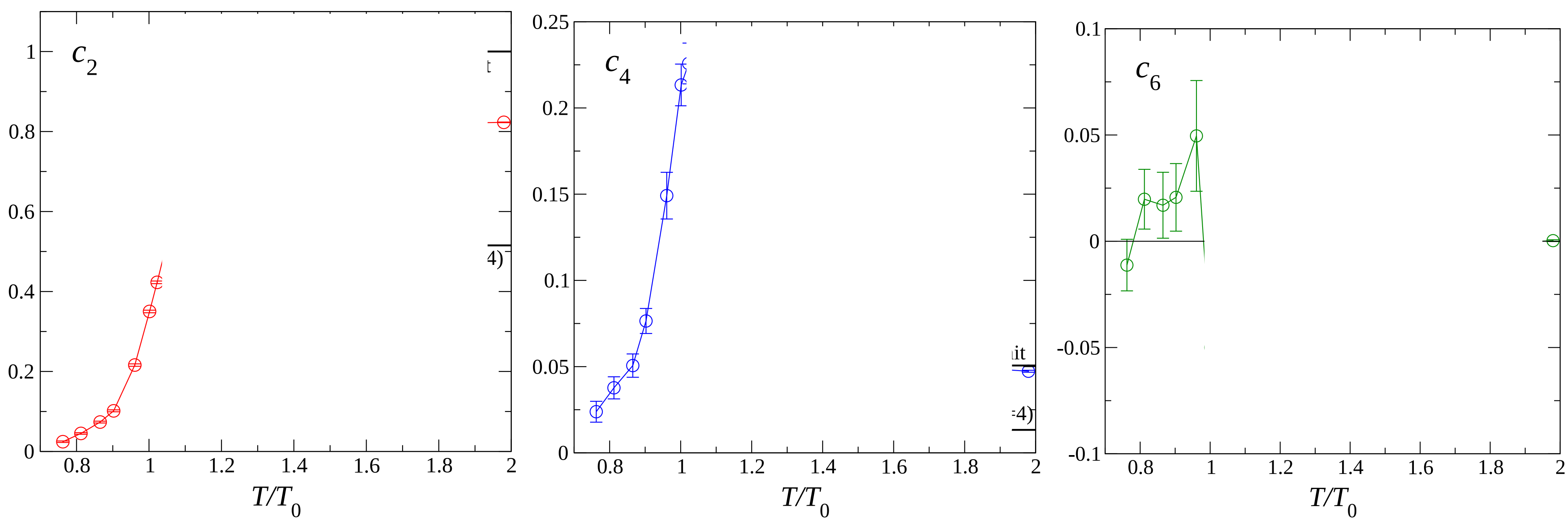

$c_n > 0$ for all n and $T \lesssim 0.95\ T_c$ $\Leftrightarrow$ singularity for real μ (positive μ^2)

Bulk thermodynamics for small μ_q/T on $16^3 \times 4$ lattices

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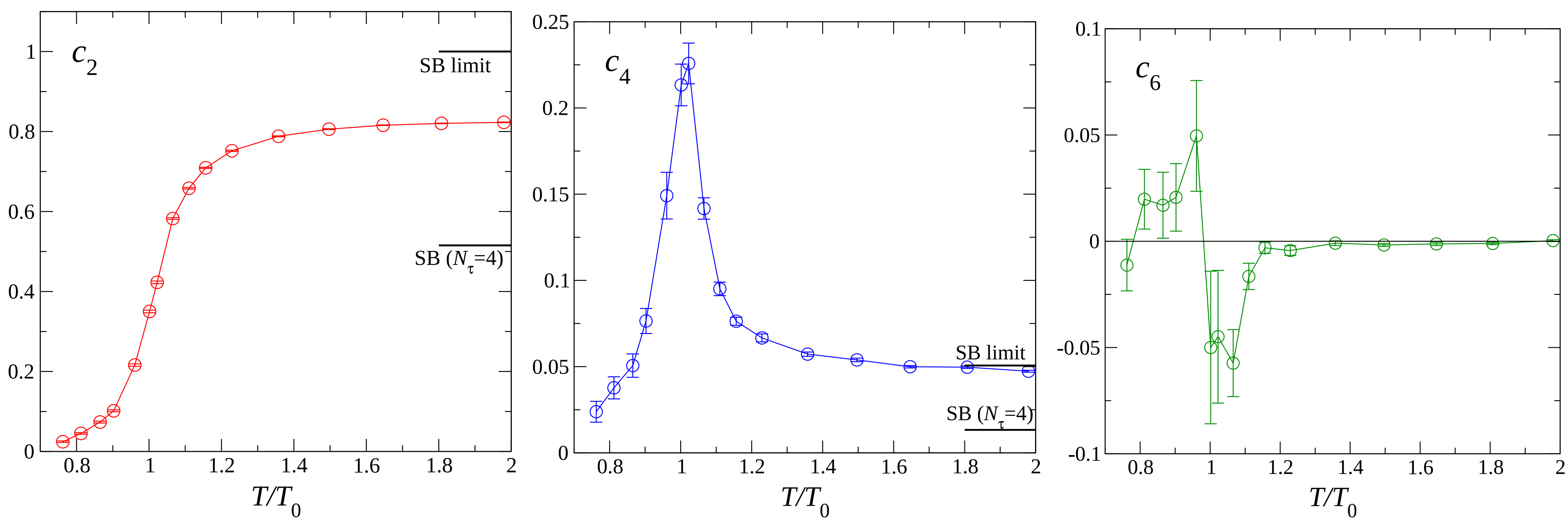
$$\frac{p}{T^4} = \frac{1}{VT^3}\ln \mathcal{Z} = \sum_{n=0}^{\infty} c_n(T)\left(\frac{\mu_q}{T}\right)^n \simeq c_0 + c_2\left(\frac{\mu_q}{T}\right)^2 + c_4\left(\frac{\mu_q}{T}\right)^4 + c_6\left(\frac{\mu_q}{T}\right)^6$$



irregular sign of c_n for $T \gtrsim T_c$ $\Leftrightarrow$ singularity in complex plane

The pressure for $\mu_q/T > 0$

C.R. Allton et al. (Bielefeld-Swansea), PRD68 (2003) 014507

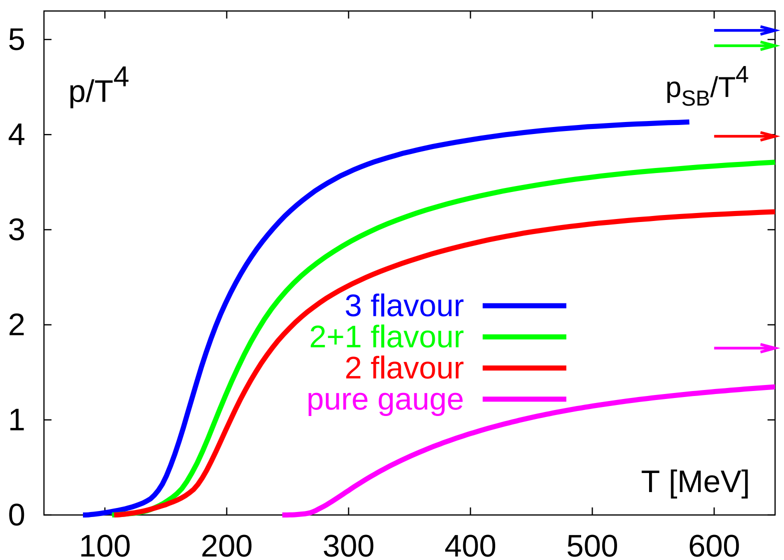
$\mu_q = 0$, $16^3 \times 4$ lattice

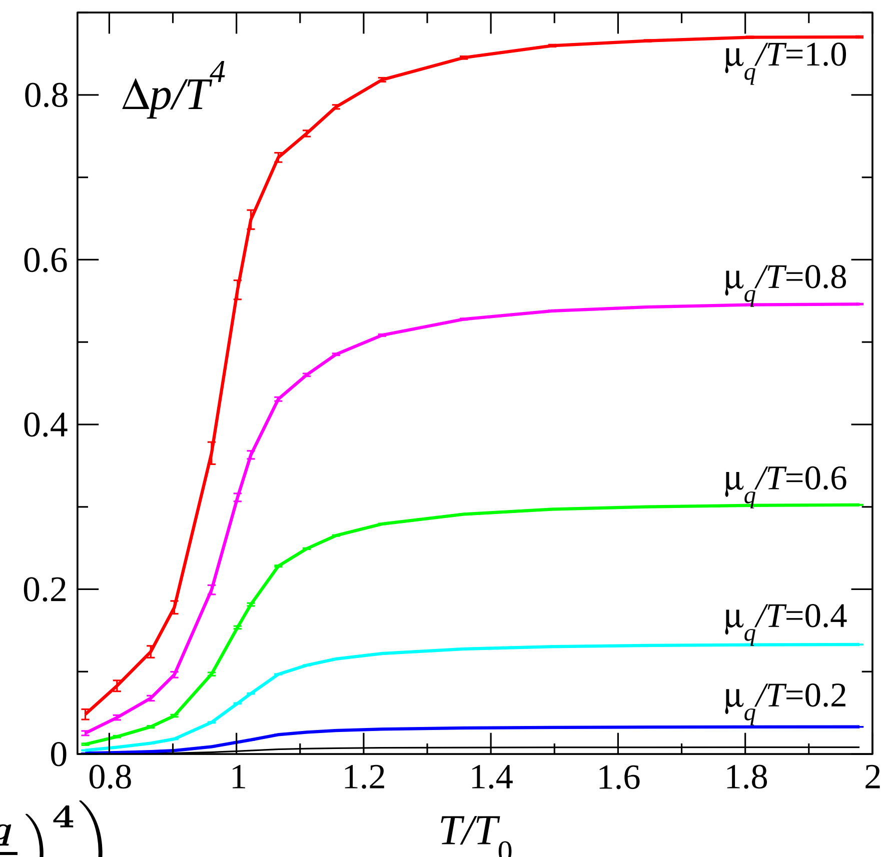
improved staggered fermions;

$n_f = 2,\ m_\pi \simeq 770\ MeV$

contribution from $\mu_q/T > 0$

Taylor expansion, $\mathcal{O}((\mu/T)^4)$





high-T, ideal gas limit

$$\frac{p}{T^4}\Big|_\infty = n_f\left(\frac{7\pi^2}{60} + \frac{1}{2}\left(\frac{\mu_q}{T}\right)^2 + \frac{1}{4\pi^2}\left(\frac{\mu_q}{T}\right)^4\right)$$

RHIC: $\mu_q/T \lesssim 0.1$

The pressure for $\mu_q/T > 0$

C.R. Allton et al. (Bielefeld-Swansea), PRD68 (2003) 014507

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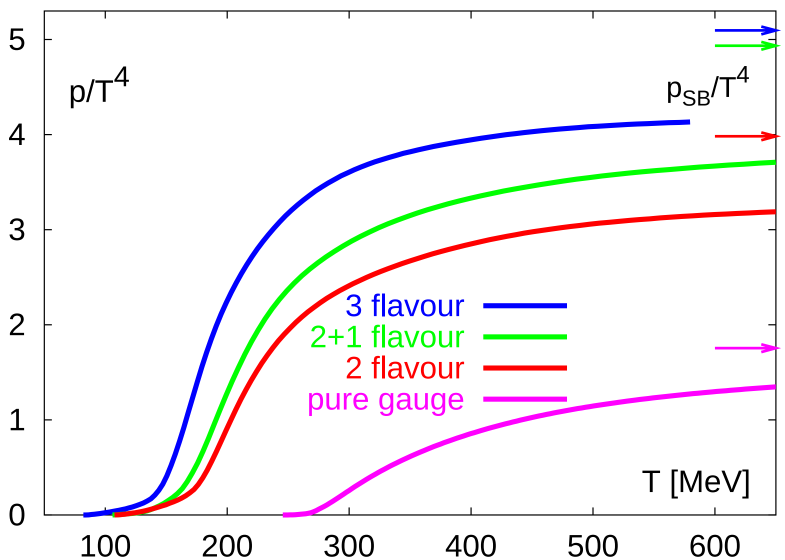
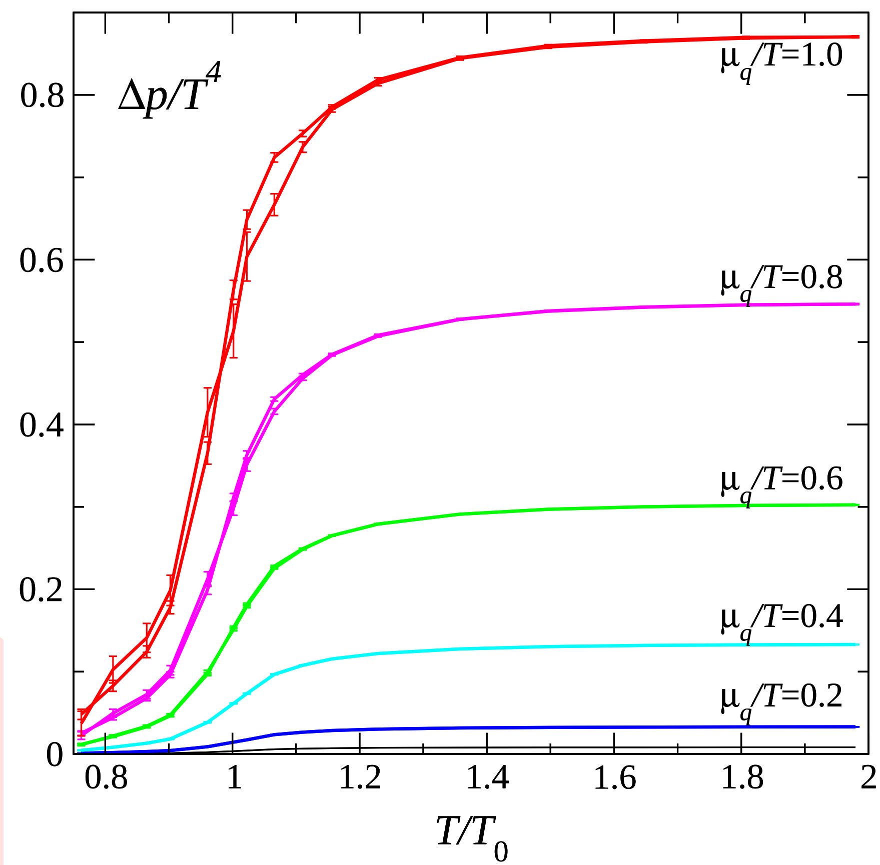


PRD71 (2005) 054508

contribution from $\mu_q/T > 0$

NEW: Taylor expansion, $\mathcal{O}((\mu/T)^6)$

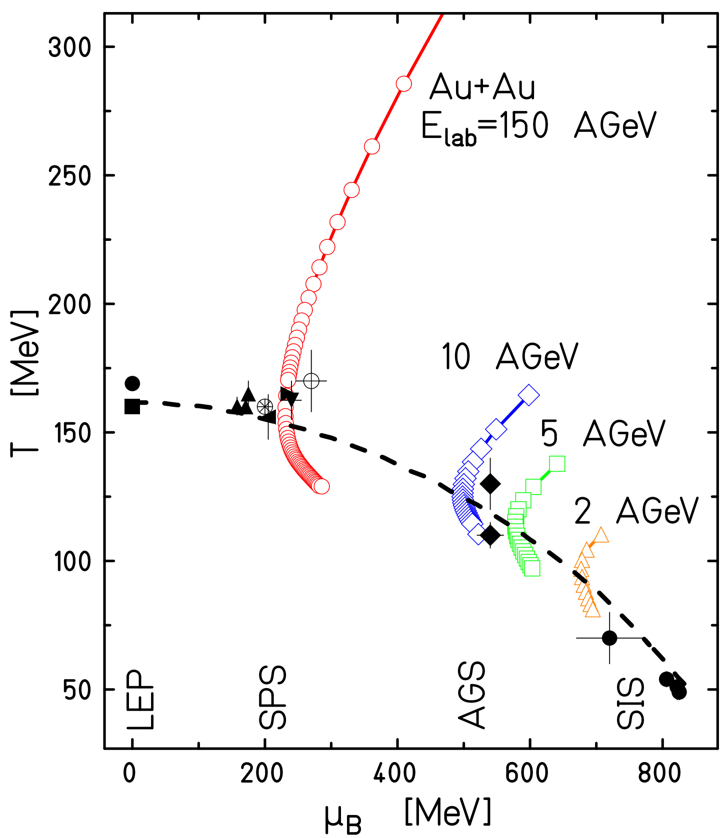


pattern for $\mu_q = 0$ and $\mu_q > 0$ similar;
quite large contribution in hadronic phase;
$\mathcal{O}((\mu/T)^6)$ correction small for $\mu_q/T \lesssim 1$

RHIC: $\mu_q/T \lesssim 0.1$

EoS on HIC trajectories

- dense matter created in a HI-collision expands and cools at fixed entropy and baryon number
  $\Rightarrow$ lines of constant S/N_B in the QCD phase diagram



for example:

isentropic expansion,
"mixed phase model":

V.D. Toneev, J. Cleymans, E.G. Nikonov, K. Redlich, A.A. Shanenko, J. Phys. G27 (2001) 827

EoS on HIC trajectories

- dense matter created in a HI-collision expands and cools at fixed entropy and baryon number
  $\Rightarrow$ lines of constant S/N_B in the QCD phase diagram
  - high T: ideal gas

$$\frac{S}{N_B} = 3\,\frac{\frac{32\pi^2}{45 n_f} + \frac{7\pi^2}{15} + \left(\frac{\mu_q}{T}\right)^2}{\frac{\mu_q}{T} + \frac{1}{\pi^2}\left(\frac{\mu_q}{T}\right)^3}$$

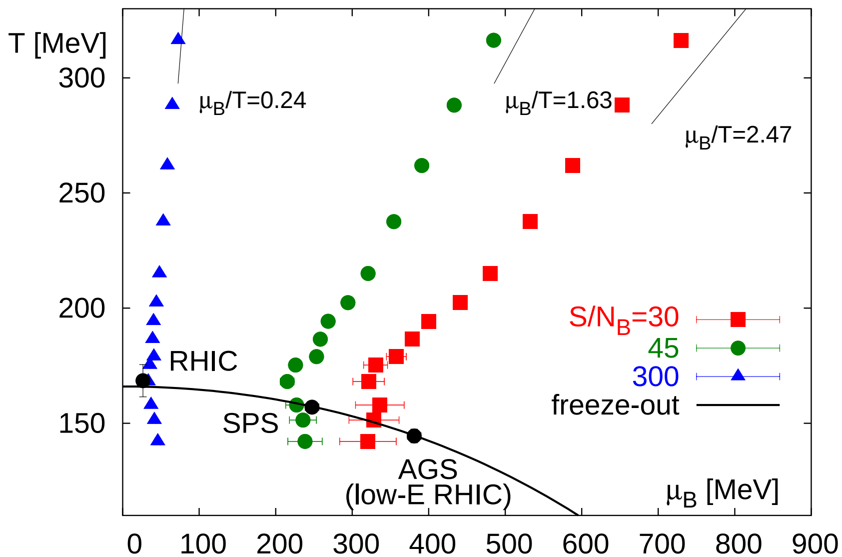
$S/N_B =$ constant $\Leftrightarrow$ μ_q/T constant

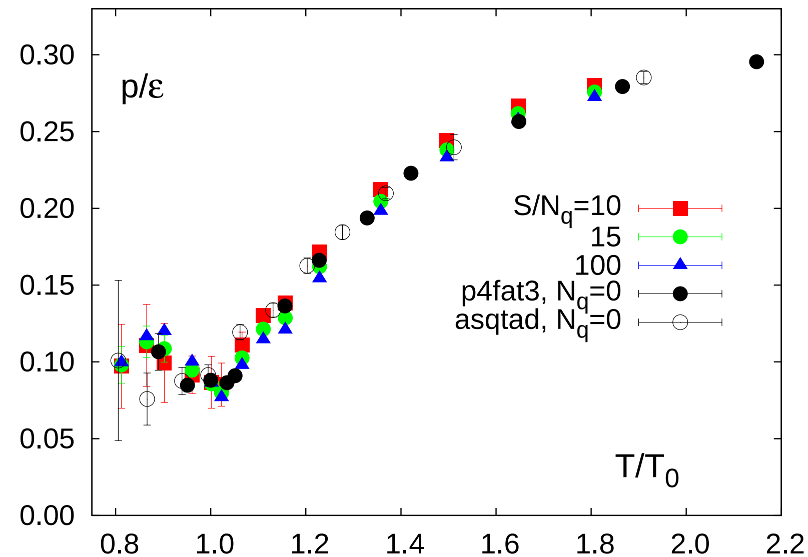
  - low T: nucleon + pion gas

$T \rightarrow 0$: $\mu_q/T \sim c/T$

Isentropic Equation of State: p/ϵ

S. Ejiri, F. Karsch, E. Laermann and C. Schmidt, Phys. Rev. D73 (2006) 054506



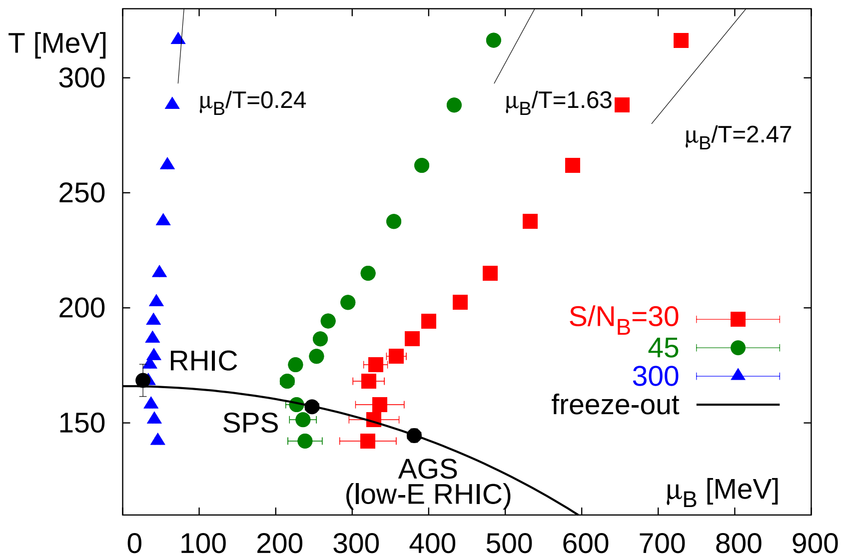


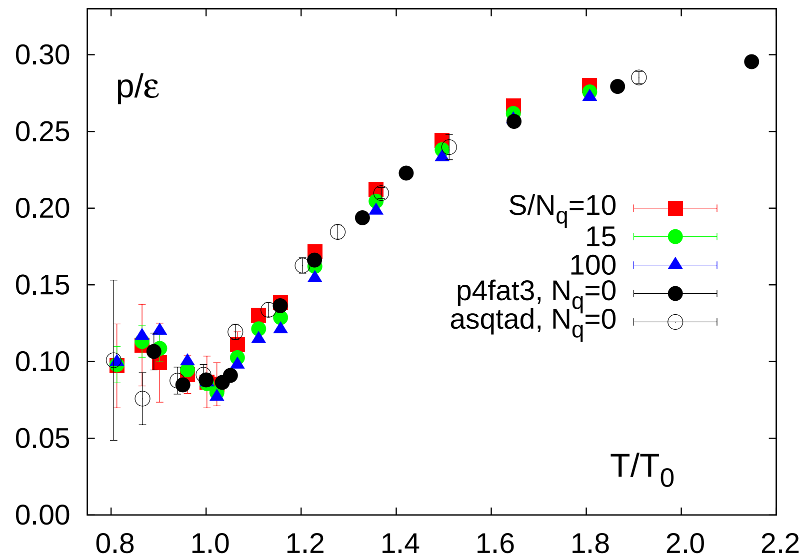
- p/ϵ vs. ϵ shows almost no dependence on S/N_B
- softest point: $p/\epsilon \simeq 0.075$
- phenomenological EoS for $T_0 \lesssim T \lesssim 2T_0$

$$\frac{p}{\epsilon} = \frac{1}{3}\left(1 - \frac{1.2}{1 + 0.5\ \epsilon\ \mathrm{fm}^3/\mathrm{GeV}}\right)$$

Isentropic Equation of State: p/ϵ

S. Ejiri, F. Karsch, E. Laermann and C. Schmidt, Phys. Rev. D73 (2006) 054506





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$\mu > 0$:

so far analyzed only for $m_\pi \simeq 770$ MeV

$$\frac{p}{\epsilon} = \frac{1}{3}\left(1 - \frac{1.2}{1 + 0.5\ \epsilon\ \mathrm{fm}^3/\mathrm{GeV}}\right)$$

awaits confirmation in (2+1)-flavor QCD with light quarks

Conclusions

- non-perturbative QGP

  the QGP is non-perturbative up to high temperatures;
  the running of α_s reflects "remnants of confinement"

- bulk thermodynamics

  the transition between a HG and the QGP is signaled by a rapid change in the energy density;
  calculations with different $\mathcal{O}(a^2)$ improved staggered fermions yield a consistent description of the high temperature phase;

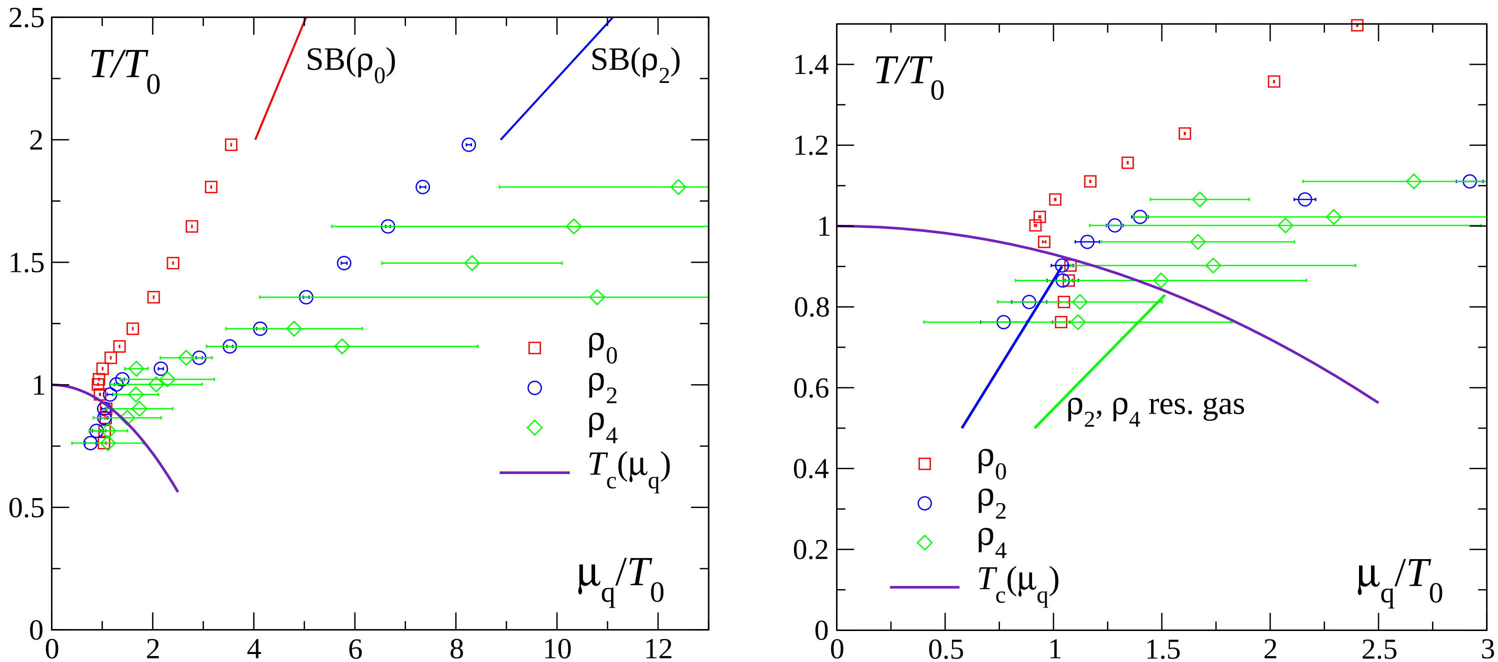
- the transition temperature

  at the physical point of (2+1)-flavor QCD our calculation of T_c yields

$$T_c = 192(7)(4)\text{MeV}$$

Radius of convergence:
lattice estimates vs. resonance gas

- Taylor expansion $\Rightarrow$ estimates for radius of convergence $\rho_{2n} = \sqrt{\left|\frac{c_{2n}}{c_{2n+2}}\right|}$



$T < T_0$: $\rho_n \simeq 1.0$ for all n $\Rightarrow$ $\mu_B^{crit} \simeq 500$ MeV

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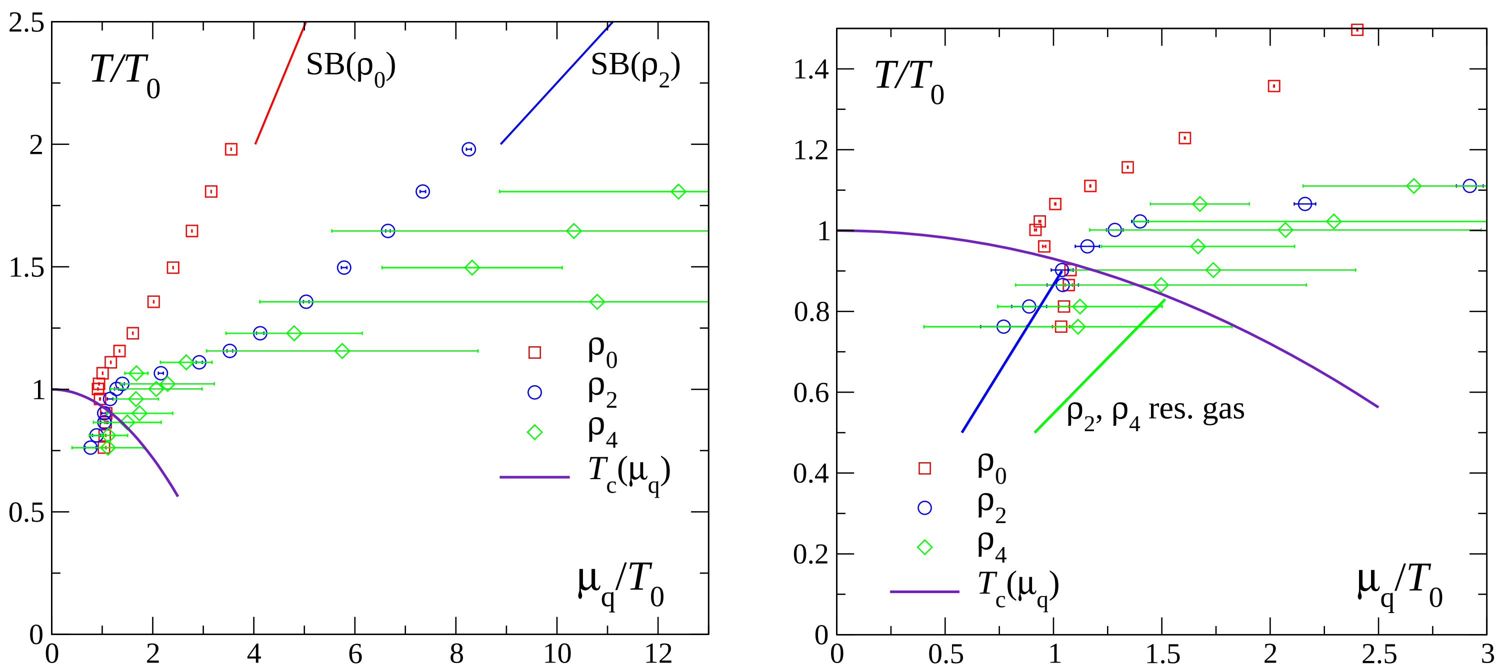

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HOWEVER still consistent with resonance gas!!!

HRG analytic, LGT consistent with HRG $\Rightarrow$ infinite radius of convergence not yet ruled out

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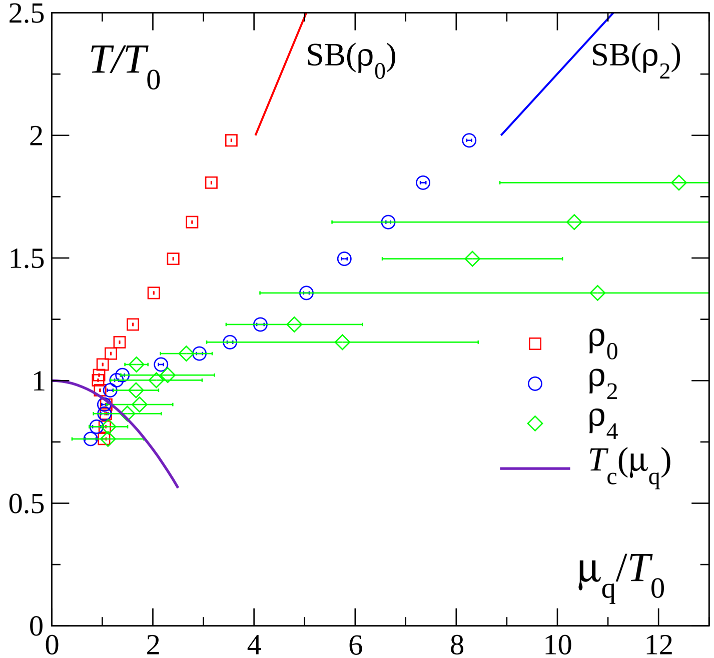
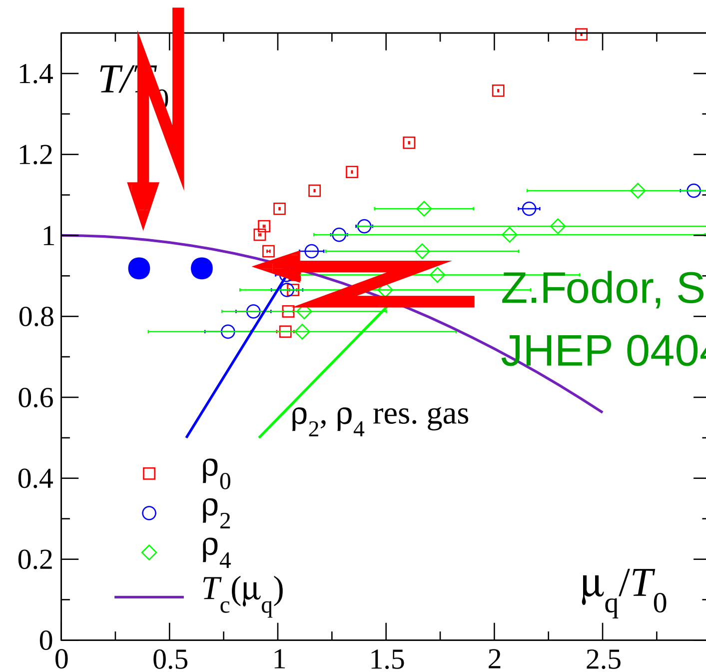
R.V. Gavai, S. Gupta, Phys. Rev. D71 (2005) 114014





Z.Fodor, S.D.Katz
JHEP 0404 (2004) 050

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