Heavy quark bindings at high temperature

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टाटा मूलभूत अनुसंधान संस्थान
TATA INSTITUTE OF FUNDAMENTAL RESEARCH

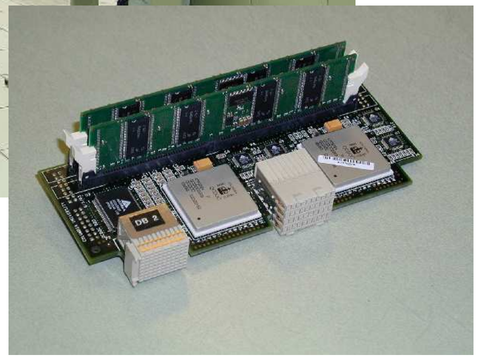
Sourendu Gupta

February 9, 2007

"Machines" for lattice QCD - used by RBC-Bielefeld

apeNEXT

QCDOC



APEmille

Installation in Bielefeld

1999/2001	144 Gflops	APEmille (2 crates)
2005/2006	5 Tflops	apeNEXT (6 racks)

Motivation - Heavy quark bound states above deconfinement

Quarkonium suppression as a probe for thermal properties of hot and dense matter [Matsui and Satz]

- heavy quark potential gets screened
- screening radius related to parton density

$$r_D \sim \frac{1}{g\sqrt{n/T}}$$

- at high T screening radius smaller than size of a quarkonium state

Typical length scales of heavy quark bound states: $1/\Lambda_{QCD} \sim 1$ fm

- screening has to be strong enough to modify short distance behaviour
- detailed analysis of "heavy quark potentials"
  - temperature and r dependence
  - screening properties above deconfinement
  - What is the correct effective potential at finite temperature ?

$$F_1(r,T) = U_1(r,T) - T\, S_1(r,T)$$

Heavy quark bound states above deconfinement

Strong interactions in the deconfined phase $T \gtrsim T_c$

Possibility of heavy quark bound states?

Charmonium ($\chi_c, J/\psi$) as thermometer above T_c

Suppression patterns of charmonium/bottomonium

$\Longrightarrow$ Potential models

$\rightarrow$ heavy quark potential (T=0)

$$V_1(r) = -\frac{4}{3}\frac{\alpha(r)}{r} + \sigma\, r$$

$\rightarrow$ heavy quark free energies ($T > T_c$)

$$F_1(r,T) \simeq -\frac{4}{3}\frac{\alpha(r,T)}{r}\, e^{-m(T)r}$$

$\rightarrow$ heavy quark internal energies ($T \neq 0$)

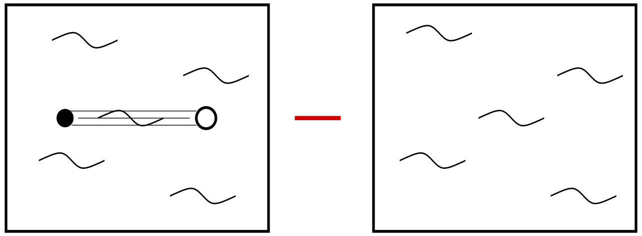
$$F_1(r,T) = U_1(r,T) - T\, S_1(r,T)$$

$\Longrightarrow$ Charmonium correlation functions/spectral functions

Polyakov loop correlation function and free energy:

L. McLerran, B. Svetitsky (1981)

$$\frac{\mathcal{Z}_{\mathbf{Q\bar{Q}}}}{\mathcal{Z}(\mathbf{T})} \simeq \frac{1}{\mathcal{Z}(\mathbf{T})}\int \mathcal{D}\mathbf{A}\ldots\, \mathbf{L}(\mathbf{x})\, \mathbf{L}^{\dagger}(\mathbf{y})\, \exp\left(-\int_{\mathbf{0}}^{\mathbf{1/T}} \mathbf{dt} \int \mathbf{d^3x}\mathcal{L}\left[\mathbf{A},\ldots\right]\right)$$



$$= \quad -\frac{\mathbf{F_{Q\bar{Q}}(r,T)}}{\mathbf{T}}$$

$\mathbf{Q\bar{Q}} = \mathbf{1},\ \mathbf{8},\ \mathbf{av}$

Lattice data used in our analysis:

$\mathbf{N_f = 0}$:
$32^3 \times 4, 8, 16$-lattices
(*Symanzik*)

O. Kaczmarek,
F. Karsch,
P. Petreczky,
F. Zantow (2002, 2004)

$\mathbf{N_f = 2}$:
$16^3 \times 4$-lattices
(*Symanzik, p4-stagg.*)
hybrid-R

$m_\pi/m_\rho \simeq 0.7$ $(m/T = 0.4)$

O. Kaczmarek, F. Zantow (2005),
O. Kaczmarek et al. (2003)

$\mathbf{N_f = 3}$:
$16^3 \times 4$-lattices
(*stagg., Asqtad*)
hybrid-R

$m_\pi/m_\rho \simeq 0.4$

P. Petreczky,
K. Petrov (2004)

$\mathbf{N_f = 2+1}$:
$24^4 \times 4$-lattices
(*Symanzik, p4fat3*)
RHMC

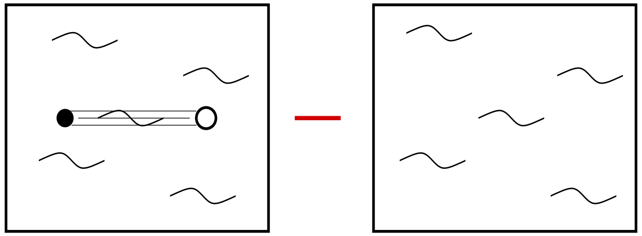
$m_\pi \simeq 220$ **MeV, phys.** m_s

OK, RBC-Bielefeld
preliminary

Polyakov loop correlation function and free energy:

L. McLerran, B. Svetitsky (1981)

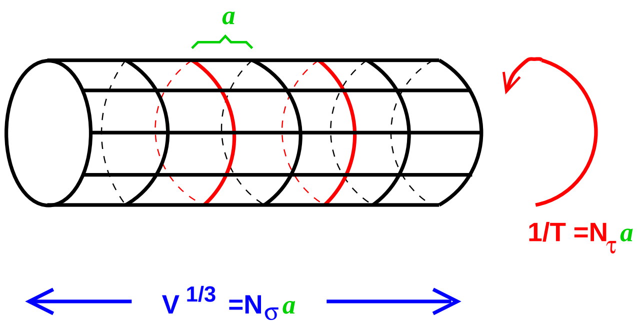
$$\frac{\mathcal{Z}_{\mathbf{Q\bar{Q}}}}{\mathcal{Z}(\mathbf{T})} \simeq \frac{1}{\mathcal{Z}(\mathbf{T})}\int \mathcal{D}\mathbf{A}\ldots\,\mathbf{L}(\mathbf{x})\,\mathbf{L}^{\dagger}(\mathbf{y})\,\exp\left(-\int_{\mathbf{0}}^{\mathbf{1/T}}\mathbf{dt}\int \mathbf{d^3x}\mathcal{L}\left[\mathbf{A},\ldots\right]\right)$$



$$= -\frac{\mathbf{F_{Q\bar{Q}}(r,T)}}{\mathbf{T}}$$

$$\mathbf{Q\bar{Q}} = \mathbf{1},\ \mathbf{8},\ \mathbf{av}$$

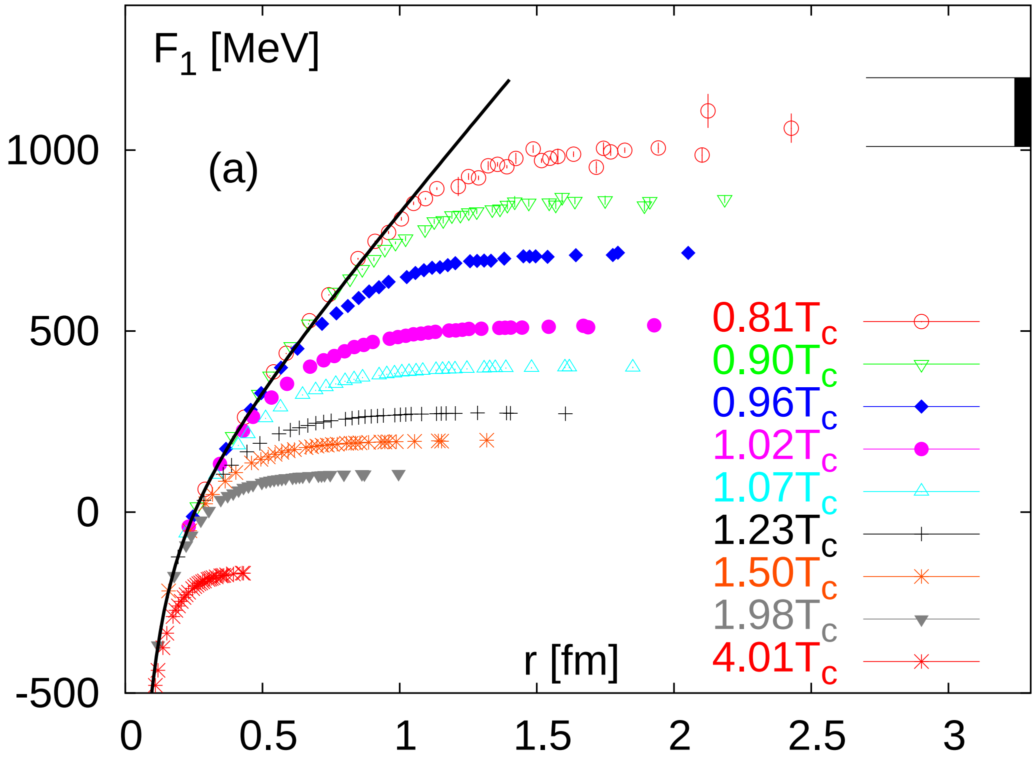
O. Philipsen (2002)
O. Jahn, O. Philipsen (2004)



$$-\ln\left(\langle \tilde{\mathrm{Tr}}L(\mathbf{x})\tilde{\mathrm{Tr}}L^{\dagger}(\mathbf{y})\rangle\right) = \frac{F_{\bar{q}q}(r,T)}{T}$$

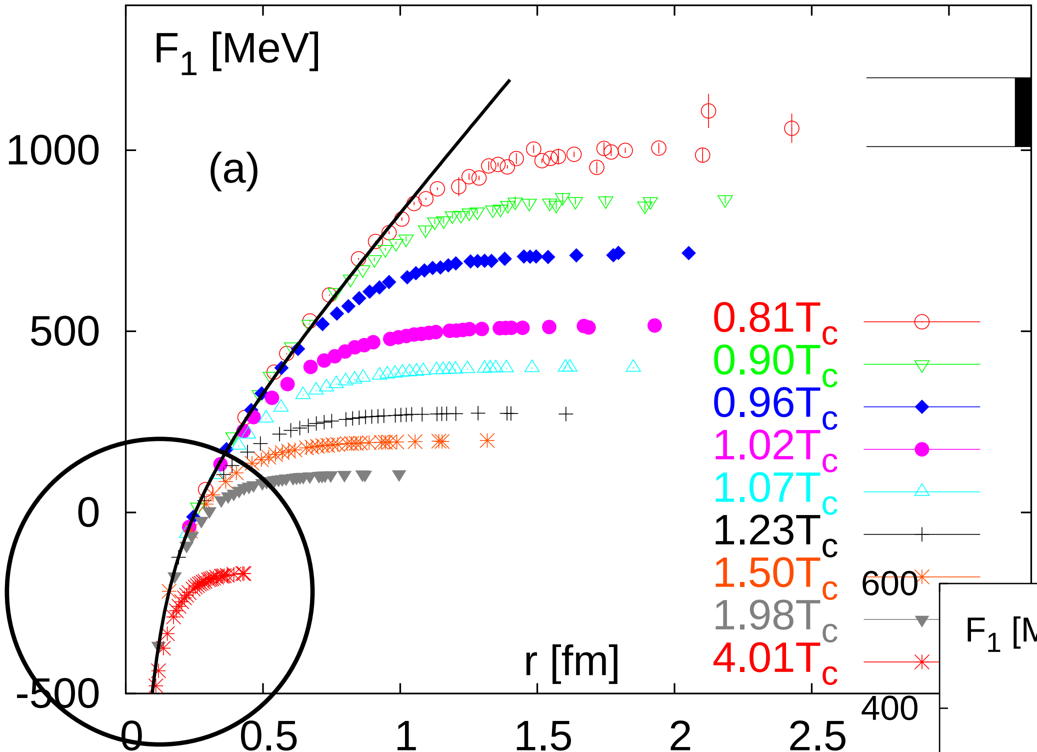
$$-\ln\left(\langle \tilde{\mathrm{Tr}}L(\mathbf{x})L^{\dagger}(\mathbf{y})\rangle\right)\Big|_{GF} = \frac{F_1(r,T)}{T}$$

$$-\ln\left(\frac{9}{8}\langle \tilde{\mathrm{Tr}}L(\mathbf{x})\tilde{\mathrm{Tr}}L^{\dagger}(\mathbf{y})\rangle - \frac{1}{8}\langle \tilde{\mathrm{Tr}}L(\mathbf{x})L^{\dagger}(\mathbf{y})\rangle\Big|_{GF}\right) = \frac{F_8(r,T)}{T}$$



Renormalization of $F(r,T)$
by
matching with T=0 potential

$$e^{-F_1(r,T)/T} = \left(Z_r(g^2)\right)^{2N_\tau} \langle \mathrm{Tr}\ (L_x L_y^\dagger) \rangle$$



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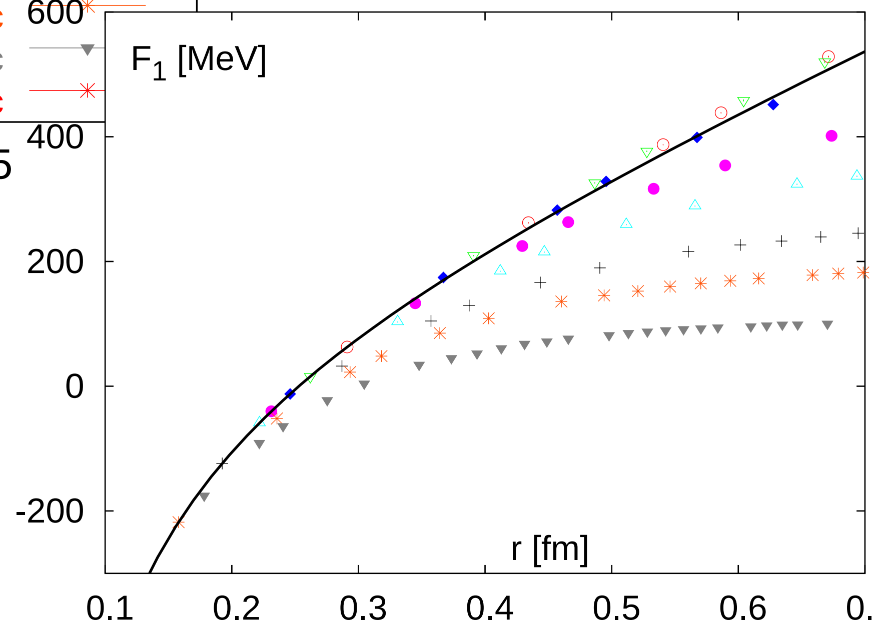
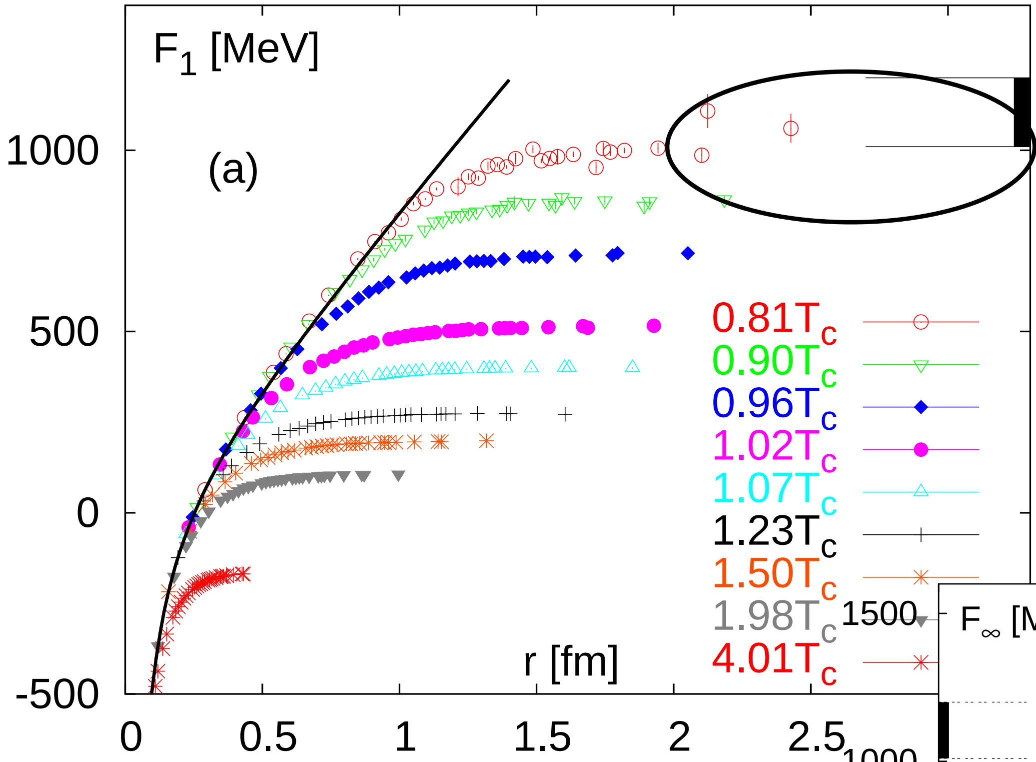


T-independent

$r \ll 1/\sqrt{\sigma}$

$F(r,T) \sim g^2(r)/r$



String breaking

$T < T_c$

$F(r\sqrt{\sigma} \gg 1, T) < \infty$

T-independent

$r \ll 1/\sqrt{\sigma}$

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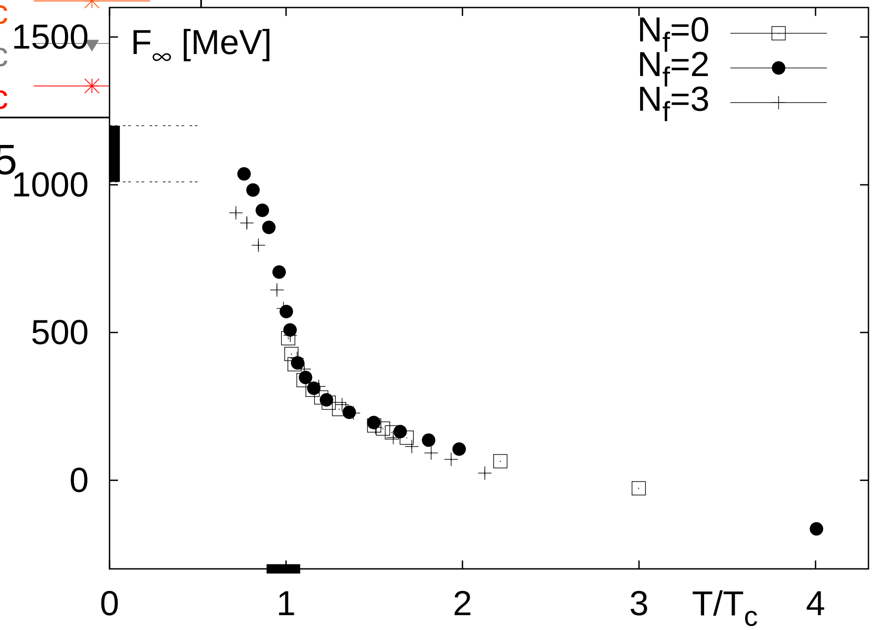
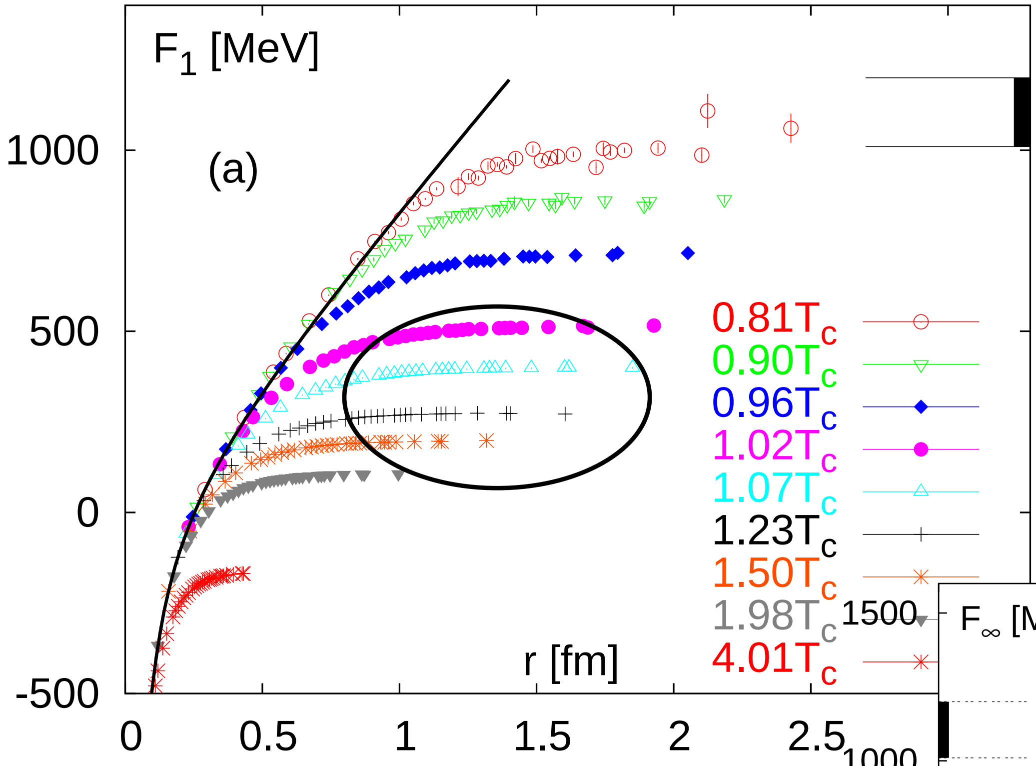




String breaking

$T < T_c$

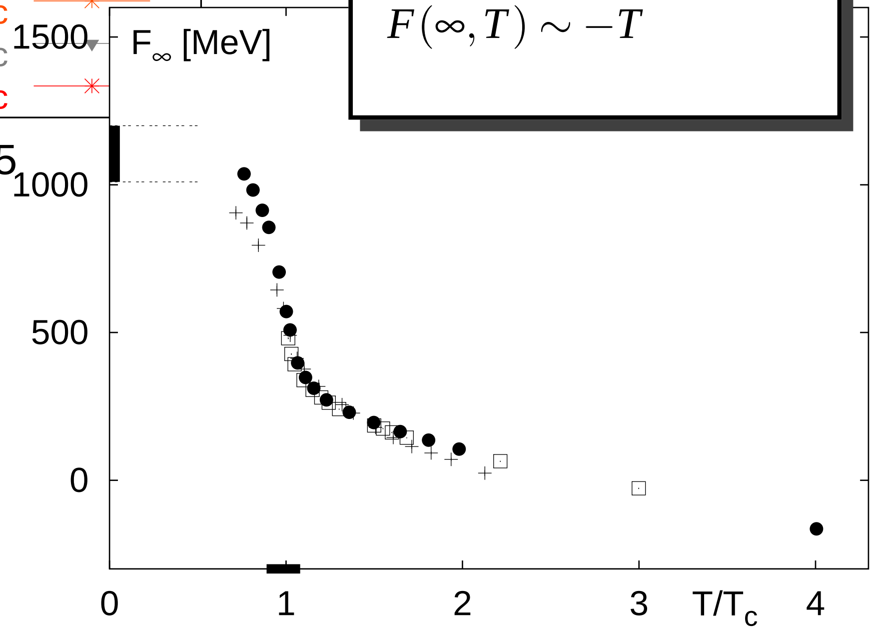
$F(r\sqrt{\sigma} \gg 1, T) < \infty$

high-T physics

$rT \gg 1$; screening

$\mu(T) \sim g(T)T$

$F(\infty, T) \sim -T$



T-independent

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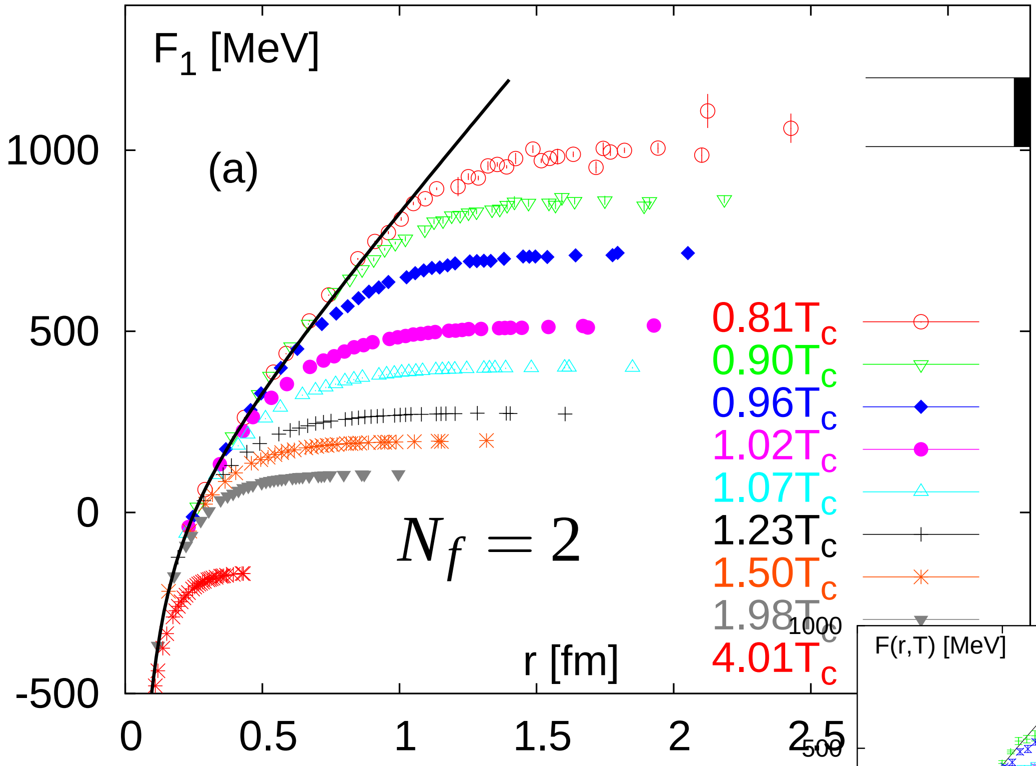

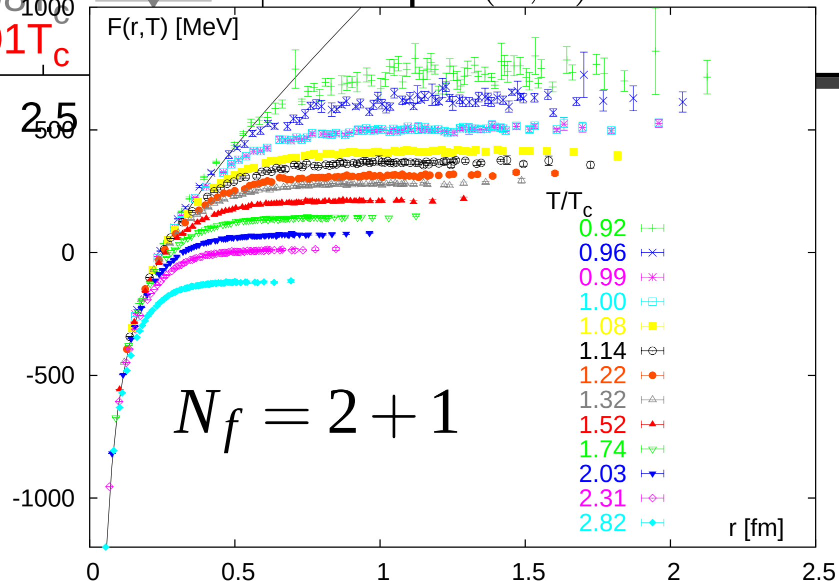

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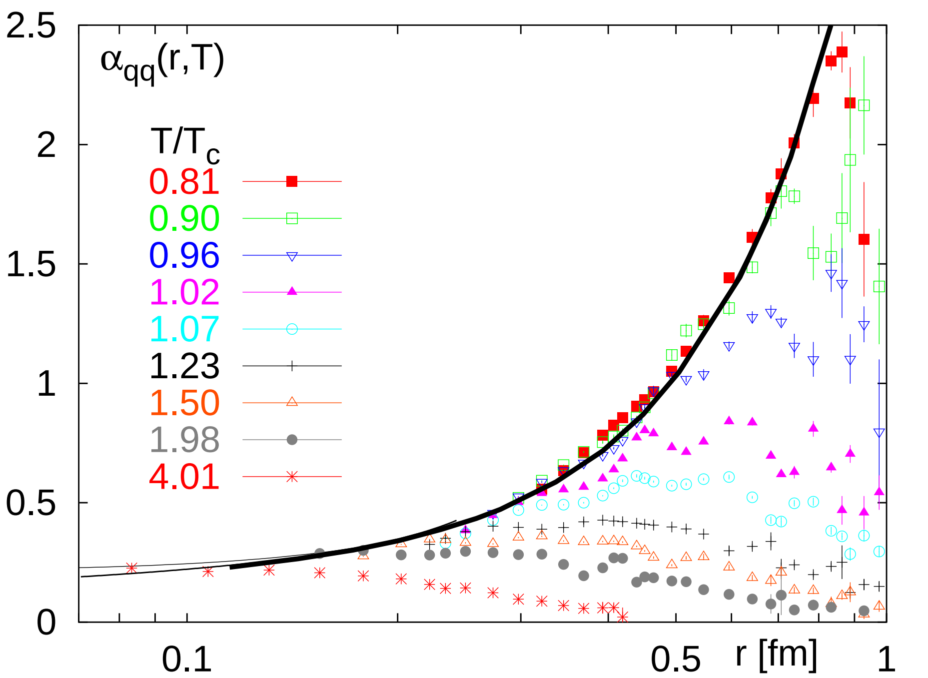
***T*-independent**

$r \ll 1/\sqrt{\sigma}$

$F(r,T) \sim g^2(r)/r$

Temperature depending running coupling



Free energy in perturbation theory:

$$F_1(r,T) \equiv V(r) \simeq -\frac{4}{3}\frac{\alpha(r)}{r} \quad \text{for} \quad r\Lambda_{QCD} \ll 1$$

$$F_1(r,T) \simeq -\frac{4}{3}\frac{\alpha(T)}{r} e^{-m_D(T)r} \quad \text{for} \quad rT \gg 1$$

QCD running coupling in the qq-scheme

$$\alpha_{qq}(r,T) = \frac{3}{4} r^2 \frac{dF_1(r,T)}{dr}$$

non-perturbative confining part for $r \gtrsim 0.4$ fm

$$\alpha_{qq}(r) \simeq 3/4 r^2 \sigma$$

present below and just above T_c

remnants of confinement at $T \gtrsim T_c$

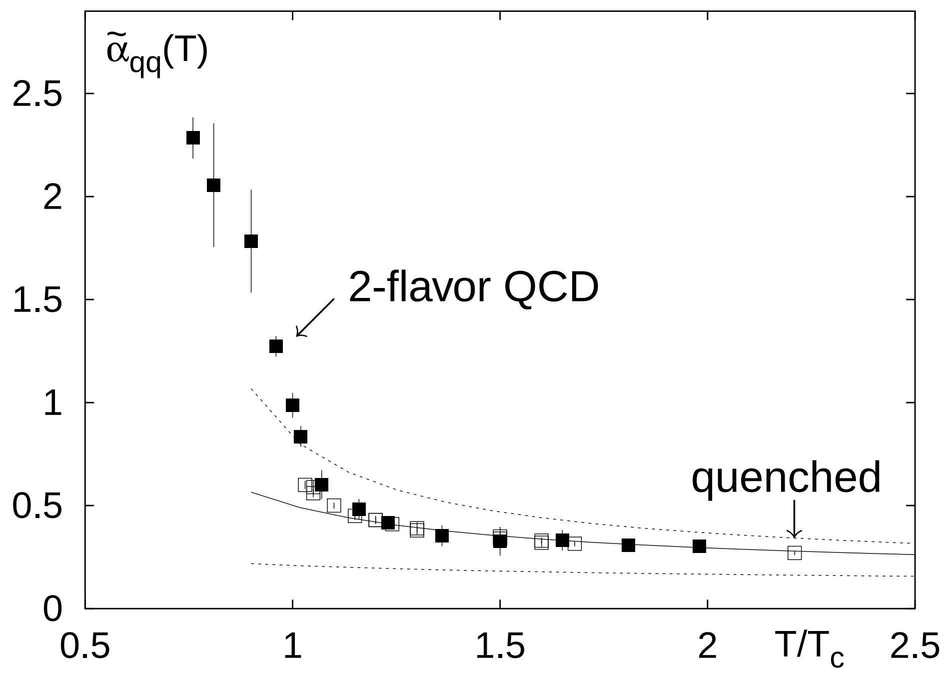
temperature effects set in at smaller r with increasing T

maximum due to screening

$\Longrightarrow$ At which distance do T-effects set in ?

$\Longrightarrow$ definition of the screening radius/mass

$\Longrightarrow$ definition of the T-dependent coupling



define $\tilde{\alpha}_{qq}(T)$ by maximum of $\alpha_{qq}(r,T)$:

$$\tilde{\alpha}_{qq}(T) \equiv \alpha_{qq}(r_{max}, T)$$

perturbative behaviour at high T:

$$g^{-2}_{\text{2-loop}}(T) = 2\beta_0 \ln\left(\frac{\mu T}{\Lambda_{\overline{MS}}}\right) + \frac{\beta_1}{\beta_0} \ln\left(2\ln\left(\frac{\mu T}{\Lambda_{\overline{MS}}}\right)\right),$$

Using $T_c/\Lambda_{\overline{MS}} = 0.77(21)$ we find $\mu = 1.14(2)\pi$

non-perturbative large values near T_c

not a large Coulombic coupling

remnants of confinement at $T \gtrsim T_c$

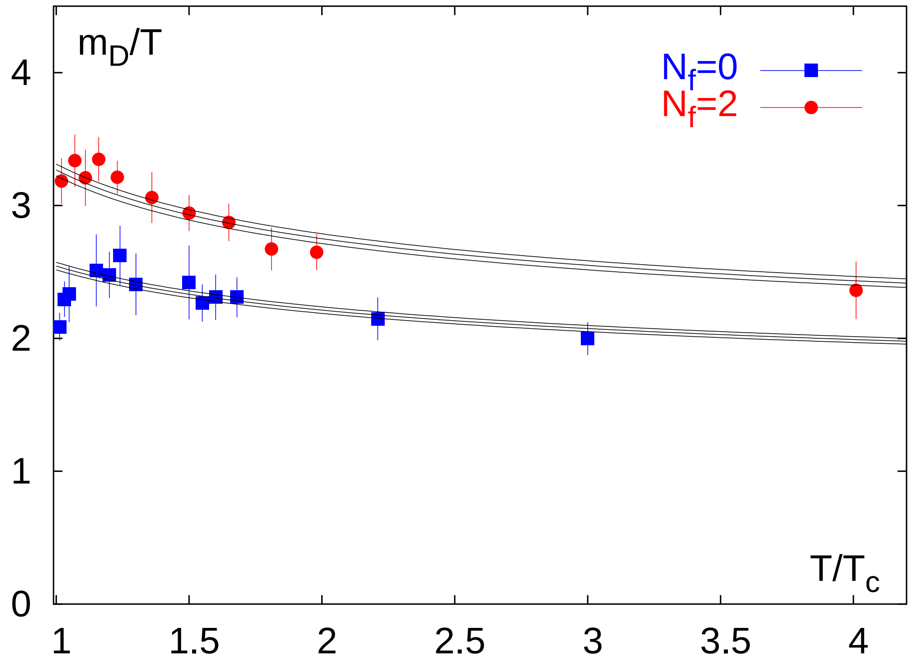
string breaking and screening difficult to separate

slope at high T well described by perturbation theory

$\Longrightarrow$ At which distance do T-effects set in ?

$\Longrightarrow$ calculation of the screening mass/radius

Screening mass - perturbative vs. non-perturbative effects



Screening masses obtained from fits to:

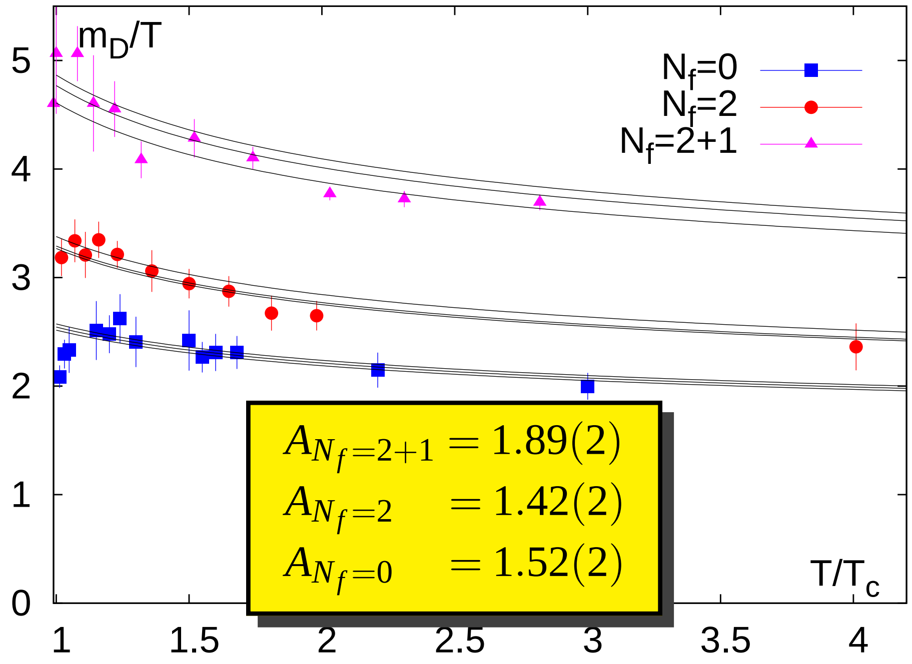
$$F_1(r,T) - F_1(r=\infty,T) = -\frac{4\alpha(T)}{3r} e^{-m_D(T)r}$$

at large distances $rT \gtrsim 1$

leading order perturbation theory:

$$\frac{m_D(T)}{T} = \left(1 + \frac{N_f}{6}\right)^{1/2} g(T)$$

Screening mass - perturbative vs. non-perturbative effects



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$$F_1(r,T) - F_1(r=\infty,T) = -\frac{4\alpha(T)}{3r} e^{-m_D(T)r}$$

at large distances $rT \gtrsim 1$

leading order perturbation theory:

$$\frac{m_D(T)}{T} = A\left(1+\frac{N_f}{6}\right)^{1/2} g(T)$$

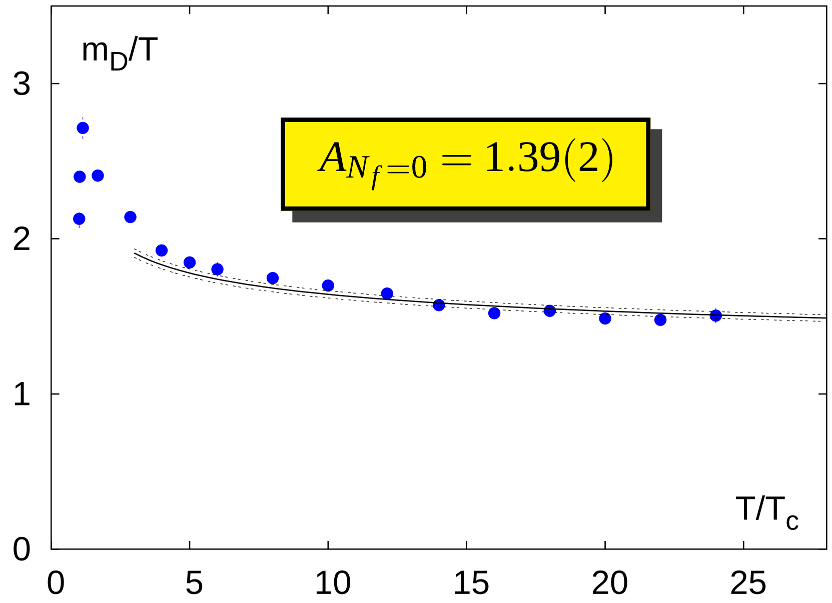
perturbative limit reached very slowly

T depcence qualitatively described by perturbation theory

But $A \approx 1.4$ - 1.5 $\Longrightarrow$ non-perturbative effects

$A \rightarrow 1$ in the (very) high temperature limit

Difference between $N_f = 0, 2$ disappears when converting to physical units

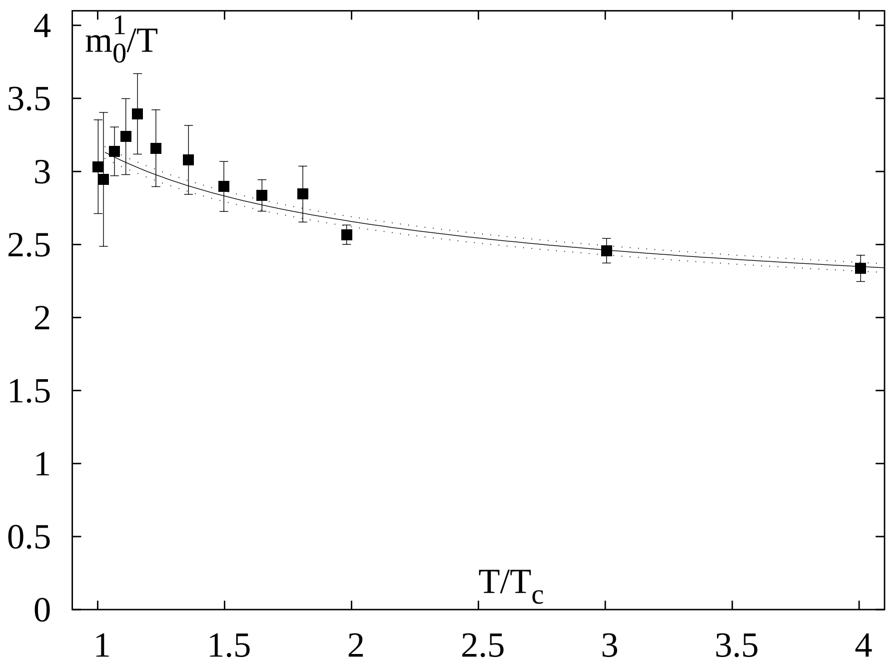


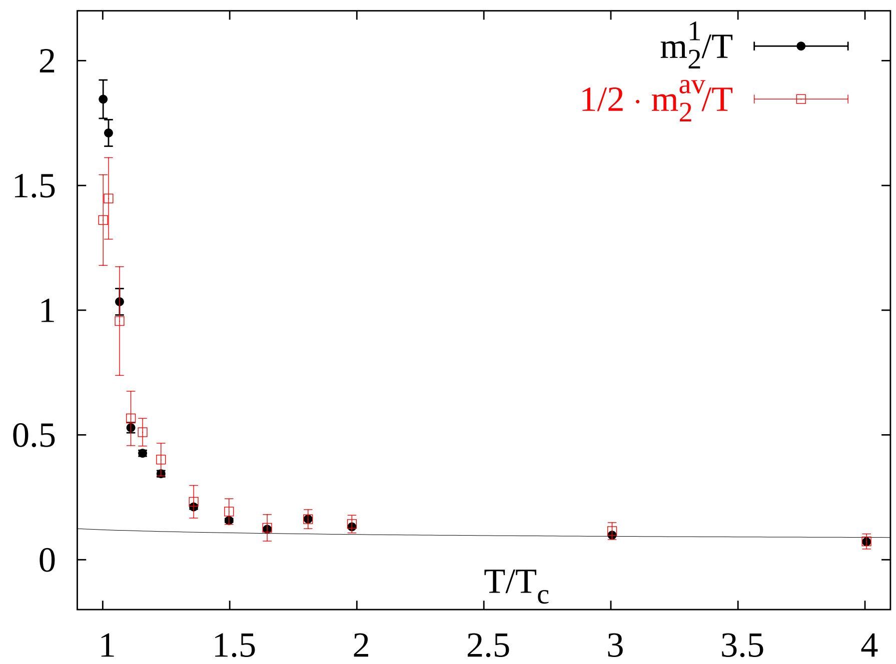
leading order perturbation theory:

$$\frac{m_D(T,\mu_q)}{T} = g(T)\sqrt{1+\frac{N_f}{6}+\frac{N_f}{2\pi^2}\left(\frac{\mu_q}{T}\right)^2}$$

Taylor expansion:

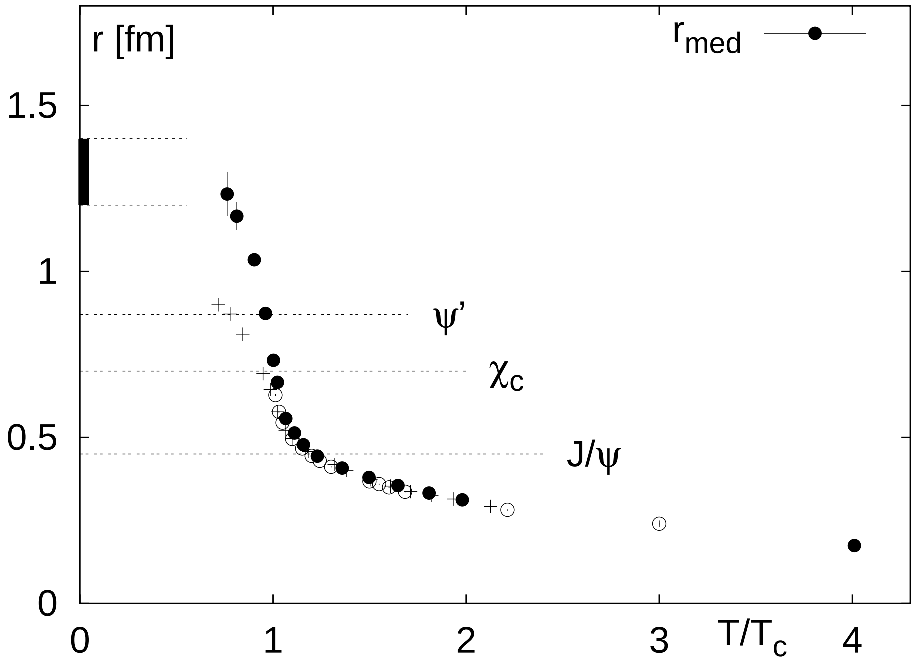
$$m_D(T) = m_0(T) + m_2(T)\left(\frac{\mu_q}{T}\right)^2 + \mathcal{O}(\mu_q^4)$$





$m_2(T)$ agrees with perturbation theory for $T \gtrsim 1.5 T_c$

non-perturbative effects dominated by gluonic sector



bound states above deconfinement?

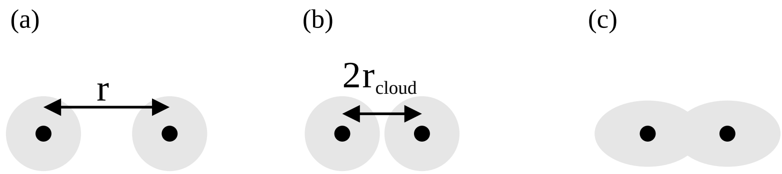
first estimate:

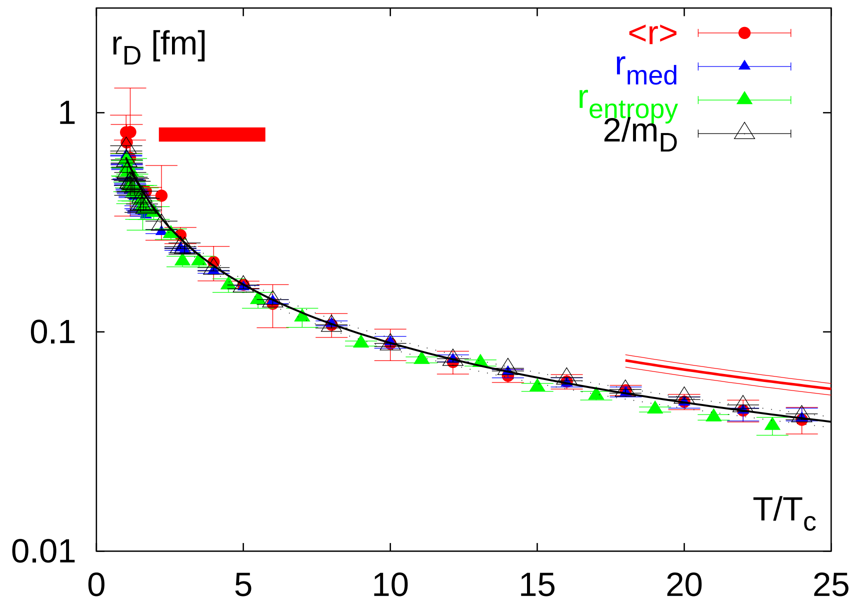
mean charge radii of charmonium states compared to screening radius

thermal modifications on ψ' and χ_c already at T_c

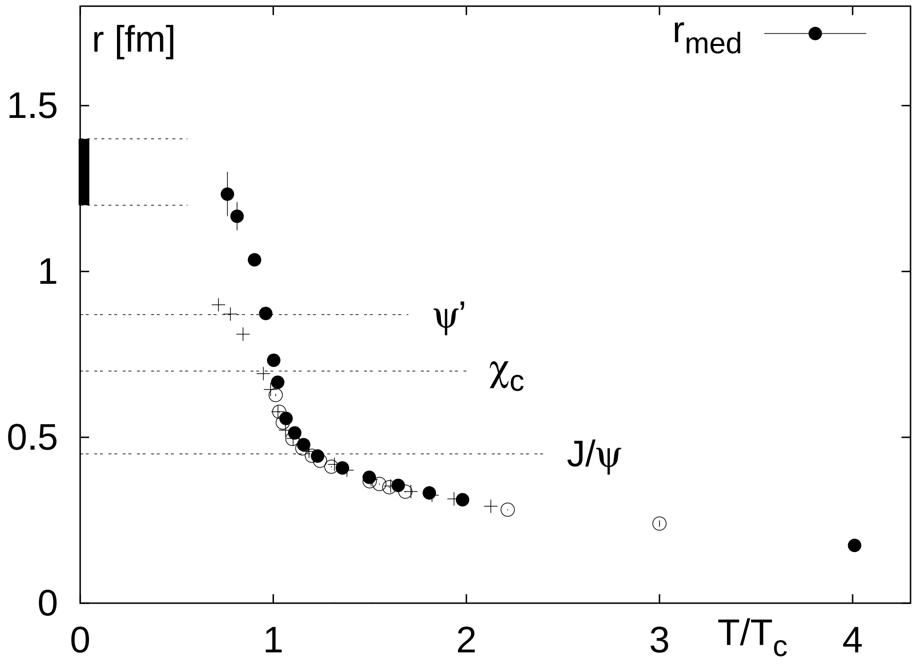
J/ψ may survive above deconfinement

$$\mathbf{r_{med}} \ : \ \mathbf{V(r_{med}) \equiv F_1(r \rightarrow \infty, T)}$$





Heavy quark bound states above T_c?



bound states above deconfinement?

first estimate:

mean charge radii of charmonium states compared to screening radius

thermal modifications on ψ' and χ_c already at T_c

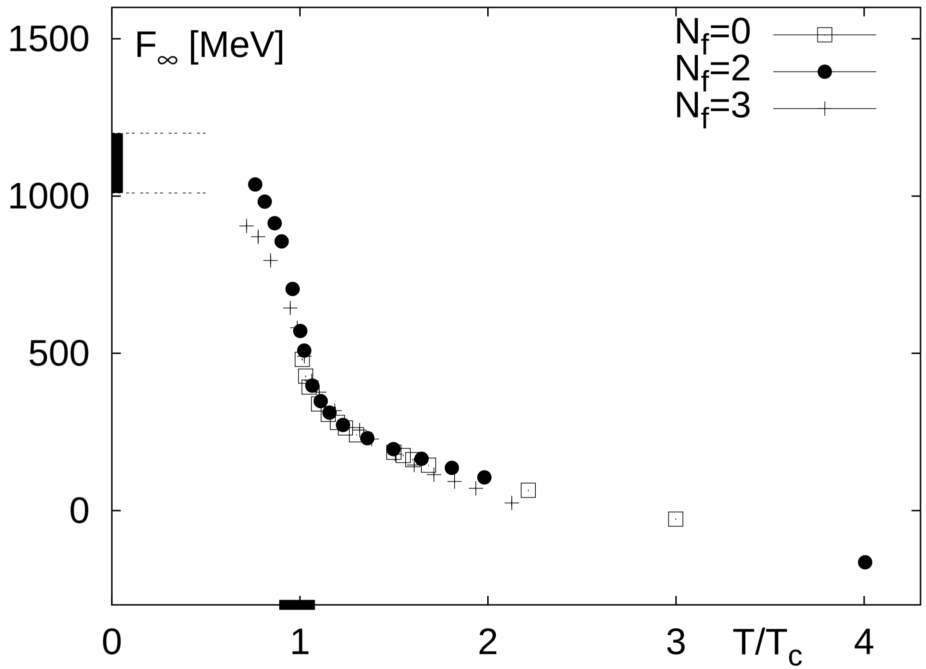
J/ψ may survive above deconfinement

Better estimates:

effective potentials in Schrödinger Equation — Potential models, effective potential $V_{eff}(r,T)$

But: Free energies vs. internal energies $F(r,T) = U(r,T) - TS(r,T)$

direct calculation using correlation functions — Maximum entropy method $\rightarrow$ spectral function



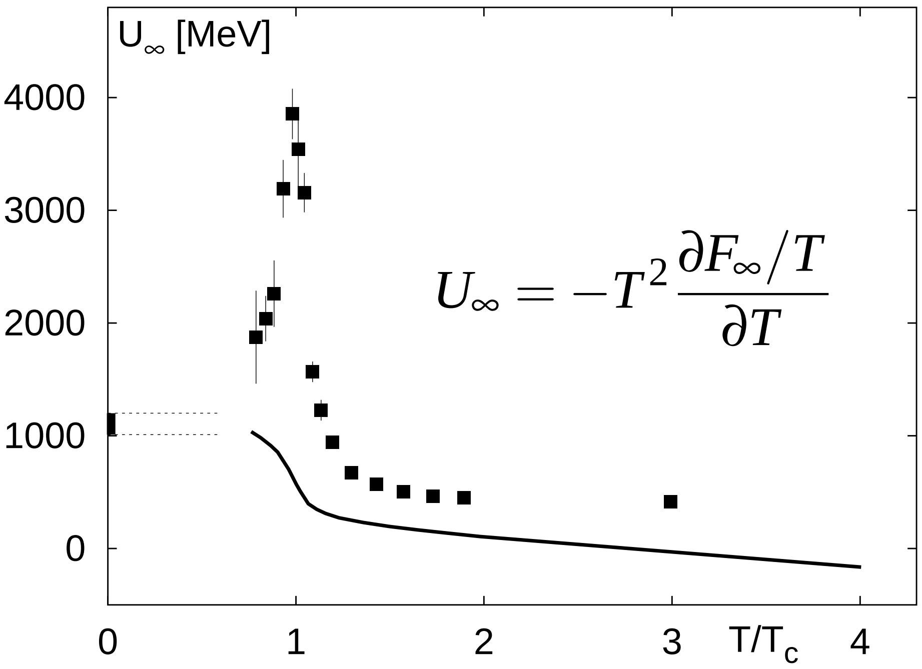
Free energies not only determined by potential energy

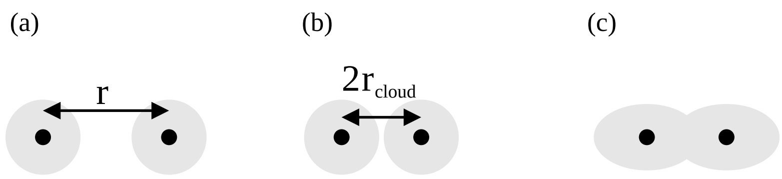
$$F_\infty = U_\infty - T S_\infty$$

Entropy contributions play a role at finite T

$$S_\infty = -\frac{\partial F_\infty}{\partial T}$$

Free energy vs. Entropy at large separations





The large distance behavior of the finite temperature energies is rather related to screening than to t he temperature dependence of masses of corresponding heavy-light mesons!

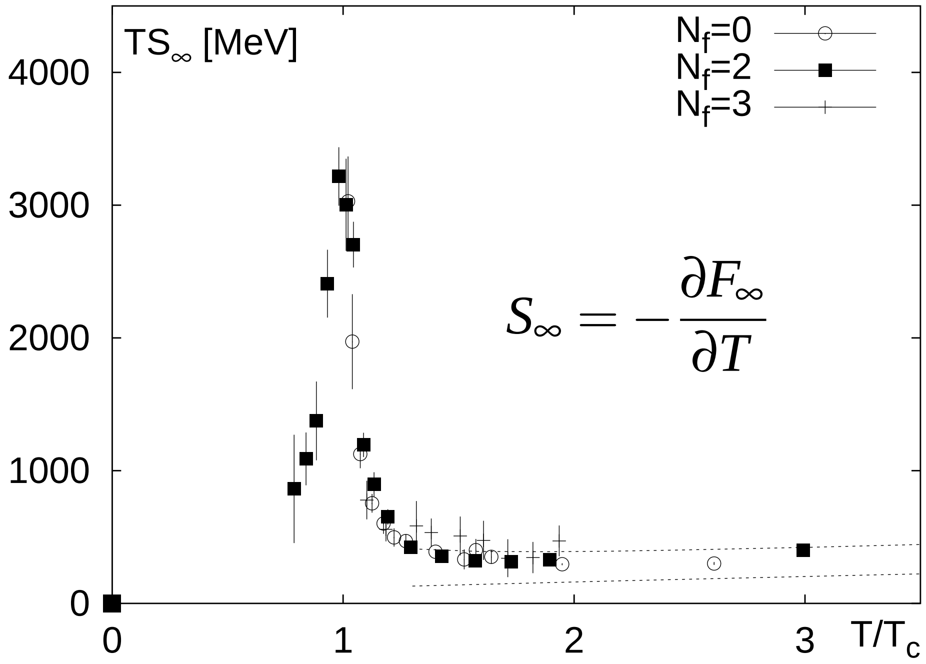
High temperatures:

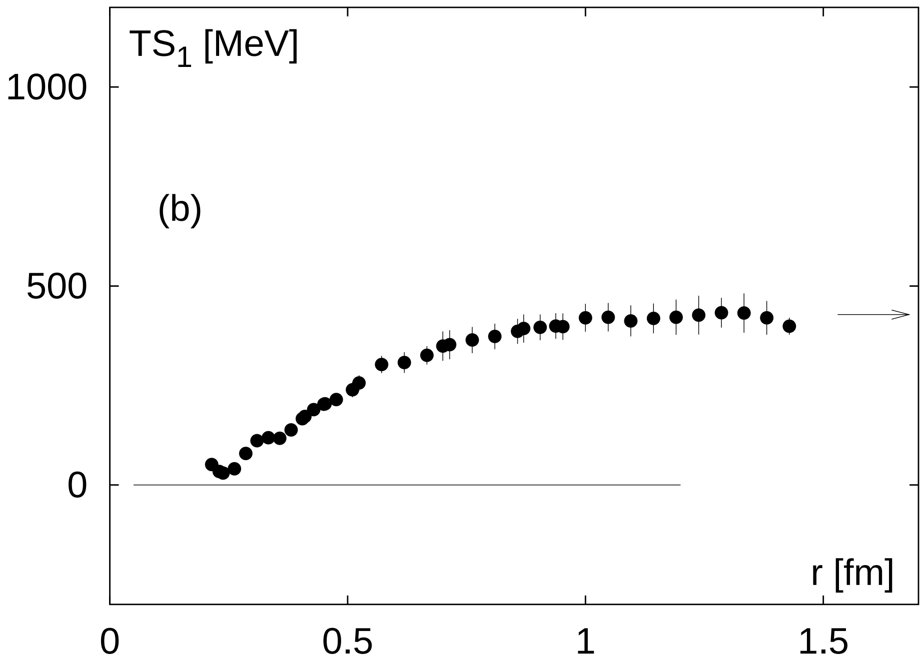
$$\mathbf{F_\infty(T)} \simeq -\frac{\mathbf{4}}{\mathbf{3}}\mathbf{m_D(T)\alpha(T)} \simeq -\mathcal{O}(g^3 T)$$

$$\mathbf{TS_\infty(T)} \simeq -\frac{\mathbf{4}}{\mathbf{3}}\mathbf{m_D(T)\alpha(T)}$$

$$\mathbf{U_\infty(T)} \simeq -\mathbf{4m_D(T)\alpha(T)}\frac{\beta(\mathbf{g})}{\mathbf{g}}$$

$$\simeq -\mathcal{O}(g^5 T)$$



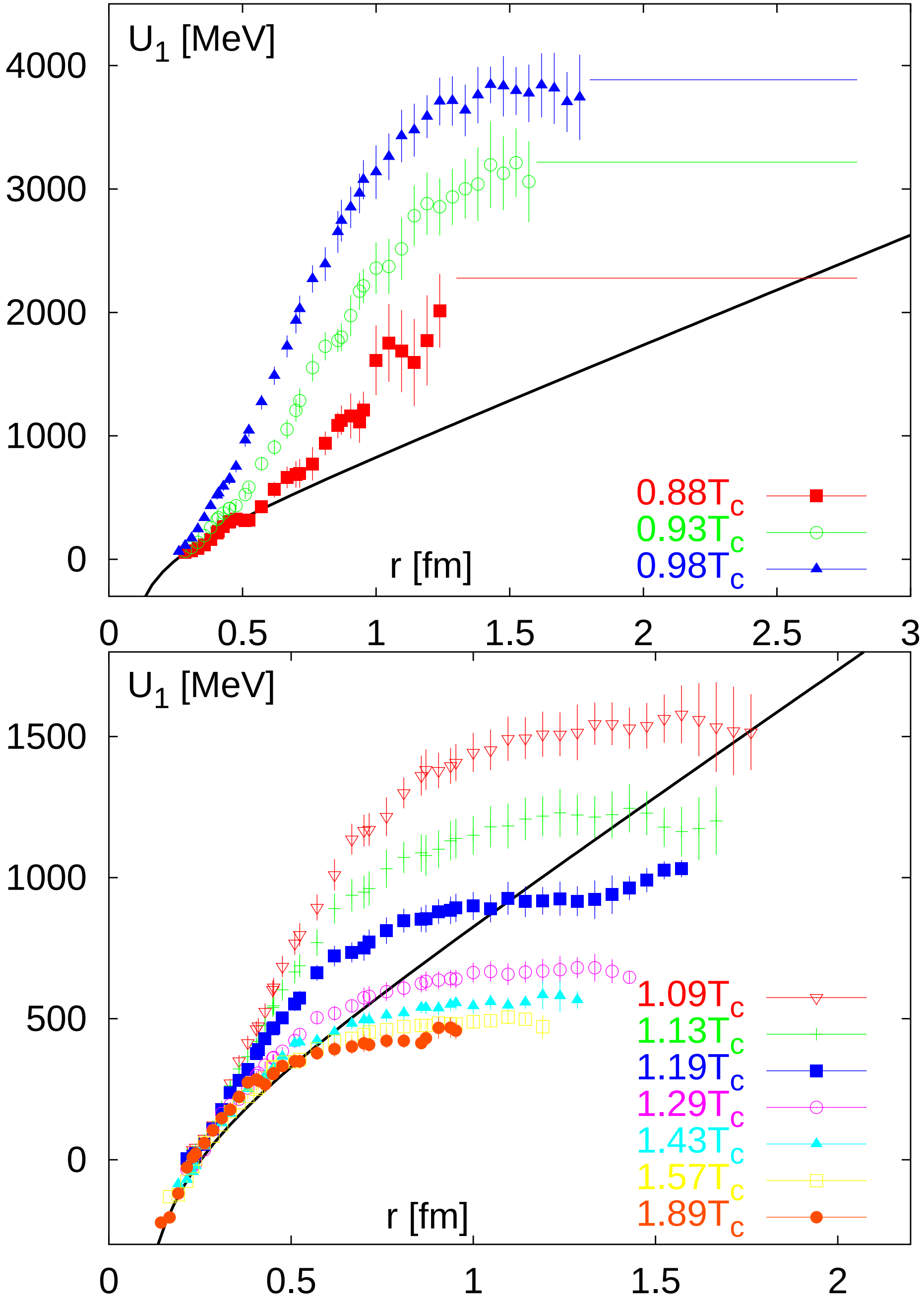


$$F_1(r,T) = U_1(r,T) - TS_1(r,T)$$
$$S_1(r,T) = \frac{\partial F_1(r,T)}{\partial T}$$
$$U_1(r,T) = -T^2 \frac{\partial F_1(r,T)/T}{\partial T}$$

Entropy contributions vanish in the limit $r \to 0$

$$F_1(r \ll 1, T) = U_1(r \ll 1, T) \equiv V_1(r)$$

important at intermediate/large distances



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Entropy contributions vanish in the limit $r \to 0$

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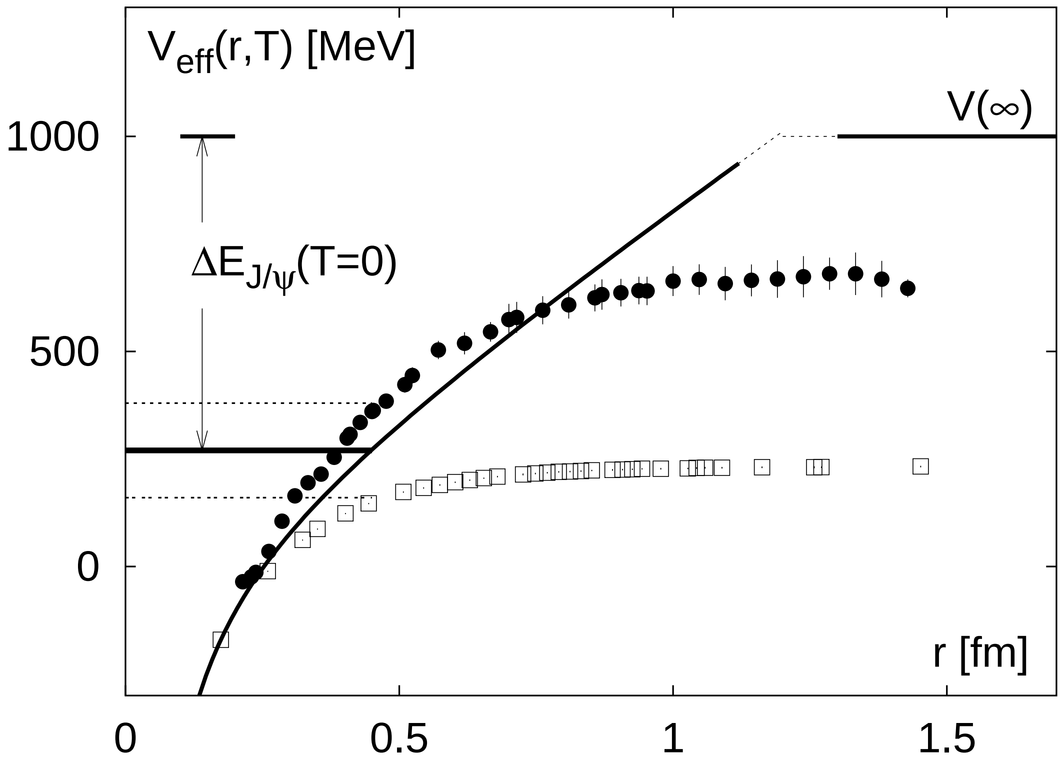
important at intermediate/large distances

$\Longrightarrow$ Implications on heavy quark bound states?

$\Longrightarrow$ What is the correct $V_{eff}(r,T)$?



steeper slope of $V_{eff}(r,T) = U_1(r,T)$

$\Longrightarrow$ J/ψ stronger bound using $V_{eff} = U_1(r,T)$

$\Longrightarrow$ dissociation at higher temperatures compared to $V_{eff}(r,T) = F_1(r,T)$

Schrödinger equation for heavy quarks:

$$\left[2m_f + \frac{1}{m_f}\Delta^2 + V_{eff}(r,T)\right]\Phi_i^f = E_i^f(T)\Phi_i^f \quad , \quad f = \text{charm}, \text{bottom}$$

T-dependent color singlet heavy quark potential mimics in-medium modifications of $q\bar{q}$ interaction

reduction to 2-particle interaction clearly too simple, in particular close to T_c

recent analysis:

using $V_{eff} = F_1$: S.Digal, P.Petreczky, H.Satz, Phys. Lett. B514 (2001)57
using $V_{eff} = V_1$: C.-Y. Wong, hep-ph/0408020
using $V_{eff} = V_1$: W.M. Alberico, A. Beraudo, A. De Pace, A. Molinari, hep-ph/0507084

state	J/ψ	χ_c	ψ'	Υ	χ_b	Υ'	χ_b'	Υ''
$E_s^i[GeV]$	0.64	0.20	0.005	1.10	0.67	0.54	0.31	0.20
T_d/T_c	1.1	0.74	0.1-0.2	2.31	1.13	1.1	0.83	0.74
T_d/T_c	~ 2.1	~ 1.2	~ 1.2	~ 5.0	~ 1.95	~ 1.65	-	-
T_d/T_c	1.75-1.95	1.13-1.15	1.10-1.11	4.4-4.7	1.5-1.6	1.4-1.5	~ 1.2	~ 1.2

Coloured bound states speculated just above T_c (SQGP) *[E.V. Shuryak,I. Zahed (2004/05)]*

- Is the interaction strong enough to support diquarks?
- Potential models for coloured bound states
- Diquark free and internal energies

Colour antitriplet (anti-symmetric) or color sextet (symmetric) state:

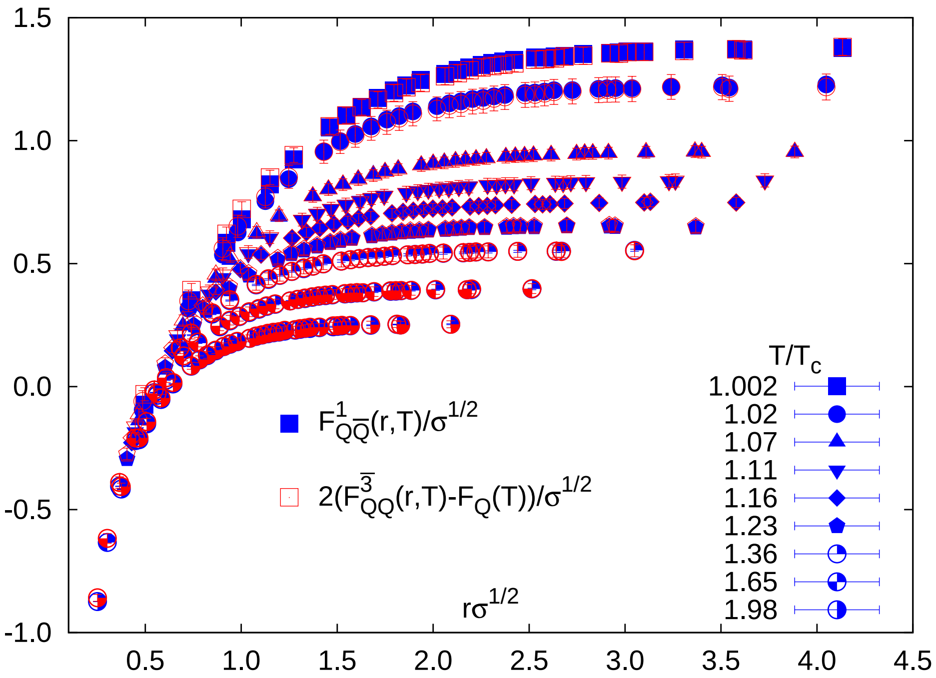
$$3 \otimes 3 = \bar{3} \oplus 6.$$

$$C_{qq}^{\bar{3}}(R,T) = \frac{3}{2}\langle \mathrm{Tr}\, L(0) \mathrm{Tr}\, L(R)\rangle - \frac{1}{2}\langle \mathrm{Tr}\, L(0) L(R)\rangle$$

$$C_{qq}^{6}(R,T) = \frac{3}{4}\langle \mathrm{Tr}\, L(0) \mathrm{Tr}\, L(R)\rangle + \frac{1}{4}\langle \mathrm{Tr}\, L(0) L(R)\rangle$$

$$F_{qq}^{\bar{3},6}(R,T) = -T \ln C_{qq}^{\bar{3},6}(R,T)$$

Diquark free energies - qq vs. $q\bar{q}$ in the deconfined phase



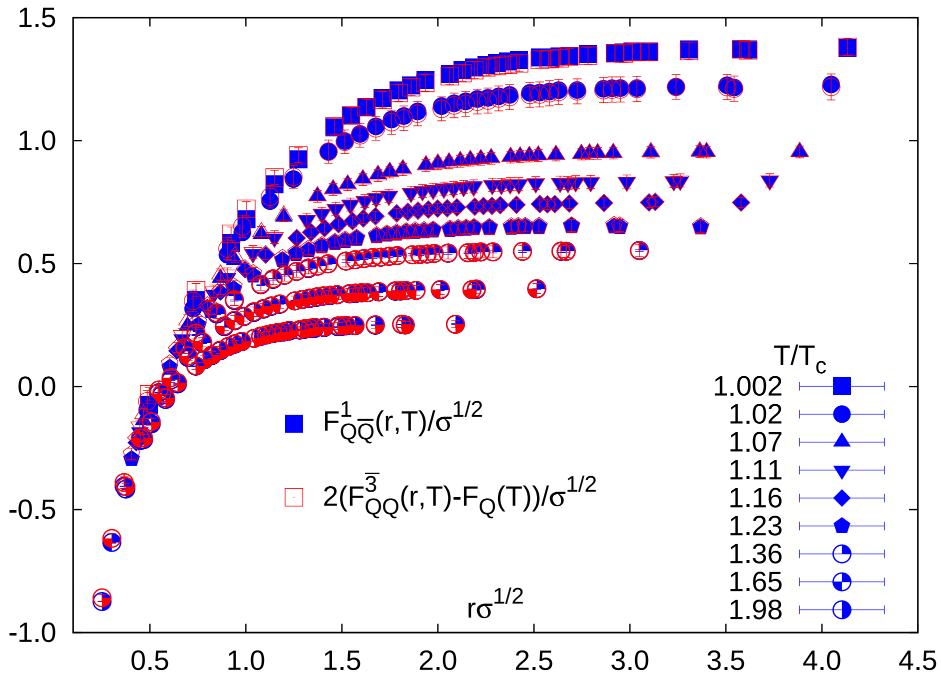
perturbative relation between diquark

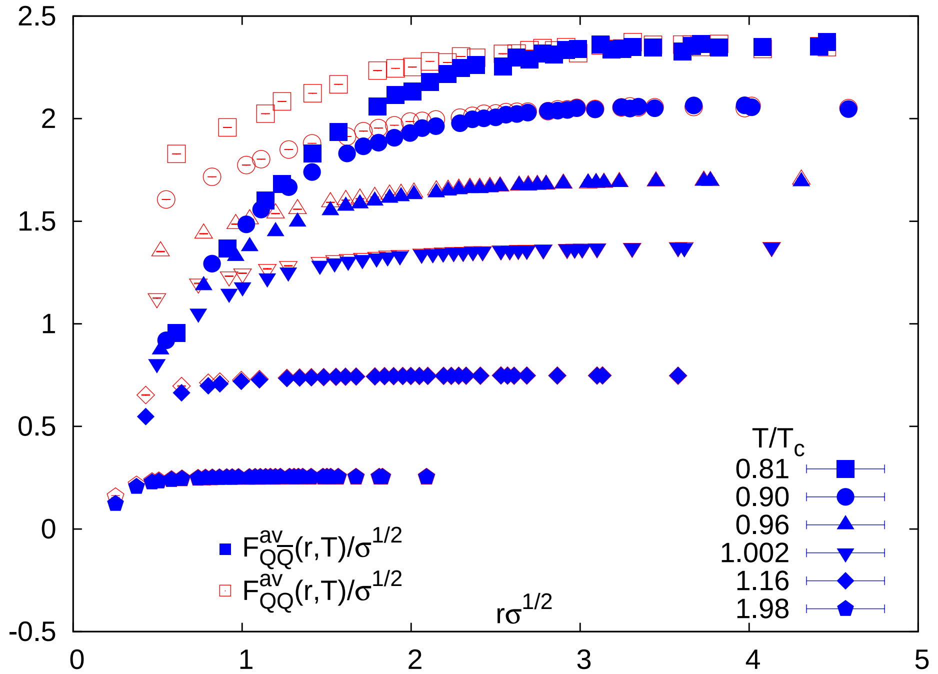
and quark-antiquark free energies

$$F^{\bar{3}}_{qq}(r,T) \simeq \frac{1}{2} F^{1}_{q\bar{q}}(r,T)$$

good approximation above T_c.

Diquark free energies - qq vs. $q\bar{q}$ in the deconfined phase





perturbative relation between diquark

and quark-antiquark free energies

$$F^{\bar{3}}_{qq}(r,T) \simeq \frac{1}{2} F^{1}_{q\bar{q}}(r,T)$$

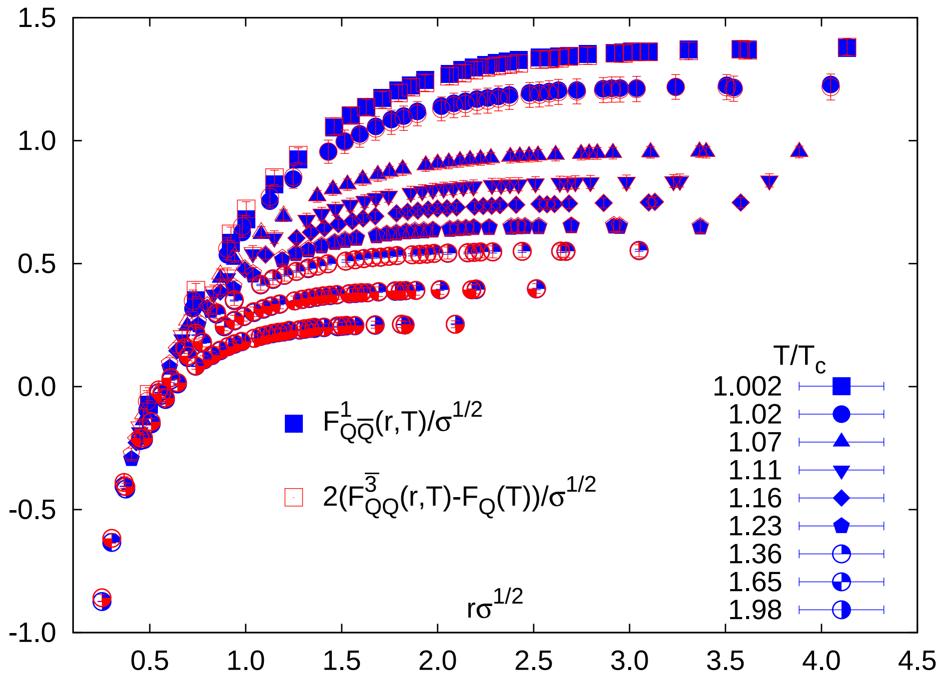
good approximation above T_c.

temperature dependence for all separations

$\Longrightarrow$ entropy contributions play a role for all r.

same asymptotic value for qq and $q\bar{q}$

$\Longrightarrow$ quarks in both systems are screened independently by the medium



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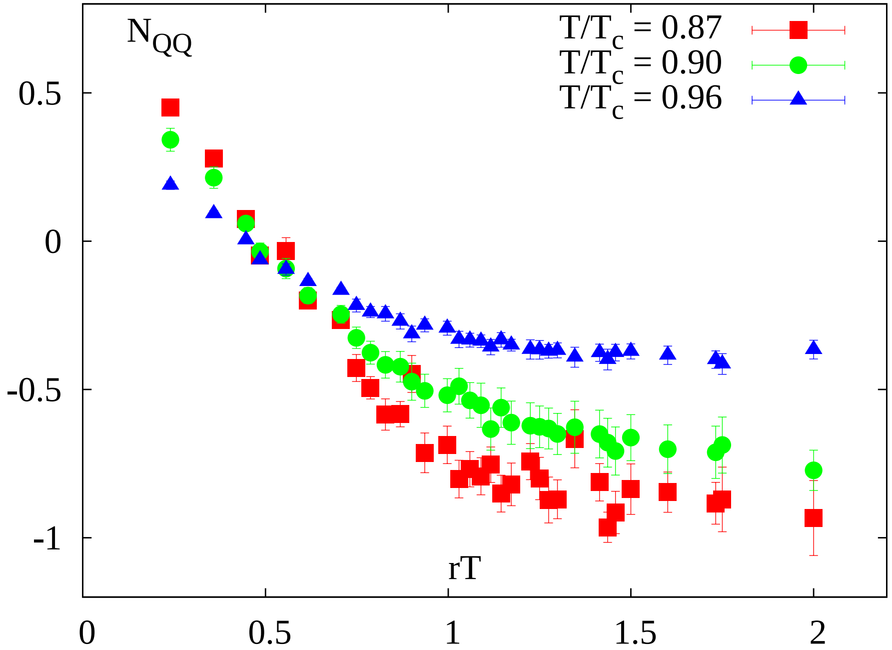
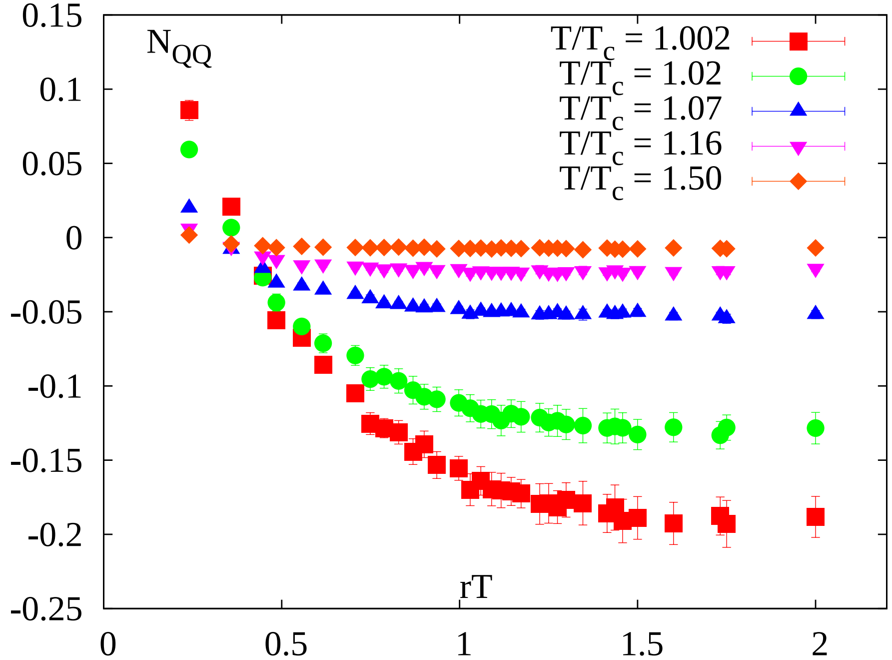
good approximation above T_c.

Dissociation temperatures for heavy $q\bar{q}$ and qq bound states:

state	$\bar{c}c\ (J/\psi)$	cc	$\bar{b}b\ (\Upsilon)$	bb
E^i_s [GeV]	0.06	0	0.3	0.07
$T_{\rm dis}/T_c$	1.5	1.0	3.2	2.1





Net quark number induced by a *qq*-pair:

$$N_{QQ}^{(c)}(r,T) = \langle N_q \rangle_{QQ} \quad = \quad \frac{\left\langle N_q L_{QQ}^{(c)}(r,T) \right\rangle}{\left\langle L_{QQ}^{(c)}(r,T) \right\rangle} \,,$$

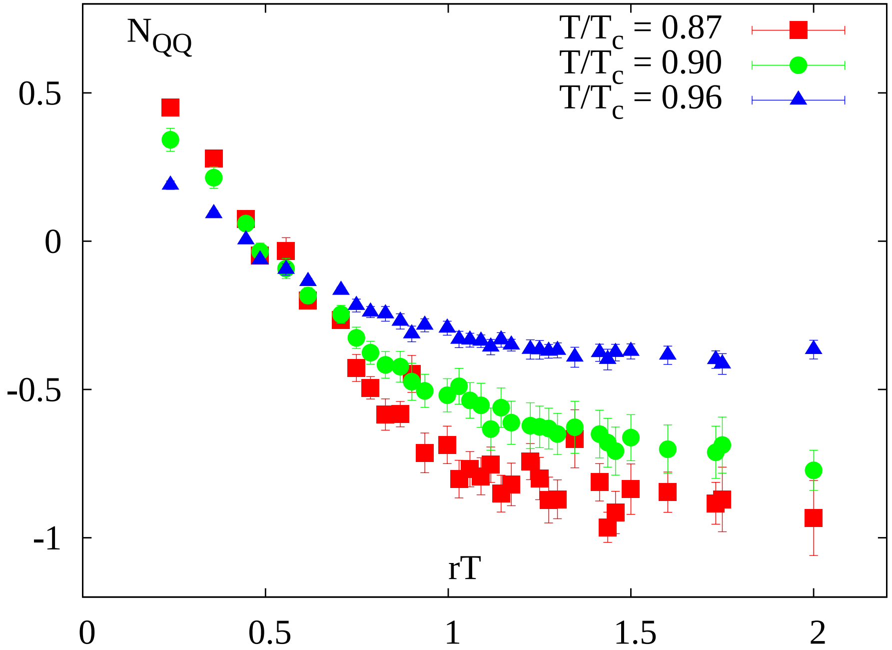
where N_q is the quark number operator,

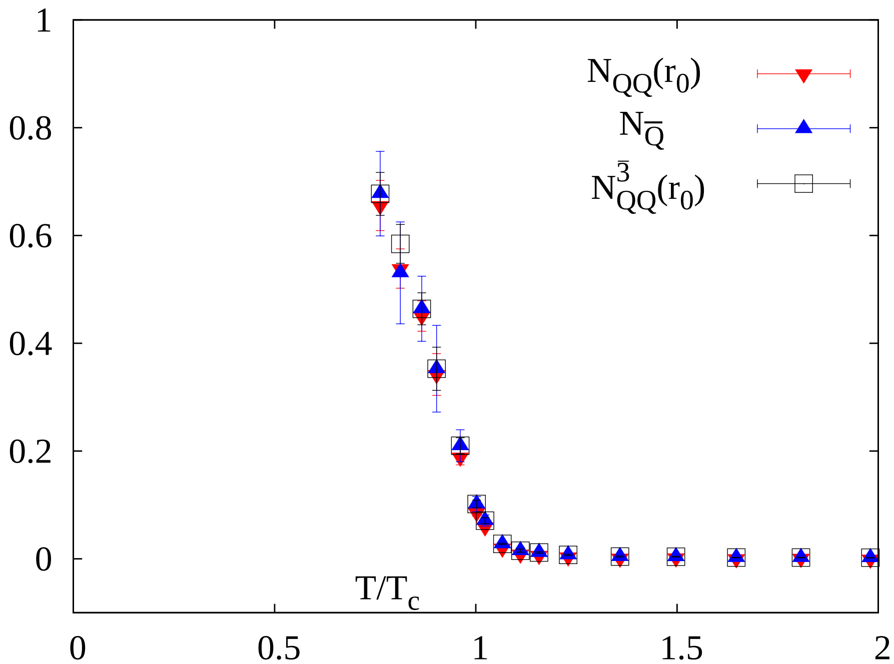
$$N_q = \frac{1}{2} \mathrm{Tr} \left[D^{-1}(\hat{m},0) \left(\frac{\partial D(\hat{m},\mu)}{\partial \mu} \right)_{\mu=0} \right] .$$

Net quark number induced by a single static quark source,

$$N_Q(T) = \langle N_q \rangle_Q = \frac{\left\langle N_q \mathrm{Tr}\, P(\vec{0}) \right\rangle}{\left\langle \mathrm{Tr}\, P(\vec{0}) \right\rangle} .$$

Diquark free energies - Screening and string breaking





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Diquark is neutralized by quarks or antiquarks

from the vacuum to be color neutral overall

$$\lim_{T \to 0} N_{QQ}(r,T) \quad = \quad \begin{cases} 1 & , r < r_c \\ -2 & , r > r_c \end{cases} \,,$$

Renormalized Polyakov loop

Using short distance behaviour of free energies

Renormalization of $F(r,T)$ at short distances

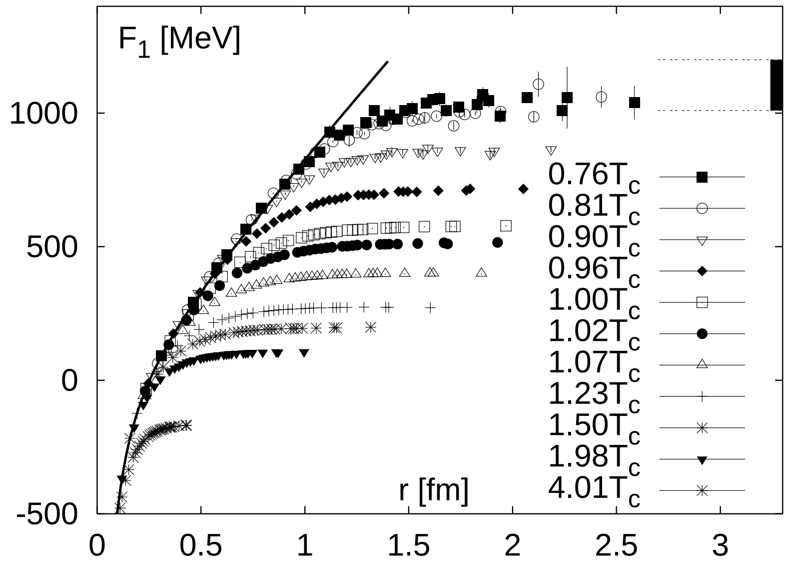
$$e^{-F_1(r,T)/T} = \left(Z_r(g^2)\right)^{2N_\tau} \langle \mathrm{Tr}\ (L_x L_y^\dagger)\rangle$$

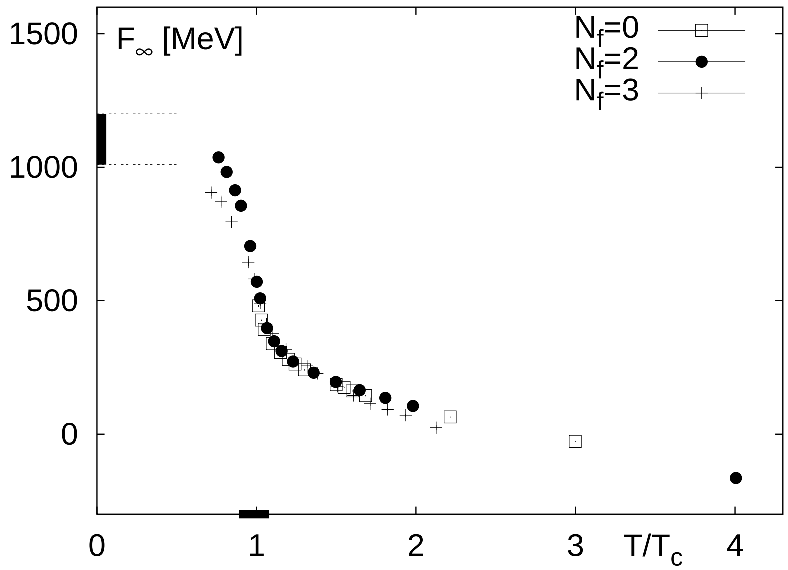
Renormalization of the Polyakov loop

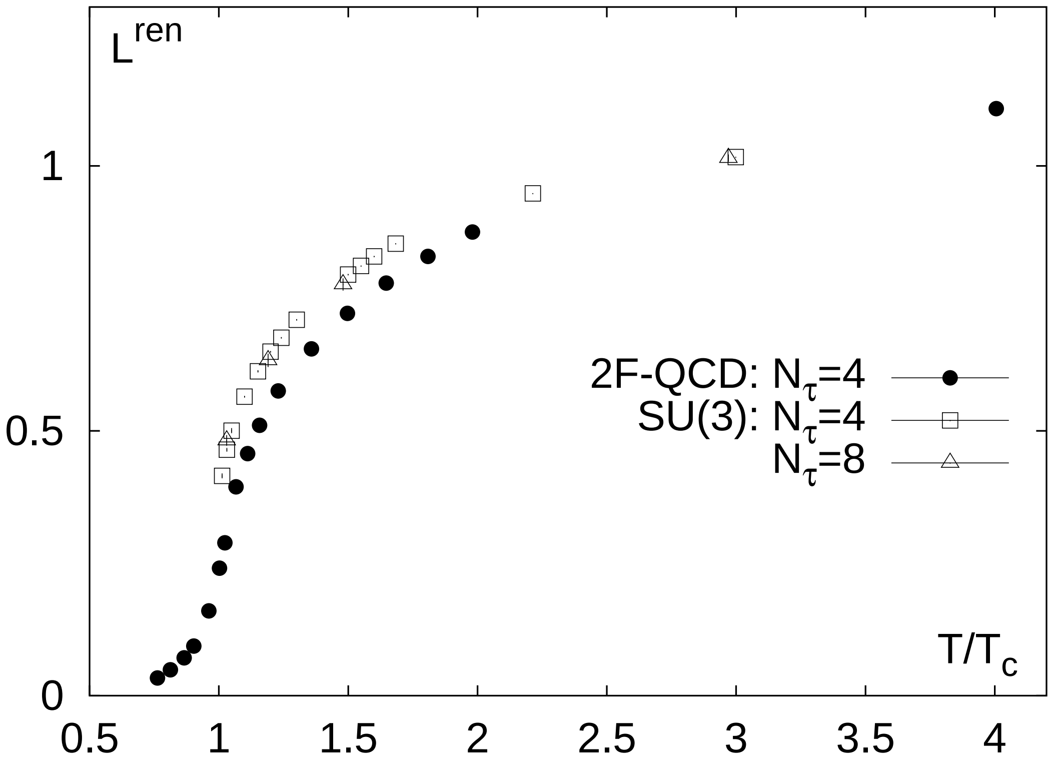
$$L_{ren} = \left(Z_R(g^2)\right)^{N_t} L_{lattice}$$

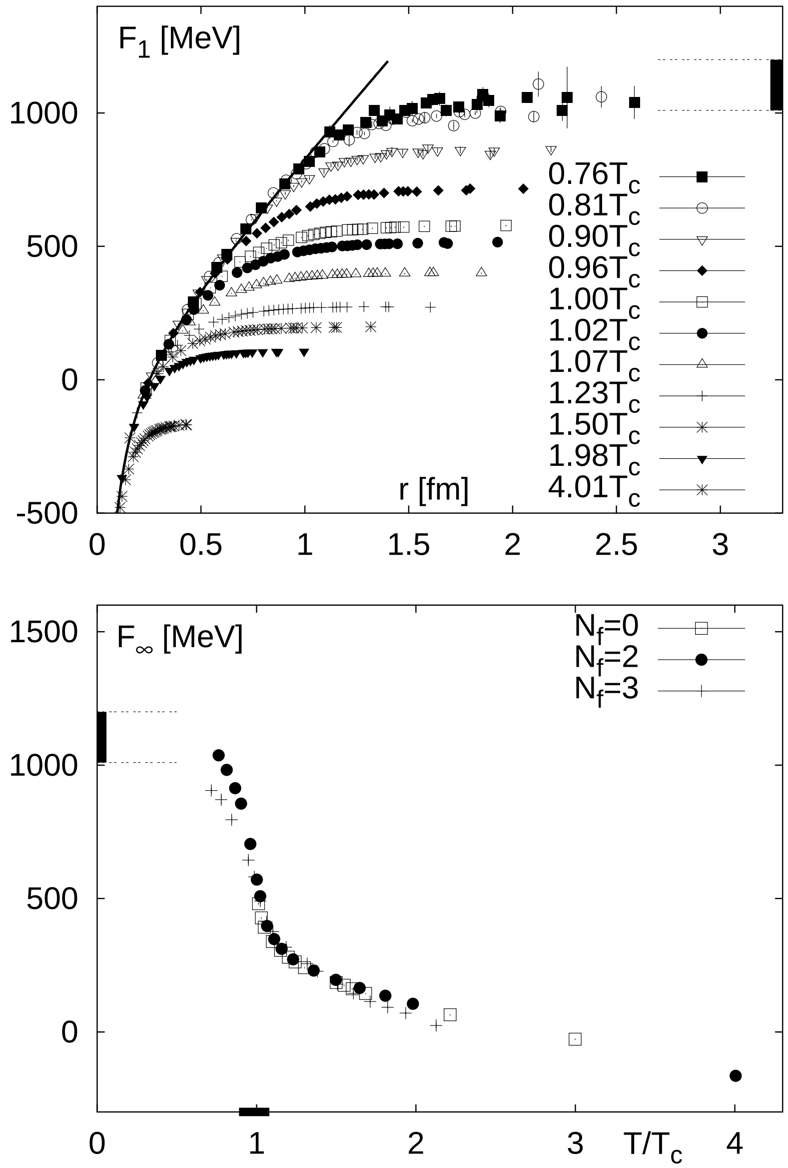
L_{ren} defined by long distance behaviour of $F(r,T)$

$$L_{ren} = \exp\left(-\frac{F(r=\infty,T)}{2T}\right)$$



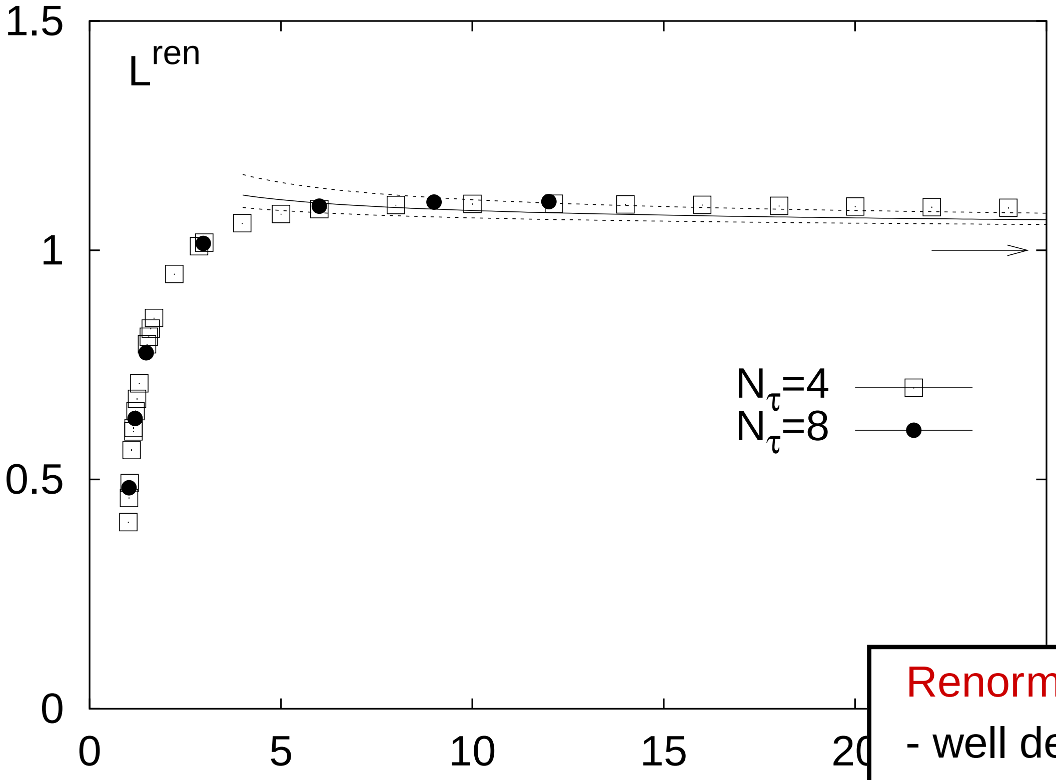






$$L_{ren} = \exp\left(-\frac{F(r=\infty,T)}{2T}\right)$$

Renormalized Polyakov loop



High temperature limit, $L^{ren} = 1$,
reached from above as expected from PT

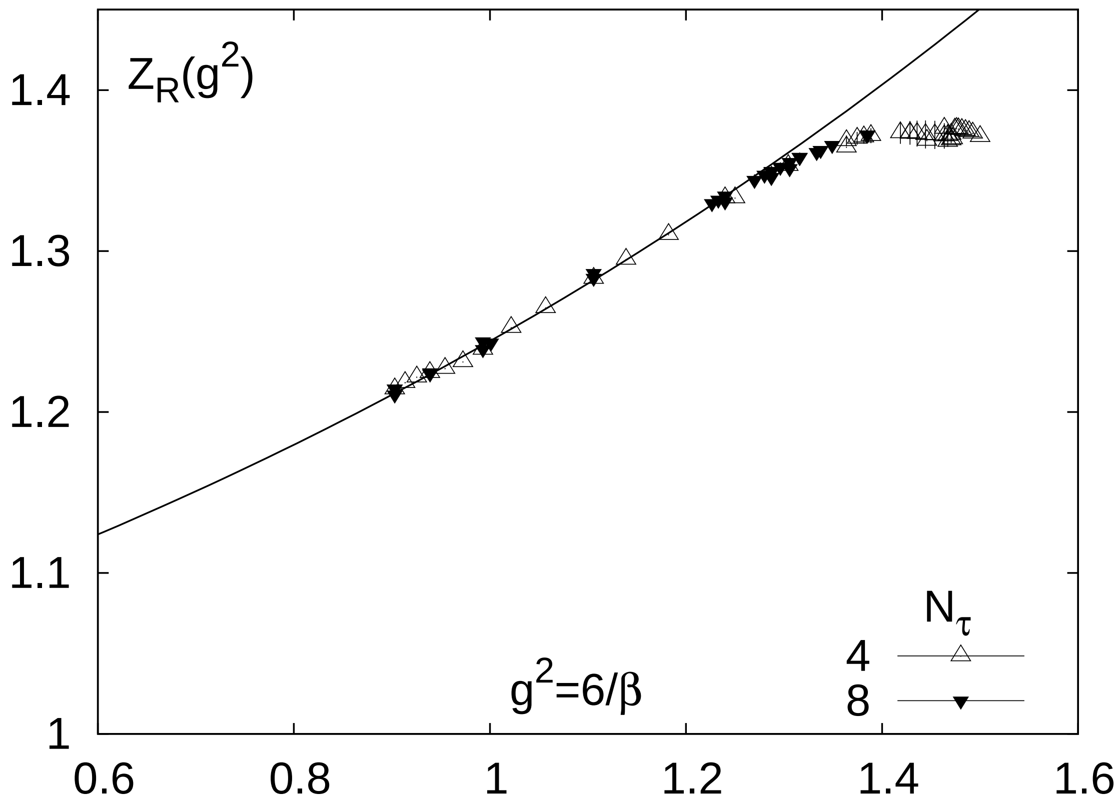
Clearly non-perturbative effects below $5T_c$

Renormalized Polyakov loop
- well defined in quenched and full QCD
- non-zero for finite quark mass
- strong increase near T_c

$$L_{ren} = \exp\left(-\frac{F(r=\infty, T)}{2T}\right)$$

Renormalization constants

Renormalization constants obtained from heavy quark free energies



The renormalization constants depend on the bare coupling, i.e. $Z_R(g^2)$

$$Z_R(g^2) \simeq \exp\left(g^2(N^2-1)/NQ^{(2)} + g^4 Q^{(4)} + o(g^6)\right)$$

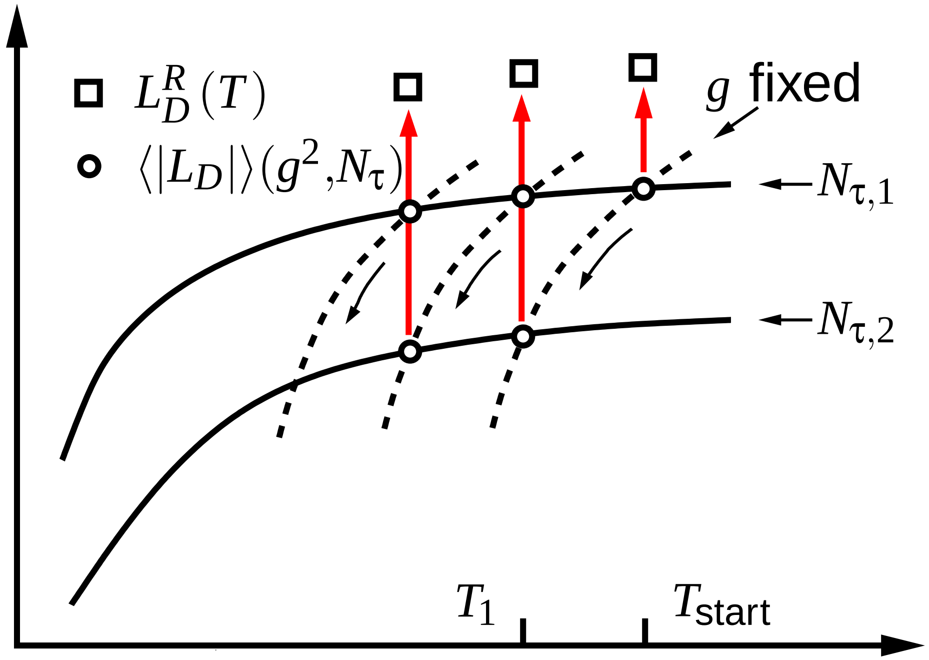
with $Q^{(2)} = 0.0597(13)$ consistent with lattice perturbation theory (Heller + Karsch, 1985)

Instead of renormalizing heavy quark free energies

Use Polyakov loops obtained at different N_τ

Assume no volume dependence ($T > T_c$)

The renormalization constants only depend on coupling, i.e. $Z(g^2)$



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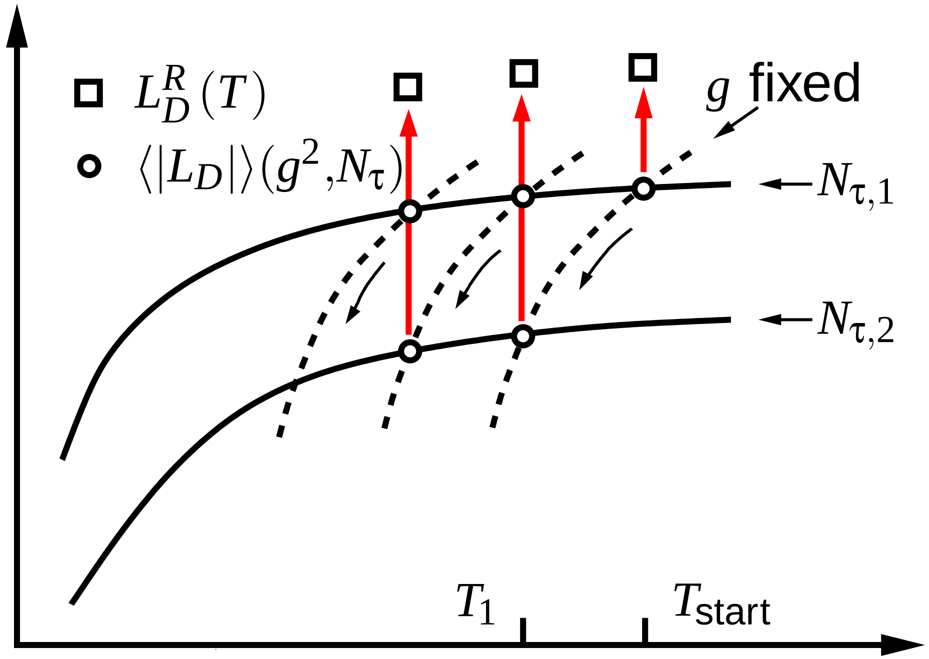

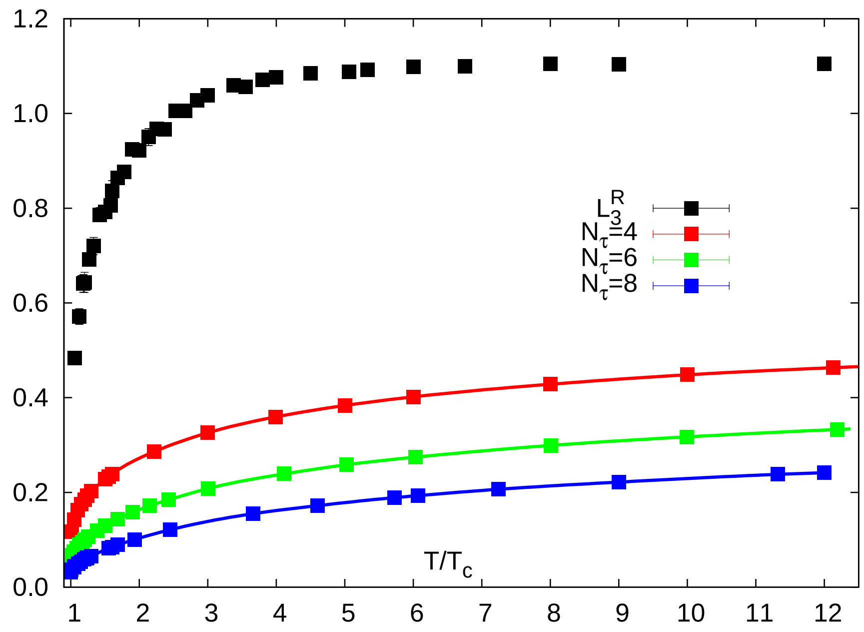

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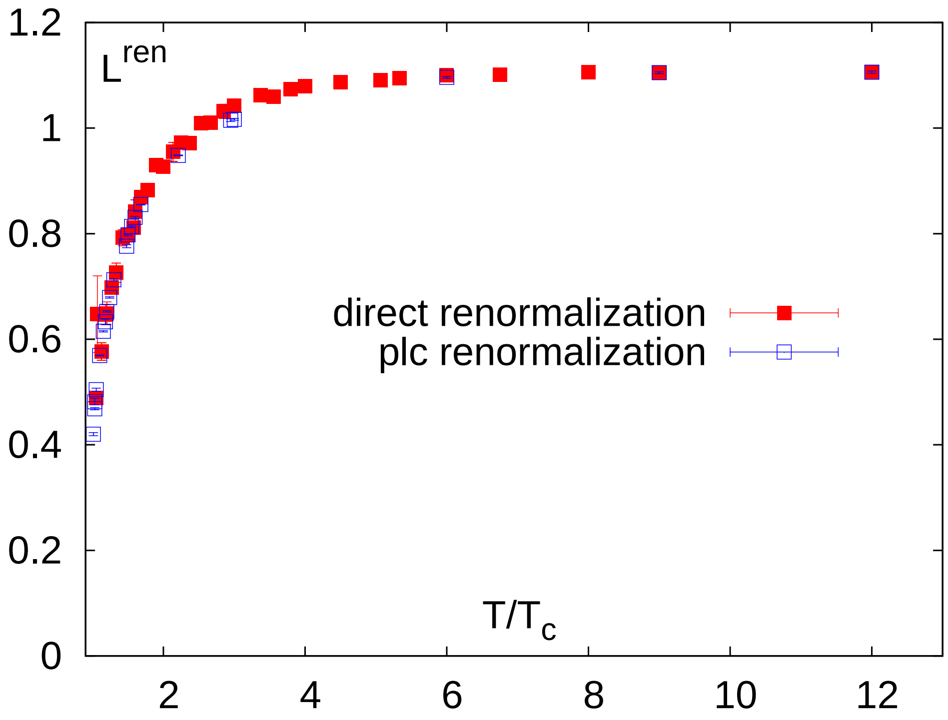
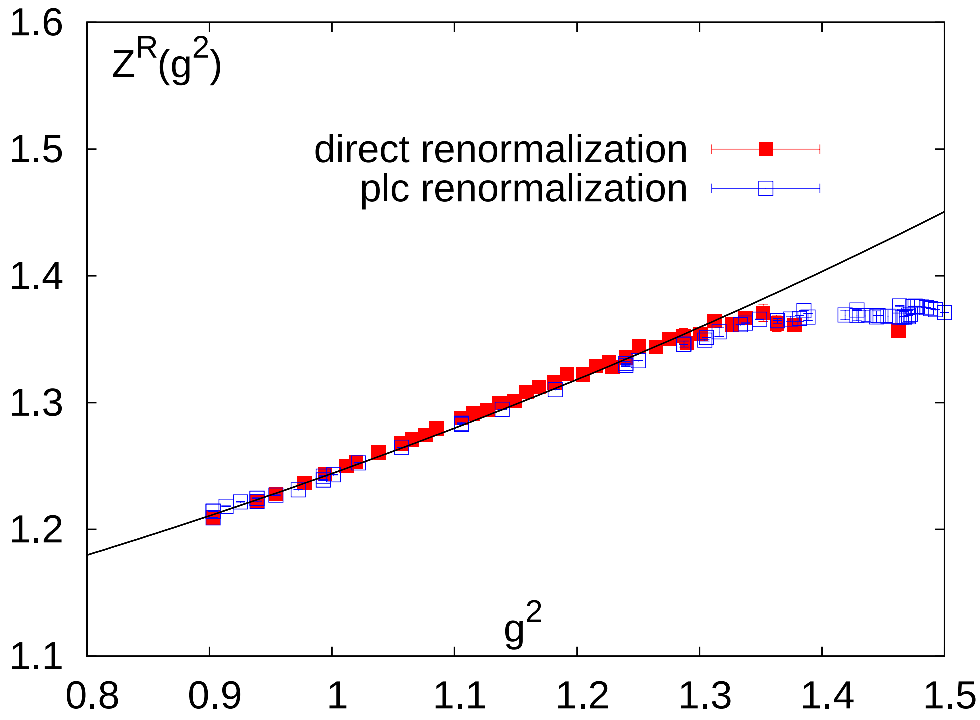




Both renormalization procedures are equivalent

solely based on gauge-invariant quantities use character property of direct product rep: $\chi_{p\otimes q}(g) = \chi_p(g)\chi_q(g)$

Z(3) symmetry: $L_D \to e^{it\phi} L_D$

triality ($Z(3)$ charge): $t \equiv p - q \mod 3$

adjoint link variable $\left[U^{D=8}\right]_{ij} := \frac{1}{2}\mathrm{Tr}\,\left[U^{D=3}\lambda_i (U^{D=3})^\dagger \lambda_j\right]$

D	(p,q)	t	$C_2(r)$	$d_D = C_D/C_F$	$L_D(x)$
3	$(1,0)$	1	$4/3$	1	L_3
6	$(2,0)$	2	$10/3$	$5/2$	$L_3^2 - L_3^*$
8	$(1,1)$	0	3	$9/4$	$\lvert L_3\rvert^2 - 1$
10	$(3,0)$	0	6	$9/2$	$L_3L_6 - L_8$
15	$(2,1)$	1	$16/3$	4	$L_3^*L_6 - L_3$
$15'$	$(4,0)$	1	$28/3$	7	$L_3L_{10} - L_{15}$
24	$(3,1)$	2	$25/3$	$25/4$	$L_3^*L_{10} - L_6$
27	$(2,2)$	0	8	6	$\lvert L_6\rvert^2 - L_8 - 1$

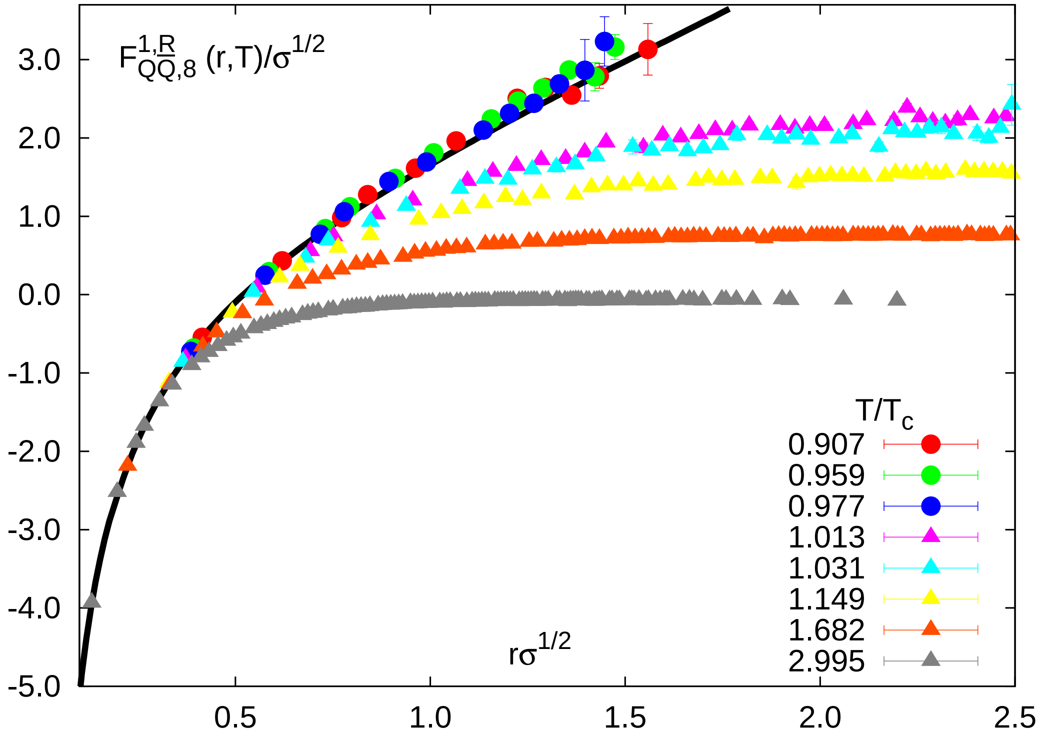
perturbation theory:

$$F_D(r,T) = -C_D \frac{\alpha_s(r)}{r} \quad \text{for} \quad r\Lambda_{\text{QCD}} \ll 1$$

renormalization of free energies:

$$e^{-F_D^1(r,T)/T} = \left(Z_r(g^2)\right)^{2d_D N_\tau} \langle \text{Tr } (L_x^D L_y^{D\dagger})$$

i.e. the renormalization constants are related by Casimir [G.Bali, Phys.Rev.D62 (2000) 114503]

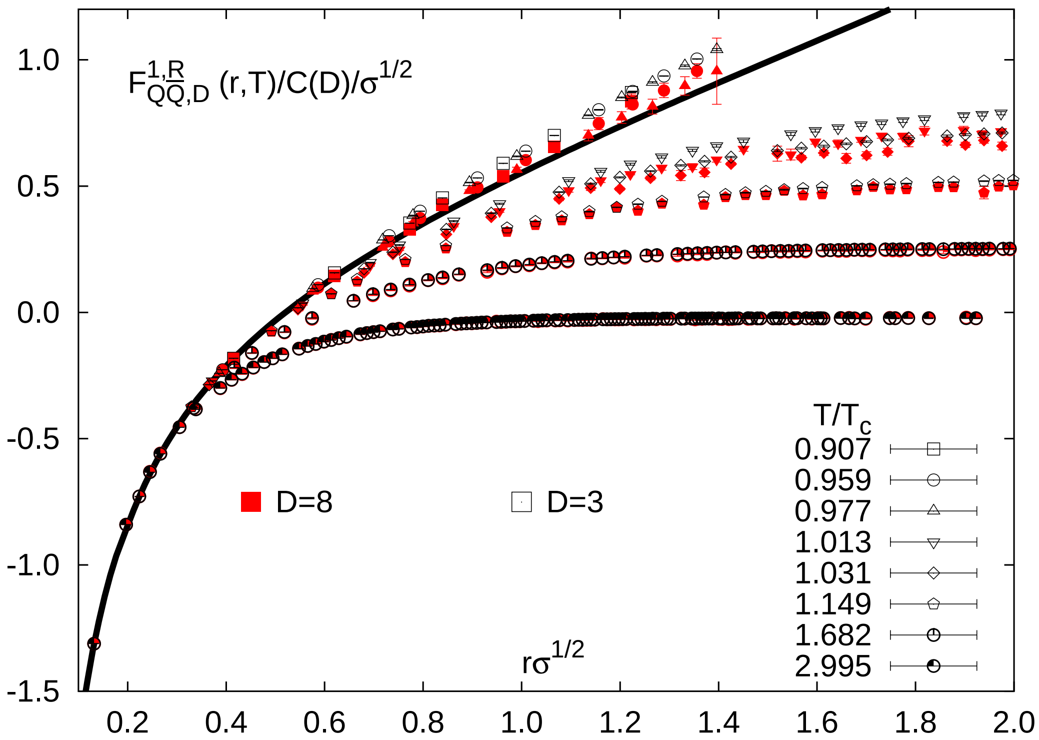


Does Casimir scaling hold beyond leading order?

Singlet free energies of static quarks in representation D=3,8:

$$\frac{F_D^{\text{sing}}(r,T)}{T} = -\ln\left(\langle \tilde{\text{Tr}} L_D(\mathbf{x}) L_D^\dagger(\mathbf{y})\rangle\right)\Big|_{GF}$$

with L_D made up of $U^{D=3}$ and $\left[U^{D=8}\right]_{ij} := \frac{1}{2}\text{Tr}\left[U^{D=3}\lambda_i (U^{D=3})^\dagger \lambda_j\right]$



Does Casimir scaling, $L_3^{1/C_F} \simeq L_D^{1/C_D}$, hold beyond two-loop order?

renormalization of the Polykov loop:

$$\langle L_D^{ren} \rangle = \left(Z_D(g^2) \right)^{N_\tau d_D} \langle L_D^{bare} \rangle,$$

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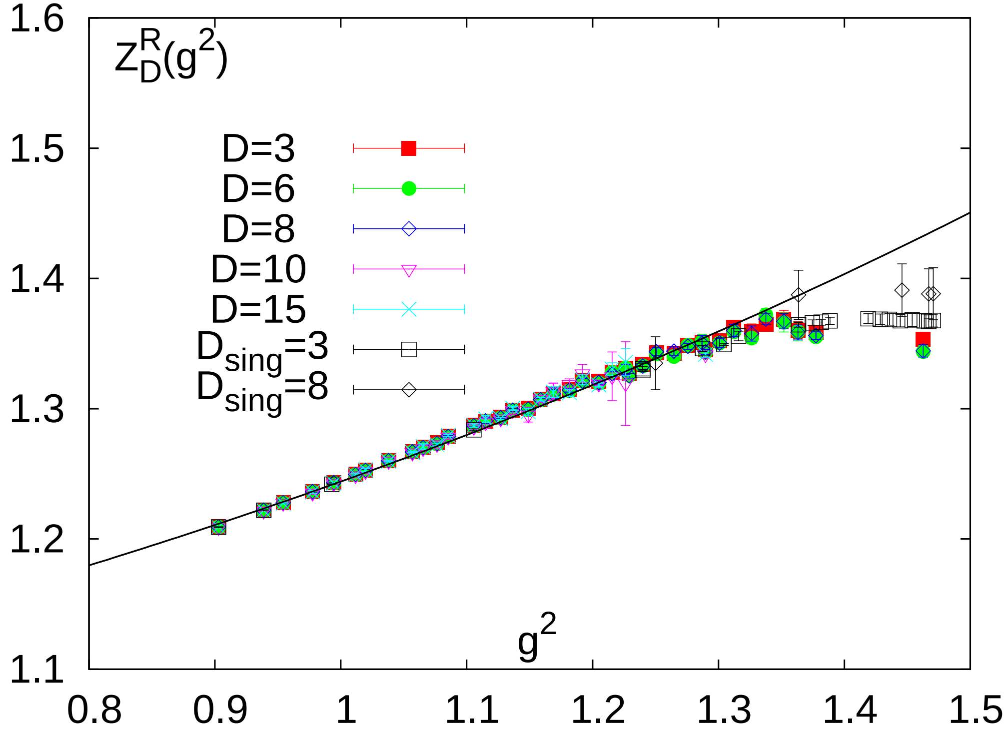

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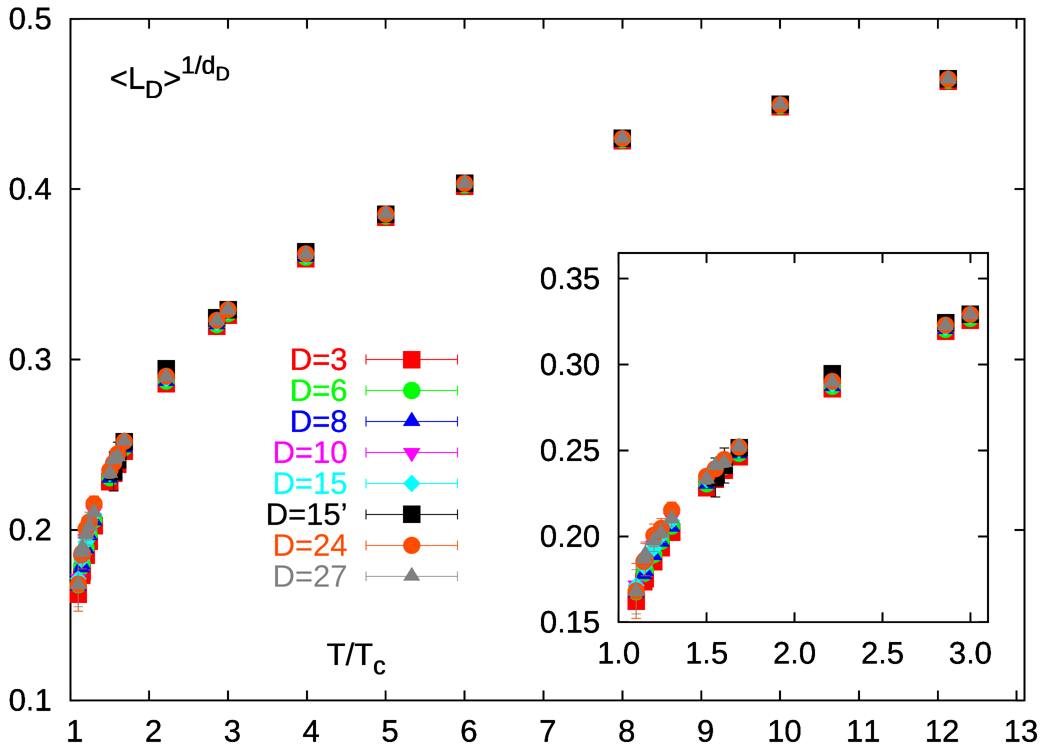
Casimir scaling equivalent for bare or renormalized Polyakov loops



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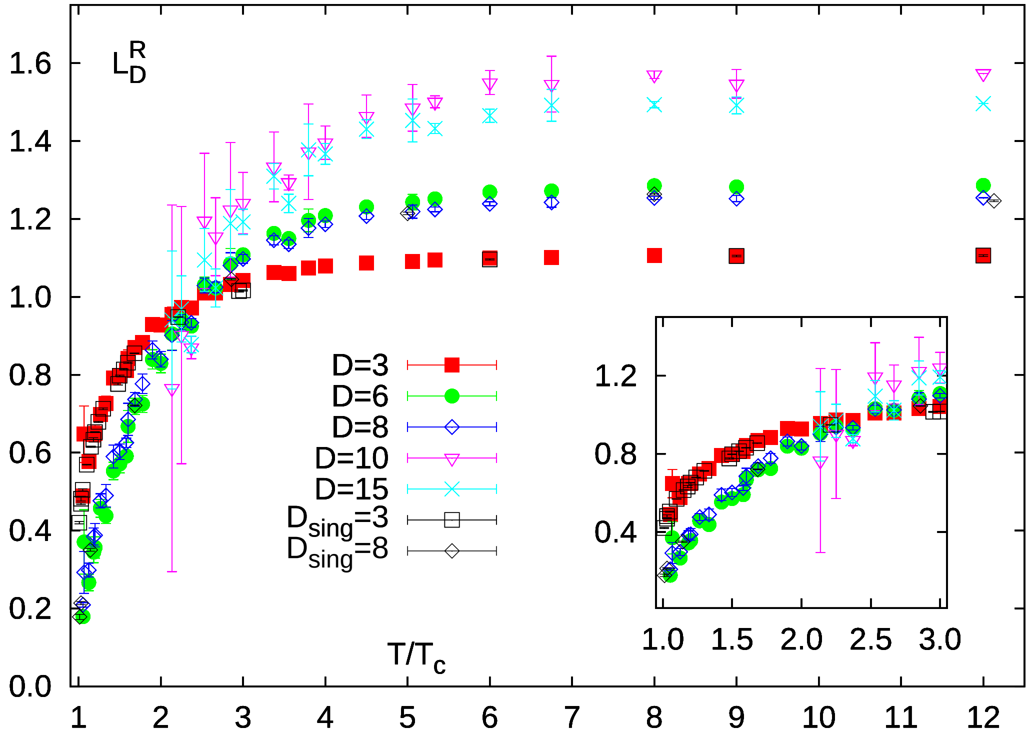
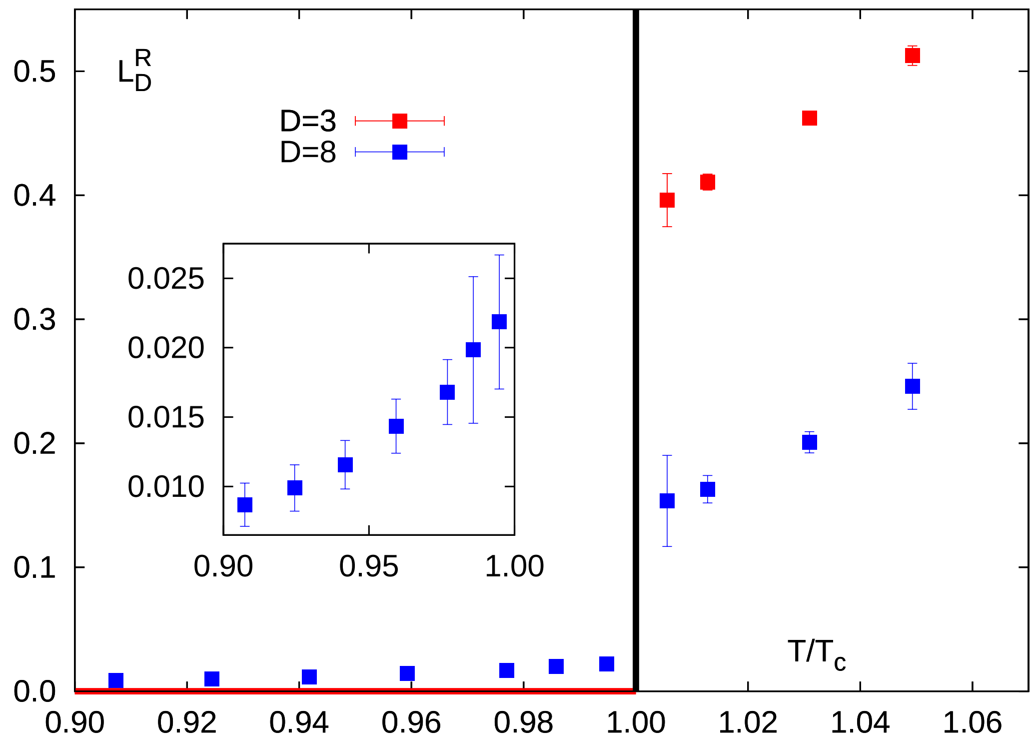


string breaking expected for representations with triality $t = 0$

non-zero for L_8^R even below T_c



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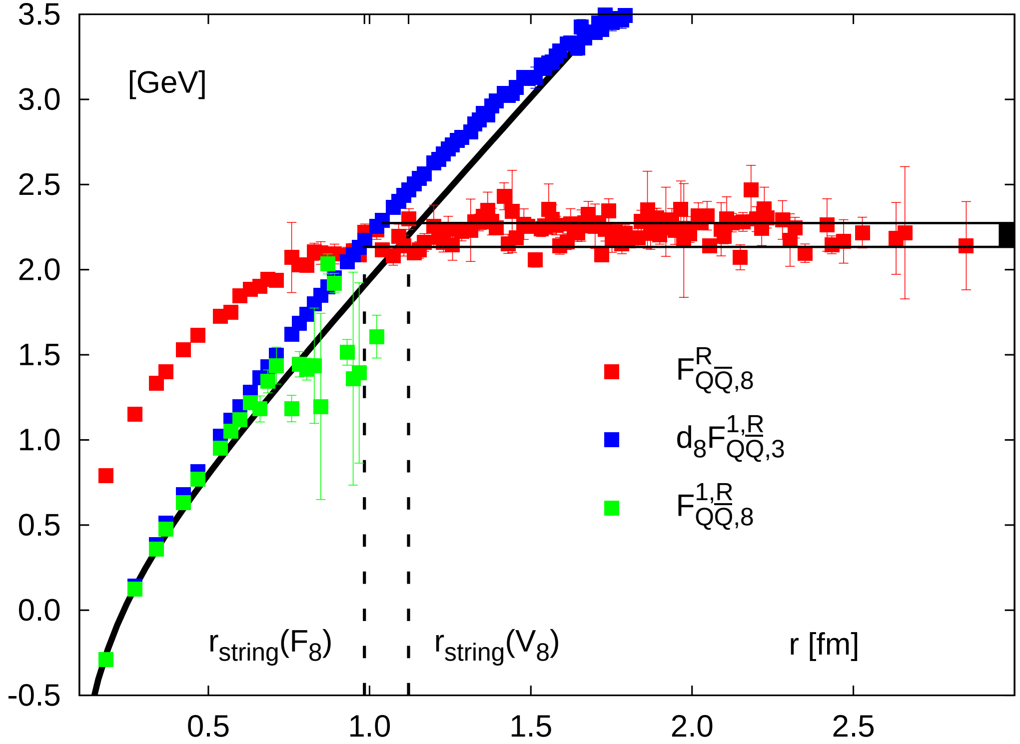

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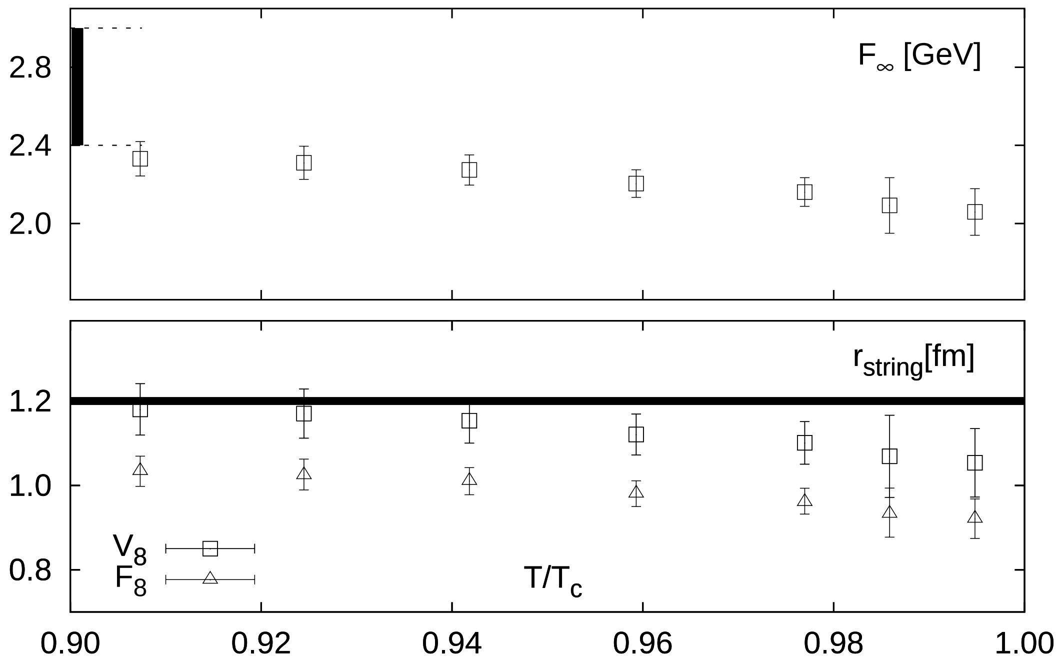
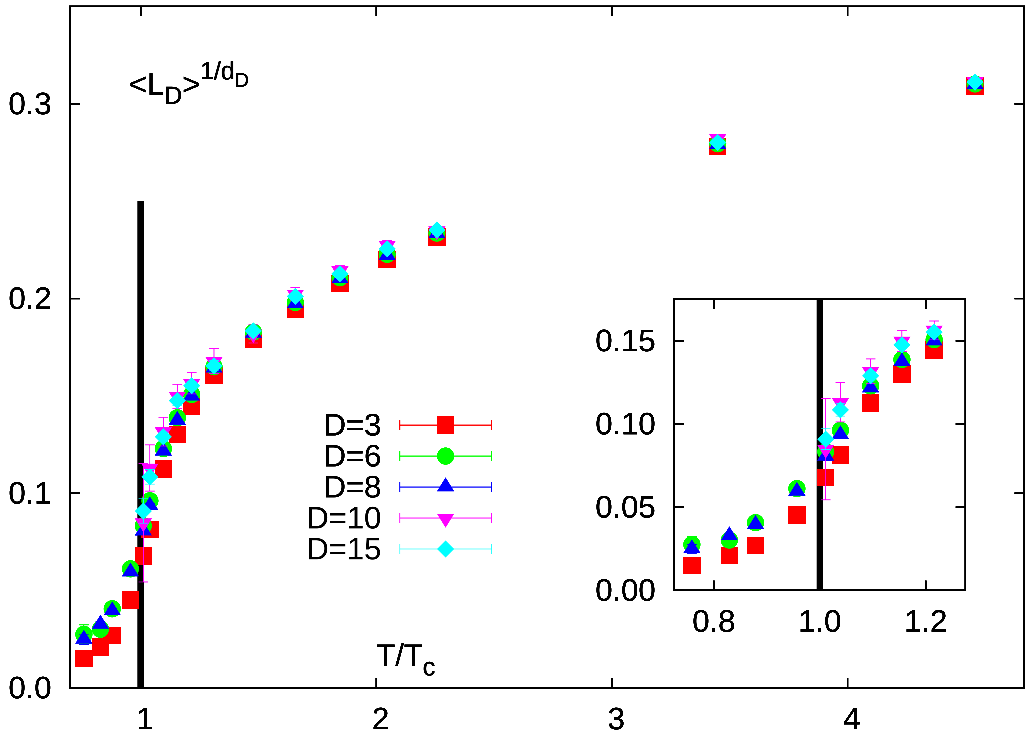
finite values of $F_8(r)$ at large distances

might be related to binding energy of gluelumps



2-flavour QCD:



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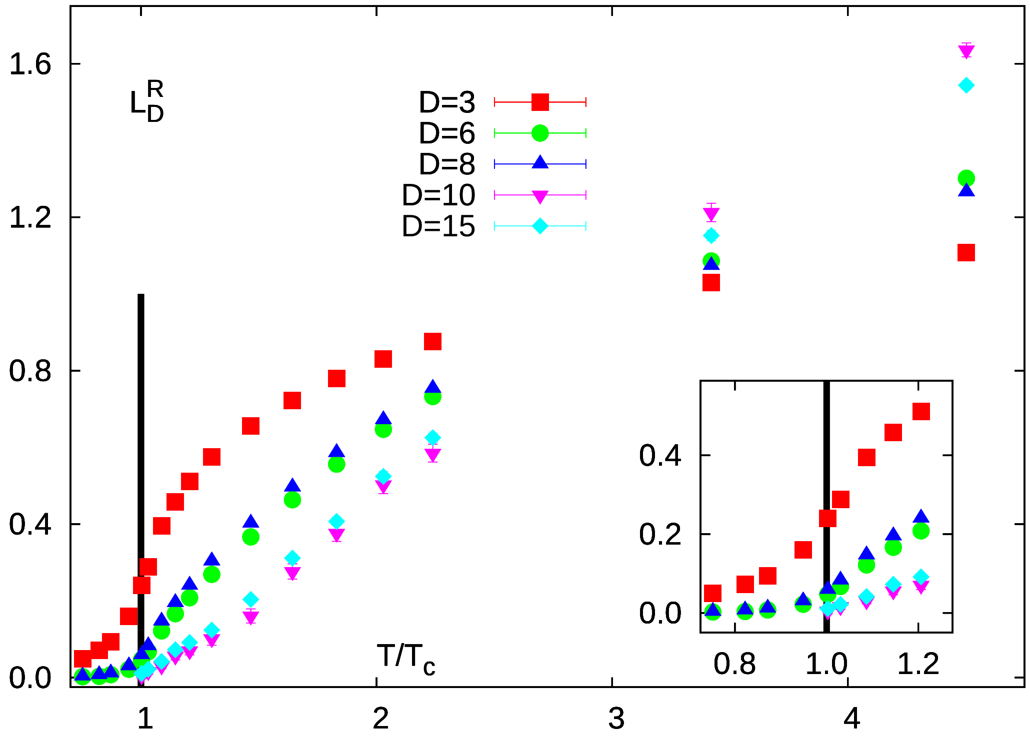

Freedom to set the scale:

$$
\begin{aligned}
V_{T=0}(r) &\longrightarrow V_{T=0}(r)+C \\
L_D^R &\longrightarrow L_D^R \cdot \exp(-C/2T) \\
Z_D^R &\longrightarrow Z_D^R \cdot \exp(-Ca(g^2)/2)
\end{aligned}
$$

Be carefull to extract T_c by slope of L^R

Susceptibility not renormalized in this way

Conclusions

Heavy quark free energies, internal energies and entropy

- Complex r and T dependence
- Running coupling shows remnants of confinenement above T_c
- Entropy contributions play a role at finite T
- Non-perturbative effects in m_D up to high T
- Non-perturbative effects dominated by gluonic sector

Bound states in the quark gluon plasma

- First estimates from potential models
- Higher dissociation temperature using V_1
- (directly produced) J/ψ exist well above T_c
- Diquarks unlikely to exist above T_c

Renormalized Polyakov loop

- Two consistent renormalization procedures
- Renormalization constant depend on the bare coupling
- Applicable for higher representations
- Casimir scaling good approximation at high T